THE EFFECT OF SHOT-NOISE ON THE START-UP OF THE FUNDAMENTAL AND HARMONICS IN FREE-ELECTRON LASERS *

H.P. Freund[1], L. Giannessi[2] and W.H. Miner[1], Jr.
[1]Science Applications International Corp., McLean, VA 22102, U.S.A., [2]ENEA CR Frascati, Via E. Fermi45, 00044 Frascati, Italy.

Abstract

Radiation start-up is important in the simulation of virtually all free-electron laser (FEL) configurations. We discuss two different shot noise models that are implemented in both 1-D wiggler-averaged (PERSEO) and non-wiggler-averaged (MEDUSA1D) simulation codes, and a 3-D, non-wiggler-averaged (MEDUSA) formulation. Three-dimensional correction factors have been included in the MEDUSA1D and PERSEO, and all three codes are in remarkable agreement on the fundamental and harmonics.

INTRODUCTION

Radiation start-up in free-electron lasers (FELs) is important in simulating all configurations including oscillators [1,2] and amplifiers [3-6] in both seeded-MOPA (Master Oscillator Power Amplifier) and SASE (Self-Amplified Spontaneous Emission) modes. Both oscillators and SASE amplifiers start up from shot-noise, which is due to the random fluctuations in the electron phase distribution. The seed in a MOPA usually overwhelms the shot-noise; however, the noise must be treated correctly to model harmonics.

The inclusion of shot noise in Particle-in-Cell (PiC) simulations has been described by McNeil *et al*. [6]; however, because of the computational requirements imposed by PiC simulations, most FEL simulation codes average Maxwell's equations over the fast wave time-scale, and this is the type of formulation we will consider. Within the context of this fast-time-scale averaging, there are two models corresponding to either wiggler-averaged- or non-wiggler-averaged orbit dynamics. Wiggler-averaged-orbit codes include GINGER [5], GENESIS [6], and PERSEO [7] among others. The non-wiggler-averaged-orbit approximation has been used in both 1-D [8] and 3-D [9,10] in the MEDUSA1D and MEDUSA codes respectively.

The simplest way to include shot-noise is to introduce a random component to the initial phases, ψ_0, of the macro-particles such that $|\langle\exp(i\psi_0)\rangle| = 1/\sqrt{N_e}$, where N_e is the number of correlated electrons. The rationale for choosing N_e differs depending upon whether the simulation is performed in the steady-state or with slippage included. It is assumed in steady-state that each "beamlet" interacts identically with the radiation so that only one such beamlet is needed. The number of electrons per slice, n_e, is given by $n_e = I_b\lambda/e\upsilon_b$, where I_b is the beam current, λ is the wavelength, e is the electronic charge, and υ_b is the bulk axial velocity. The total number of interacting electrons, N_e, includes contributions from multiple slices and is given by [11]

$$N_e \cong 4.3\,\frac{L_g}{\lambda_\omega}\,n_e = 4.3\,\frac{L_g}{\lambda_\omega}\,\frac{I_b\lambda}{e\upsilon_b}\ , \tag{1}$$

where λ_w denotes the wiggler period, and L_g is the field exponentiation length. In time-dependent simulations, we choose an integration window that is many "slices" in length, although the number of "slices" used in simulation is usually less than this total. We find by trial and error that it is usually necessary to have at least one slice approximately every three wavelengths. Thus, each slice included in the simulation is separated by some distance $\Delta\lambda_s$, and the number of interacting electrons is

$$N_e = \frac{\Delta\lambda_s}{\lambda}\,n_e = \frac{I_b\Delta\lambda_s}{e\upsilon_b}\ . \tag{2}$$

We present three new contributions to the literature dealing with shot noise in FELs. First, we describe and directly compare two algorithms for describing shot noise in FELs, which differ from those discussed in the literature. Second, we compare the shot noise algorithms in two very different simulation codes one of which uses wiggler-averaged orbit dynamics (PERSEO) and the other (MEDUSA) does not. Third, we self-consistently include harmonics.

THE NON-WIGGLER-AVERAGED FORMULATION: MEDUSA

The 1-D (MEDUSA1D) and 3-D (MEDUSA) non-wiggler-averaged-orbit formulations treat the electron dynamics in the same way and the shot-noise model is identical in both simulation codes. In the absence of shot-noise, particles are loaded into the 2-D phase space of the 1-D formulation or the 6-D phase space of the 3-D formulation using Gaussian quadrature in each of the degrees of freedom. This is deterministic and we assume that the additional degrees of freedom describing the beam (γ_0 in one dimension, and x_0, y_0, p_{x0}, p_{y0}, and γ_0 in three dimensions) do not vary with the initial phase ψ_0. As a result, each choice of ψ_0 is associated with an identical distribution of particles in the additional degrees of freedom. We choose an initial loading in ψ_0 such that

*Work supported in part by the Office of Naval Research in the USA and in part by the EU Commission in the sixth framework programme, Contract No. 011935-EUROFEL.
[1]henry.p.freund@saic.com

$$\frac{1}{2\pi}\int_0^{2\pi} d\psi_0 \exp\left(i\psi_0\right) = 0\,, \qquad (3)$$

for one choice of $(x_0,y_0,p_{x0},p_{y0},\gamma_0)$ so that it will also vanish for all such choices, and we will have ensured a "quiet-start". Particles are loaded in ψ_0 using an N^{th} order Gaussian quadrature. It is found empirically that Gaussian quadrature ensures that the average $\langle\exp(i\psi_0)\rangle$ vanishes to within round off error with about 8 particles in phase. If it is desired to include up to the h^{th} harmonic, then more particles are needed and a rule of thumb is that h-times as many particles are needed as for the fundamental. Since $\langle\exp(ih\psi_0)\rangle$ also vanishes the "quiet-start" is implicitly obtained for the fundamental and all harmonics.

We start with the unperturbed choice for the initial phase $\{\psi_{0j}\}$ to include shot-noise and introduce a perturbation to obtain a new distribution $\{\psi'_{0j}\}$. The procedure is similar, but not identical, to that used by Fawley [4] and we write

$$\psi'_{0j} = \psi_{0j} + \sum_{h=1}^{h_{max}} \delta\psi_h \sin\left[h\left(\psi_{0j} - \phi\right)\right], \qquad (4)$$

where $\delta\psi_h$ ($\ll 1$) is chosen to describe the Poisson statistics, and ϕ is chosen randomly over the interval $[0,2\pi]$. Note that $\langle\sin(h\psi_0)\rangle = \langle\cos(h\psi_0)\rangle = 0$ and that $\langle\sin^2(h\psi_0)\rangle = \langle\cos^2(h\psi_0)\rangle = ½$, hence

$$\left\langle \sin\left(h\psi'_0\right)\right\rangle \cong -\frac{1}{2}h\delta\psi_h \sin\left(h\phi\right), \qquad (5)$$

$$\left\langle \cos\left(h\psi'_0\right)\right\rangle \cong \frac{1}{2}h\delta\psi_h \cos\left(h\phi\right), \qquad (6)$$

to lowest order in $\delta\psi_h$. If $h\delta\psi_h = \delta\psi_1$ and $\delta\psi_1 = 2/\sqrt{N_e}$, then

$$\left|\left\langle \exp\left(ih\psi'_0\right)\right\rangle\right| \cong \frac{1}{2}\delta\psi_1 = \frac{1}{\sqrt{N_e}}\,, \qquad (7)$$

and we recover the correct Poisson statistics.

THE WIGGLER-AVERAGED FORMULATION: PERSEO

PERSEO [11] is the name of a library of functions devoted to the simulation of FEL dynamics in the *Mathcad* environment. Functions for the generation of phase space variables, for the solution of the pendulum-like equation and for manipulating the phase space in a number of devices are available. These functions are combined in order to model specific FEL configurations. The particle loading algorithm uniformly distributes n particles in the interval $(0,2\pi)$ and has a null harmonic content at the frequency $2\pi c/\lambda_0$. The introduction of the desired harmonic content in the beam distribution is obtained by shifting the particle positions from the equally spaced configuration. A random shift doesn't ensure the correct statistics at the higher harmonics. By applying the following rule

$$\psi_k = \psi_m + \frac{2\pi}{n}\left(1-\delta\right)\left(k - \frac{n-1}{2}\right), \qquad (8)$$

where $k = 0, \ldots, n-1$. It can be shown that the Fourier coefficient of the distribution at the h^{th} harmonic has phase ψ_m and amplitude δ independent of the harmonic order. The normalized Fourier coefficient is

$$\begin{aligned} b_h &= \frac{1}{n}\int d\psi \exp\left(ih\psi\right)\sum_k \delta\left(\psi - \psi_k\right) \\ &= \frac{1}{n}\exp\left(ih\psi_m\right)\frac{\sin\left[2\pi h\left(1-\delta\right)\right]}{\sin\left[\frac{2\pi h}{n}\left(1-\delta\right)\right]}\,. \end{aligned} \qquad (9)$$

In the limit of large n and $\delta \to 0$ we have $b_k \cong b = \delta\exp(ih\psi_k)$.

A specific function in PERSEO library accepts as input the complex coefficient b and returns a set of particles distributed in phase. At startup the evolution of the different harmonic components in an FEL are not coupled. For this reason the correlation between the bunching factors at different harmonics do not provide substantial physical effects. In comparison with the method implemented in MEDUSA and other methods existing in literature, the bunching coefficient is not affected by any statistical uncertainty and by itself does not reproduce any Poisson statistical behavior expected from shot noise. The user from the Mathcad environment has the ability to select amplitude and phase of the fundamental harmonic bunching. In order to reproduce noise factors resembling the shot noise, the amplitude is generated from the Poisson number generator within Mathcad. According to the sampling of the longitudinal current the noise factor amplitude and rms fluctuations are proportional to $[I(\zeta)\Delta\zeta/ce]^{1/2}$ i.e. to the root of the number of "real" electrons represented by the specific time slice at position ζ. The phase is randomly selected in the interval $(0,2\pi)$. This permits the reproduction of different statistical behavior related to the current variations.

NUMERICAL ANALYSIS

We now describe the simulation results obtained using both PERSEO and MEDUSA1D and MEDUSA. In 3-D, we compare PERSEO and MEDUSA1D with MEDUSA where we have included scale factors in the 1-D formulations intended to describe the filling factor, effective increase in the energy spread due to the emittance, and diffraction. The fundamental parameters are as follows. The electron beam energy and peak current are 200 MeV and 110 A respectively, with a radius of 95.3 microns, an energy spread of 0.01% and a normalized emittance of 1 mm-mrad. The wiggler period is 2.8 cm with a peak on-axis amplitude of 7.46 kG. In 3-D, we assume that the wiggler has two-plane focusing. The fundamental resonance is at about 265 nm.

In comparing PERSEO and MEDUSA1D in 1-D, we do not include any *ad hoc* 3-D corrections for the emittance, filling factor, or diffraction. Hence, the energy spread is assumed to be 0.01%, and the filling factor is unity.

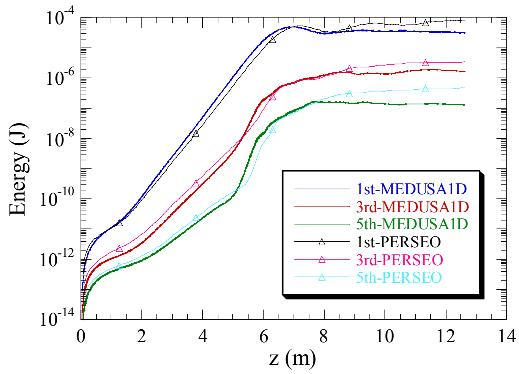

Figure 1: Comparison of time-dependent simulation results for PERSEO and MEDUSA1D in the SASE regime.

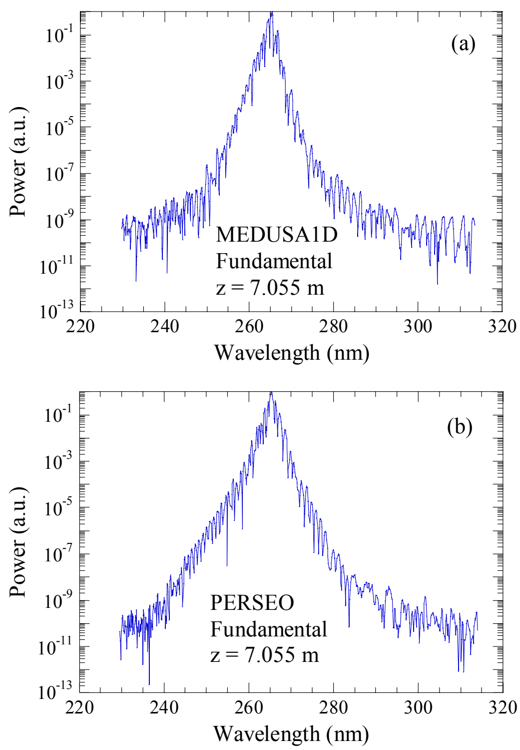

Figure 2: Fundamental spectra at saturation (7.055 m) from (a) MEDUSA1D and (b) PERSEO.

We describe a full SASE configuration where the fundamental and the harmonics start up from noise. The evolution of the pulse energies of the fundamental and the 3rd and 5th harmonics in the case of a full time-dependent SASE simulation is shown in Fig. 1. Here we note that PERSEO and MEDUSA1D are in good agreement for the fundamental up to and somewhat past the saturation point at about 7.1 m and an energy of 71 μJ. The evolution of the harmonic energies are also in close agreement up to and somewhat beyond saturation.

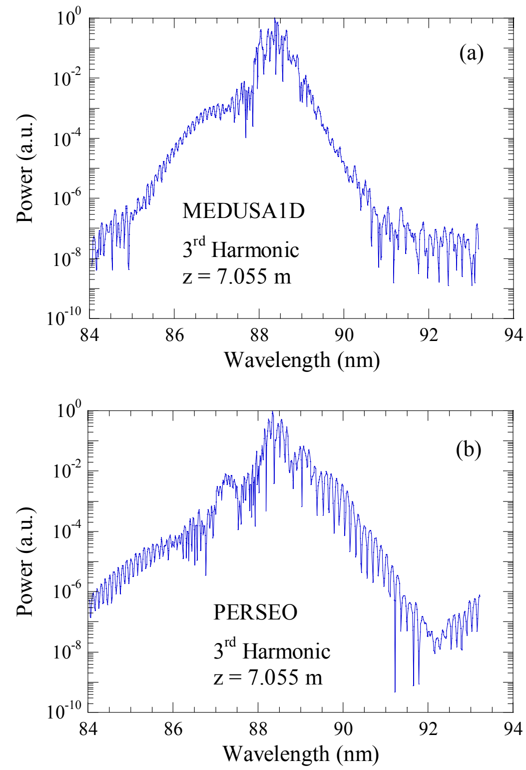

Figure 3: Third Harmonic Spectra at 7.055 m from (a) MEDUSA1D and (b) PERSEO.

The fundamental and 3rd harmonic spectra from MEDUSA1D and PERSEO at the saturation point of the fundamental (7.055 m) are shown in Figs. 2 and 3. As in the case of the spectra for the SASE example, the comparisons between MEDUSA1D and PERSEO are very good and exhibit comparable pulse widths and noise floors.

We now compare 3-D simulations using MEDUSA with results from PERSEO and MEDUSA1D. The parameters we use in the three-dimensional simulations differ somewhat from what we used in 1-D. In this case, we assume a beam kinetic energy of 100.75 MeV, a peak beam current of 270 A, a normalized emittance of 4 mm-mrad, and an energy spread of 0.1%. The bunch is assumed to have a parabolic profile with a duration of 2.5 psec, which yields a bunch charge of 450 pC. The planar wiggler has equal focusing in both planes with a period of 3.89 cm, a total length of 10 m, and a peak, on-axis amplitude of 3.03 kG (K_{rms} = 0.778), which yields a resonance at a wavelength of 795 nm. We treat a seeded amplifier for the three-dimensional simulations with a pulse length equal to that of the electron bunch and a peak power of 4 kW, and this yields an seed energy of 6.67 nJ.

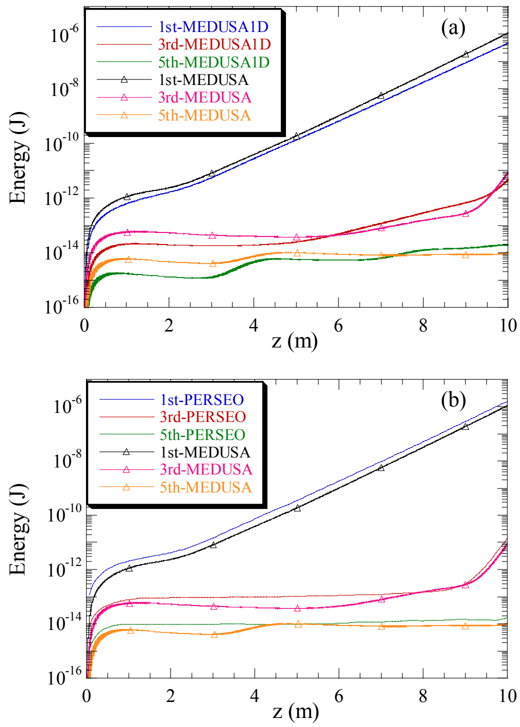

Figure 4: Comparison of time-dependent simulations between MEDUSA and (a) MEDUSA1D, and (b) PERSEO.

We simulate a SASE example where the fundamental and the harmonics start up from noise. We include a simulation window of 7 psec in width with a total of 700 slices. The evolution of the fundamental and the 3rd and 5th harmonics as found in MEDUSA and MEDUSA1D are shown in Fig. 4a. Overall, the agreement between the 1-D and the 3-D simulations is reasonably good. Note that the wiggler length is too short to reach saturation. Because of this, the fundamental is in the linear regime where the energy has not grown to a level where the harmonics experience strong nonlinear growth, although this has begun for the 3rd harmonic near the end of the wiggler. A similar comparison between MEDUSA and PERSEO is shown in Fig. 4b where the agreement between the two codes is also reasonably good.

SUMMARY AND DISCUSSION

In this paper, we have discussed two different algorithms for describing shot-noise in FELs that treat the start-up of both the fundamental and harmonic radiation. Comparison between PERSEO and MEDUSA1D, are very good for both the fundamental and harmonics. This is all the more remarkable considering that the two formulations are differ in almost every aspect Further, 3-D correction factors were included in MEDUSA1D and PERSEO and comparison with MEDUSA also shows reasonable agreement.

REFERENCES

[1] P. Sprangle *et al*., Phys. Rev. Lett. **50**, 1775 (1983).

[2 S. Benson and J.M. J. Madey, Nucl. Instrum. Meth. **A237**, 55 (1985).

[3] C. Penman and B.W.J. McNeil, Optics Commun. **90**, 82 (1992).

[4] W.M. Fawley, Phys. Rev. ST-AB **5**, 070701 (2002).

[5] S. Reiche, Nucl. Instrum. & Meth. **A429**, 243 (1999).

[6] B.W.J. McNeil *et al*., Phys. Rev. ST-AB **6**, 070701 (2003).

[7] L. Giannessi, in Proceedings of the 28th *International Conference on Free-Electron Lasers* (www.JACoW.org, 2006), p.91.

[8] H.P. Freund, Phys. Rev. A **27**, 1977 (1983).

[9] H.P. Freund *et al*., IEEE J. Quantum Electron. **27**, 243 (2000).

[10] H.P. Freund, Phys. Rev. ST-AB **8**, 110701 (2005).

[11] L. Giannessi, L. Giannessi, in Proceedings of the 26th *International Conference on Free-Electron Lasers* (www.JACoW.org, 2004), p. 37.