

# STUDY OF SHORT SEED PULSE EFFECTS IN HIGH GAIN HARMONIC GENERATION FREE ELECTRON LASER\*

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## Abstract

To generate the hard x-ray free electron laser (FEL), multi-stage high gain harmonic generation (HG) will employ seed pulse with longitudinal length down to tens of femtoseconds. Thus, strong slippage effects and broad bandwidth effects are introduced. In this paper, we study the short seed pulse effects of HG. A set of multi-frequency equations is derived to appropriately model the interaction between the seed laser and the electrons in the modulator. The short seed pulse effects on the electron beam's energy modulation, the output wavelength tuning, the output pulse length and the output peak power in HG are theoretically and numerically investigated. It shows that the short seed pulse significantly influences HG performances, and the influences should be taken into accounts in the design of seeded FEL scheme.

## INTRODUCTION

Multi-stage HG [1] with "fresh bunch" technique [2] seems a feasible way to generate fully coherent radiations spanning to Angstrom wavelength range. For the sake of the low factor of the frequency up-conversion in single-stage HG, generally 5~6 stages HG are necessary to reach hard x-ray spectral region when starting from the optical wavelength. Meanwhile, in order to obtain high peak current at the entrance of the undulator system, the 1 nC, 10 ps electron bunch from the photo-injector needs to be compressed by a factor of 20~30. Thus in multi-stage HG for generating hard x-ray FEL, the practical pulse length of the initial seed laser may be in tens of fs order.

Generally, the bandwidth of the seed laser is much less than FEL gain bandwidth in current HG. According to the principle of Fourier transform, short pulse lasers have broad bandwidth. Typically, ultra-short pulse lasers are obtained by the optical compression [3], resulting in an even broader bandwidth than theoretical estimate. Due to the seed laser's broad bandwidth, the particle dynamics and the radiation propagation can not be properly pictured by monochromatic FEL equations.

## NUMERICAL APPROACHES

In current state-of-the-art FEL code, the slowly varying envelope approximation is utilized in time and space for radiation propagation, which can't be used to properly describe ultra-short laser pulse in time domain. And since a narrow bandwidth of frequencies around the central frequency is used to model the radiation, some interesting physical process may be lost. Thus, appropriate numerical approaches of the short pulse laser seeded HG is of great importance.

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## Multi-frequency FEL equations

In order to investigate the short seed pulse effects, the monochromatic electromagnetic field in the FEL equation should be replaced by the broad bandwidth radiations. According to mode-locking technique [3], the ultra-short pulse laser field can be treated as a linearly polarized electromagnetic field with multi-frequency modes. We denote each angular frequency by

$$\omega_q = \omega_0 + q\Delta\omega \quad (1)$$

where  $\Delta\omega$  is frequency interval of the adjacent mode and  $q = 0$  is the index of the central mode with an angular frequency of  $\omega_0$ . There are  $2N_f+1$  frequency modes with the initial phase of 0. We denote each slowly varying envelope function by  $E_q$ , and the corresponding wave numbers is  $k_q = \omega_q/c$ . Thus, the field may be written as

$$E_x(z, t) = \frac{1}{2} \sum_{q=-N_f}^{N_f} E_q(z) e^{i(k_q z - \omega_q t)} + c.c. \quad (2)$$

The multi-frequency longitudinal electron dynamics is governed by the equations

$$\frac{d\gamma_j}{dz} = -\frac{K}{4\gamma mc^2} \sum_{q=-N_f}^{N_f} E_q e^{i\frac{\omega_q}{\omega_r} \theta_j} [JJ]_q + c.c., \quad (3)$$

$$\frac{d\theta_j}{dz} = k_w \left(1 - \frac{\gamma^2}{\gamma_0^2}\right) \cong 2k_w \frac{\gamma_j - \gamma_0}{\gamma_0}, \quad (4)$$

in which  $K \equiv eA_w/mc$  is the undulator magnetic parameter of the modulator,  $\lambda_w$  is the period length of the modulator magnet and  $k_w = 2\pi/\lambda_w$ .  $\theta = k_w z + k_r z - \omega_r t + \xi \sin(2k_w z)$  describes the FEL bunching action, where  $k_r$  is the resonant wave number in the modulator,  $\omega_r = ck_r$  and  $\xi = K^2/(4+2K^2)$ .  $[JJ]_q$  is the modified difference of the Bessel functions, given as

$$[JJ]_q = e^{i\pi(\frac{\omega_q}{\omega_r}-1)} \left( J_0 \left[ \frac{\xi \omega_q}{\omega_r} \right] - J_1 \left[ \frac{\xi \omega_q}{\omega_r} \right] \right). \quad (5)$$

In further, we introduce the FEL universal scaling [4] and the accurate propagation of the dimensionless multi-frequency electromagnetic envelope [5]. Thus, the multi-frequency FEL dynamics in the modulator is expressed as

$$\frac{d\theta_j}{dz} = p_j, \quad (6)$$

$$\frac{dp_j}{dz} = - \sum_{q=-N_f}^{N_f} A_q e^{i \frac{\omega_q}{\omega_r} \theta_j} [JJ]_q + c.c., \quad (7)$$

$$\frac{dA_q}{dz} = \frac{1}{2N_f + 1} [JJ]_q \langle e^{-i\theta_j} \rangle, \quad (8)$$

After the modulator, when the energy-modulated electron beam enters the dispersive section, the dispersive section gives rotation on the longitudinal phase space and change the energy modulation to density modulation.

### Development of new time-dependent FEL code

Usually, HGHG modulator works at small-gain region. Therefore, the dimensionless electromagnetic envelope can be approximated as a constant along the modulator for quick computation. Moreover, since strong slippage effects involved and the frequency up-conversion, the bandwidth of the radiator radiation is usually 10% less than that of the seed laser, then the FEL dynamics in the radiator may be determined by the conventional universal FEL equations [4].

The initial distribution of short pulse seed laser with the peak field amplitude  $A$  can be given as

$$\frac{A_q}{A} = f(q) \quad (9)$$

where  $\sum f = 1$  is forced to be satisfied. Thus, the temporal characters of short pulse laser is characterized by  $\Delta\omega/\omega_0$ ,  $N_f$  and  $f(q)$  together. The pulse length and the Fourier-transform-limited bandwidth of short pulse laser may be solved by numerical calculations.

A one-dimensional, time-dependent code on the basis of the multi-frequency FEL equations derived above is built to numerically study HGHG seeded by short pulse laser. The electron bunch is represented by uniform slices in time domain and the seed laser pulse is represented by uniformly separated modes in spectral domain. In order to involve the slippage effect, a  $-2\pi$  offset is performed for each electron phase after traversing an undulator period. In order to check the validity of the numerical simulation approaches mentioned above, we studied the output pulse shape dependence on the seed laser. The results match perfectly the output pulse shape dependence on the seed laser pulse. Moreover, HGHG and SASE contribution are clearly recognized in the simulation.

## EFFECTS ON SDUV FEL

Shanghai deep ultraviolet (SDUV) [6] FEL is a 262nm HGHG test facility. The first radiation beam is expected

to be observed in August, 2009. The main parameters of the scheme are listed in Table 1.

Table 1: Parameters of harmonic operation of SDUV FEL

Parameters	Value
Seed laser wavelength	786 nm
Electron beam energy	160 MeV
Peak current	300 A
Normalized emittance	6 mm-mrad
Local energy spread	$1 \times 10^{-4}$
Modulator period length	50 mm
Modulator length	0.80 m
Modulator gap	alterable
Strength of the dispersive section	1.00
Radiator period length	25 mm
Radiator gap	10 mm
Radiator resonant wavelength	262 nm

### The electron beam's energy modulation

According to the resonant relationship, after traversing one undulator period, the radiation slips over the electron beam forward by one radiation wavelength. Generally the seed pulse length is much larger than the slippage length in the modulator and all the peak power of the seed laser effectively contribute to the electron beam's modulation. However, if the seed laser pulse is less than the slippage length in the modulator, the seed laser power seen by the electron beam is reduced. Thus to obtain an equivalent spatial bunching, the seed laser power should be properly enhanced as the seed pulse reducing.

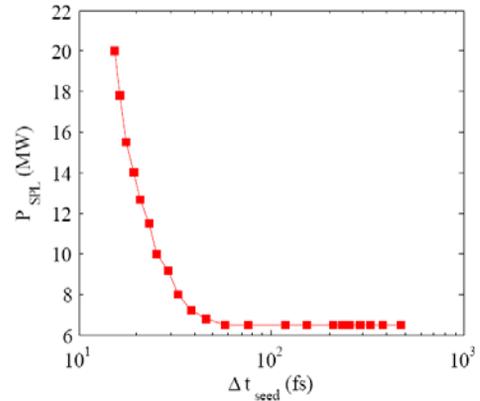


Figure 1: The dependence of the required peak power of SDUV FEL on the seed pulse length.

The pulse length of 786 nm seed laser is 10 ps in the normal operation of SDUV FEL, which is long enough so that short seed pulse effects can be neglected. According to the simulation, with the seed pulse length decreasing and the dispersive strength fixed, the proper enhancement of the peak power of the seed laser is given in Fig. 1.

### The output wavelength tuning

Output wavelength tuning is a key aspect in HGHG FEL [7]. As in any seeded scheme, the output wavelength is determined by the seed laser in HGHG FEL. In most conventional HGHG FEL, the laser radiation is split into two beams. One beam is utilized to drive photocathode RF gun and the other is used as the seed laser for HGHG FEL. Under such circumstances, the seed pulse length is usually several ps and the output wavelength is absolutely controlled by the central wavelength of the seed laser. However, ultra-short seed pulse offers bandwidth much broader than FEL gain bandwidth. When we adjust the electron beam energy or the radiator gap, the resonant wavelength in the modulator is adjusted. Then within the bandwidth range of the ultra-short seed laser, the electron beam can be bunched on the flexible resonant wavelength. Thus the output wavelength tuning is expected.

In SDUV FEL, a fixed gap radiator and an alterable gap modulator are employed. Thus, if we adjust the electron beam energy to satisfy the radiator resonant relation and adjust the modulator gap to satisfy the modulator resonant condition, with the central wavelength of the ultra-short seed laser fixed, the output wavelength tuning is expected. In the simulation with the optimal parameters for each wavelength, the output wavelength tuning is investigated. The simulated dependence of HGHG tuning range on the seed pulse length is shown in Fig. 2. The tuning range is basically in inverse proportion to the pulse length of the seed laser. More precisely, the wavelength tuning range is approximately the bandwidth of the seed laser.

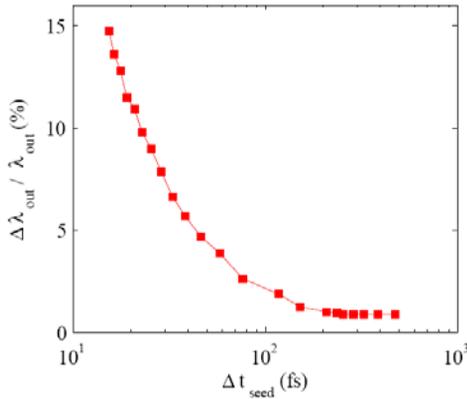


Figure 2: The dependence of the wavelength tuning of SDUV FEL on the seed pulse length.

### The output pulse length

In the normal status of current HGHG, the seed pulse length is much longer than the slippage length in the modulator, thus the slippage effect can be neglected and the output pulse is expected to maintain the temporal properties of the seed pulse. However, HGHG seeded by the short pulse laser operates in the regime where slippage effect is crucial. After the modulator, the ultra-short seed laser only modulates the electrons which it slips over. Then the electron beam enters the radiator with the bunching pulse length basically equal to the slippage

length in the modulator. Once the electron bunch enters the radiator, rapid coherent harmonic is produced. The output pulse length in the radiator has no dependence on the pulse length of the seed laser.

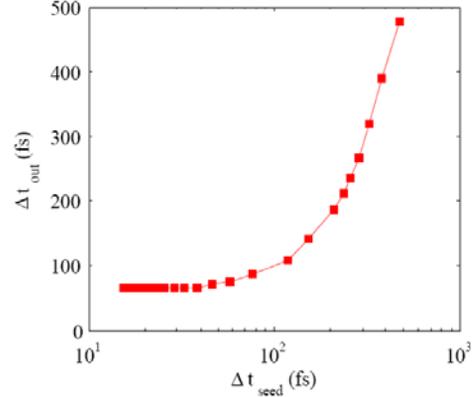


Figure 3: The dependence of the output pulse length of SDUV FEL on the seed pulse length.

The output pulse length of SDUV FEL is depicted in Fig. 3. When a long seed pulse is used, the output pulse length is approximately the pulse length of the seed laser. When the seed pulse length is less than the slippage length in the modulator, the output pulse length would be a constant value about 65 fs. In SDUV FEL case, the slippage length in the modulator is 41 fs. The difference is contributed to the complex evolution of the radiation pulse length in the radiator [8].

### The output peak power

Steady-state FEL theory predicts that, after exponential growth, the radiation power saturates at a value that scales as  $I^{4/3}$  [4], where  $I$  is the peak current of the electron beam. However, in the radiator of HGHG seeded by the ultra-short pulse, the pulse length of coherent radiation is much shorter than the slippage length in the radiator. The steady-state region where the radiation and the electron beam interact with each other can be easily destroyed. Thus the radiation grows exponentially with gain length less than predicted by the steady-state theory. When the radiation is intense enough, it enters superradiant regime where the leading edge of the radiation pulse propagates over fresh electrons which had no FEL interaction except for the negligible SASE contribution.

The simulated dependence of the output peak power of SDUV FEL on the seed pulse length is given in Fig. 4. 262 nm HGHG of SDUV FEL saturates at 5 m in the radiator with a saturation power of 200 MW when seeded by an ultra-long laser, which is labelled by point A in Fig. 4. As the seed pulse length decreases, the gain reduction of the exponential growth and the quadratic growth of the superradiant come out and compete with each other. Thus, with the seed pulse length decreasing, there exists a temporary and small increase of the output peak power, which can be explained by the quadratic growth of the superradiant that plays a more important role in power growth than FEL saturation in the steady-state theory. The

exponential growth degradation becomes dominant from point B, and the output peak power starts to decrease, ultimately, when the seed pulse length is short enough, it almost has no impact on the output peak power again, as seen in point C of Fig. 4.

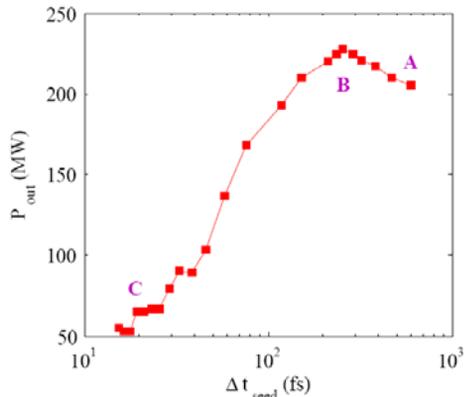


Figure 4: The dependence of the output peak power of SDUV FEL on the seed pulse length.

To clearly illustrate what we have discussed above, we plot the output peak power growth in the radiator in Fig. 5, where A, B and C represent the three points labelled in Fig. 4. It is perfectly consistent with the above statement. Further, one may observe that the quadratic growth of the superradiant still works until the growth of SASE stops the evolution of superradiant by ultimately spoiling the longitudinal phase space of the electrons.

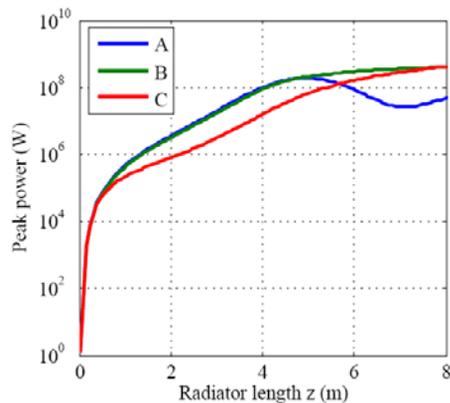


Figure 5: The output peak power growth of SDUV FEL in the radiator

## THE CONCLUDING REMARKS

In this paper, based on multi-frequency FEL equations, the short seed pulse effects of HGHG is theoretically and numerically investigated. Since the continuous progress in laser technology has made few-cycle and terawatt scale laser pulses available [3]. Because of the enhancement of HGHG coherence and wavelength tuning, HGHG seeded by few-cycle laser may be of interest. Study shows that a 262 nm HGHG seeded by a 2-cycle, 786 nm laser may have a complete wavelength tuning ability over a very

large range and a more than 10 times enhancement of the longitudinal coherence [8].

In order to exert the seeding ability of the seed laser in HGHG, the seed pulse length should be much larger than the slippage length in the modulator. In most of current single stage HGHG operating at UV and DUV spectral region, the driven-laser of radio frequency gun serves as a source of the seed laser. The seed pulse length is usually in the order of picoseconds, and the requirement can be satisfied naturally. However, in multi-stage HGHG, the requirement may not be satisfied in the first stage HGHG because of the limit of the “fresh bunch” technique.

The output pulse length of HGHG is expected to be comparable to that of the seed laser. However, when the seed pulse length is ultra-short, the output pulse length is much larger than that of the seed laser for the sake of strong slippage effect. Thus, if short pulse laser is used as the initial seed of multi-stage HGHG, the “fresh bunch” chicane must be re-optimized to delay the electron bunch appropriately in the following stages. Moreover, the total scheme should be carefully treated.

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