HIGH GAIN FEL AMPLIFICATION OF CHARGE MODULATION CAUSED BY A HADRON *

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Abstract

In scheme of coherent electron cooling (CeC) [1,2], a modulation of electron beam density induced by a co-propagation hadron is amplified in high gain FEL. The resulting amplified modulation of electron beam, its shape, form and its lethargy determine number of important properties of the coherent electron cooling. In this talk we present both analytical and numerical (using codes RON [3] and Genesis [4]) evaluations of the corresponding Green functions. We also discuss influence of electron beam parameters on the FEL response.

INTRODUCTION

The main advantage of coherent electron cooling is that it promises very short cooling time – under an hour - for high-energy hadron colliders such as RHIC and LHC [1,2]. Strong cooling, in return, has potential of significant luminosity increases in hadron and electron-hadron colliders [2].

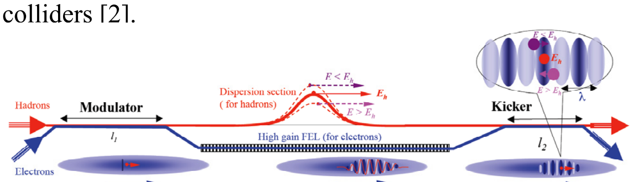

Figure 1: A scheme of a coherent electron cooling

The CeC shown in Fig.1 has three parts: The Modulator, the FEL Amplifier for electron and Longitudinal Dispersion for Hadrons, and the Kicker. In CeC electrons and hadrons should have the same relativistic factor: $\gamma_o = E_e / m_e c^2 = E_h / m_h c^2$. In short, the CeC principles of operation are as follows (see [1,2] for more details): <u>In the modulator</u>, individual hadrons attract electrons and create centers if local density modulation at or near the position of individual hadrons – see [5] for the description of the process. After a half of the e-beam plasma oscillation the electron beam density perturbation a total excess charge of –2Ze. <u>In a FEL-amplifier</u> – which is the main subject of this paper - this modulation of charge density in the electron beam is amplified with exponential FEL growth. At the exit of the FEL, a reaction on individual distortion of the distribution function will become a wave-packet of modulation with FEL wavelength $\lambda_o = \lambda_w (1 + a_w^2)/2\gamma_o^2$ period, (where λ_w and, a_w respectively, are the wiggler period and wiggler parameter). Most importantly, the modulation contains G_{FEL} -times larger charge. At the same time hadrons go *<u>through the dispersion section</u>* which correlates their energy and arrival time to the modulator: $(t - t_o)v_o = -D \cdot \delta$. Fine tuning the CeC provides for synchronization between the space-charge wave-packet induced by a hadron in such away that the hadron with central energy, E_o, arrives at *<u>the kicker section</u>* just on the top of the pancake of increased electron density (induced by the hadron), wherein the longitudinal electric field is zero. Hadrons with higher energy will arrive at the kicker ahead of their respective pancake in the electron beam, and will be pulled back (decelerated) by the coherent field of the electron beam; we note that positively charged hadrons are attracted to high-density pancakes of electrons. Similarly, a hadron with lower energy falls behind and, as a result will be dragged forward (accelerated) by the clump of electron density. This interaction reduces longitudinal phase space volume of the hadron beam, i.e. effectively cools it.

A decrement of transverse cooling, if needed, can be introduced by coupling between longitudinal and transverse degrees of freedom [1], and it can be as powerful as the longitudinal one.

The comprehensive studies of coherent electron cooling had been initiated about ten months ago. They included both theoretical and computational analysis of processes in the modulator, the FEL, the dispersion section and the kicker. Progress of these studies is described elsewhere [5]. The most important conclusion for this paper is that there a clear theoretical [6] and numerical [7] ways of determine the exact 3D time-dependent response of uniform electron plasma on the presence of a moving both in longitudinal and transverse direction (in co-moving reference frame for electron beam). Thus, one can calculate the distribution of electron beam at the entrance of the FEL modified by its interaction with the hadron(s) in the modulator.

In this short paper we focus on the amplification of this modification, i.e. the imprint of the hadrons in the electron beam - in a high-gain FEL. Further more, we will focus on the longitudinal part of the FEL Green-function (see below)

FEL'S GREEN FUNCTION

Even though evolution of the optical power in high-gain, single-pass FEL is well studied and well-described in a number of publications, the time-dependent FEL response on a δ-function type disturbance and especially evolution of the density modulation through the FEL is

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not studies thoroughly. In general terms, our goal –as illustrated in Fig. 2 – to find a kernel of integral transformation (an linear operator) for transformation of electron beam distribution while propagating in an FEL:

$$f\left(\vec{r}_\perp,\vec{p},t;z\right) = \int G\left(\vec{r}_\perp,\vec{r}_\perp{}',\vec{p},\vec{p}',t,t';z\right) f\left(\vec{r}_\perp{}',\vec{p}',t';0\right) d\vec{r}_\perp{}' d\vec{p}' dt' \quad (1),$$

where $G\left(\vec{r}_\perp,\vec{r}_\perp{}',\vec{p},\vec{p}',t,t';z\right)$ is nothing else but a Green function corresponding to a δ-function distortion of the electron-beam distribution at the entrance to the FEL (z=0):

$$\delta f\left(\vec{r}_\perp,\vec{p},t;0\right) = \delta\left(\vec{r}_\perp{}' - \vec{r}_\perp\right)\delta\left(\vec{p}' - \vec{p}\right)\delta\left(t' - t\right). \quad (2)$$

We are considering an FEL with time independent parameters, and there fore Green function does not on time explicitly, i.e. it is the function of the time difference between events:

$$G\left(....,t,t';z\right) = G\left(...,\tau = t - t';z\right) \quad (3)$$

In addition, the Green function satisfy causality conditions, i.e. it is equal to zero outside the light cone:

$$G\left(\vec{r}_\perp,\vec{r}_\perp{}',\vec{p},\vec{p}',t,t';z\right) \equiv 0 \;\; \forall \;\; \left|\vec{r} - r'\right| > c\left|t - t'\right| \quad (4)$$

with the main practical implication for an FEL that response is non-zero only within a slippage distance (time) in the FEL from the location of initial perturbation.

Naturally, the response also strongly depends on the parameter of the FEL wiggler, associated focusing, wiggler errors, etc. Finding analytical expression for Green function of an arbitrary FEL is as unrealistic and it is a natural problem for time-resolved 3D FEL codes.

Nevertheless, it is of interest for deeper understanding of the process to simplify it to a solvable problem, which allow comparison with 1D FEL solution.

One simplification can be done for a case when electron beam has only modification on in density distribution[1], but not in velocity and energy, i.e.

$$\delta f\left(\vec{r}_\perp,\vec{p},t;0\right) = \delta\left(\vec{r}_\perp{}' - \vec{r}_\perp\right)\delta\left(t' - t\right)$$

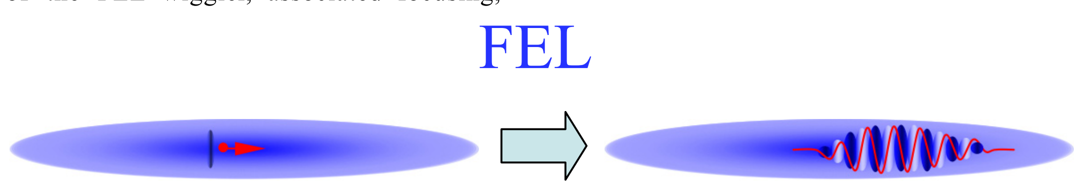
Diffraction of the optical radiation in the FEL weakens dependence on the transverse position of the initial distortion. Furthermore, the most interesting information for the kicker is the longitudinal density modulation [1], which is integral over both the momenta and transverse coordinates.

This is the reason why we decided, as the first step of these studies, to focus an 1D green-function for longitudinal density modulation:

$$\rho\left(t;z\right) = \int G\left(\tau;z\right)\rho\left(t-\tau;z\right)d\tau\,; \qquad \delta\rho\left(t;0\right) = \delta\left(t\right), \quad (5)$$

where $\rho\left(t;z\right) = \int f\left(\vec{r}_\perp,\vec{p},t;z\right) d\vec{r}_\perp{}' d\vec{p}'$. This approach allows us to compare computer simulations with 1D FEL theory and to check the validity of concept used in CeC.

[1] Such situation can occur after a half of plasma oscillation in electron beam when density distortion is at its maximum.



$$\mathcal{L}_{FEL} : f_{entrance}\left(t,\vec{r},\vec{\mathrm{v}}\right) \Rightarrow f_{exit}\left(t,\vec{r},\vec{\mathrm{v}}\right)$$

Figure 2: The process of interest for this paper – how a perturbation induced in electron beam distribution in the modulator by the hadron is amplified by a high-gain FEL?

GREEN FUNCTION OF 1D FEL

Linear theory of 1D FEL is well developed and will use here a number of results from [8], even through we will use more traditional notations.

General qualitative features of the FEL response on are also well known – the Green function is a wave-packet a rather smooth envelope modulated with the FEL frequency:

$$G\left(\tau;z\right) = \mathrm{Re}\left(\tilde{G}_z\left(\tau\right)e^{i\omega_o\tau}\right)\; \omega_0 = \frac{2\pi c}{\lambda_o}; \quad (6)$$

The total extend of the envelope is equal to the total slippage in the FEL wiggler: $\Delta = N_w\lambda_o$, while the peak of the envelope is located at about one third of the slippage length from the origin. The later is the consequence of the fact that group velocity (of a wave-packet) is equal to one third of the speed of light plus two thirds of the average longitudinal velocity of the electron in the wiggler, v_z [8]:

$$\mathrm{v}_g = \frac{c + 2\langle \mathrm{v}_z\rangle}{3} = c\left(1 - \frac{1 + a_w^2}{3\gamma_o^2}\right). \quad (7)$$

The duration of such a wave-packet (i.e., the thickness of the individual pancake stack) is equal to the coherence length of SASE FEL radiation [8,9]. In 1D theory [8] a large number of analytical solutions do exist in Fourier domain for cases including space charge and energy spread (we used Lorentzian distribution). Apart from a poor convergence and a need for careful error analysis, these tools are sufficient for calculating the Green-function of 1D FEL. In 1D theory everything naturally is scaled by the gain length [8] and Fig. 3 shows the Green function in an a ten-gain-lengths FEL as function of the slippage in these units.

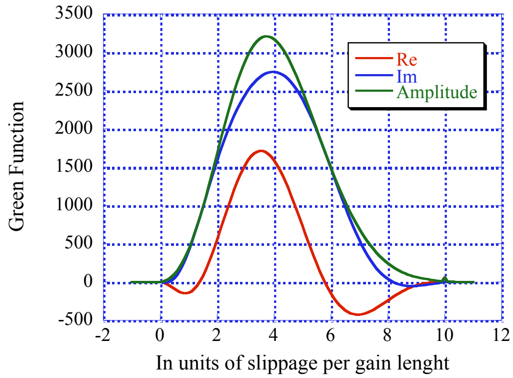


Figure 3: The amplitude, the real and the imaginary part of the Green function envelope after propagation in FEL with ten-gain length. For comparison with 3D FEL in the following section – their one gain length is equal and 2 m or 40 periods for amplitude. Thus, the unit slippage in this figure corresponds to 40 optical wavelengths.

Detailed studies of the green function show that its maximum located at 3.744 slippage units, i.e. just a bit further that expected 3 and 1/3 slippage units. The Green function (which oscillates) had effective RMS length [1] of 1.48 slippage units.

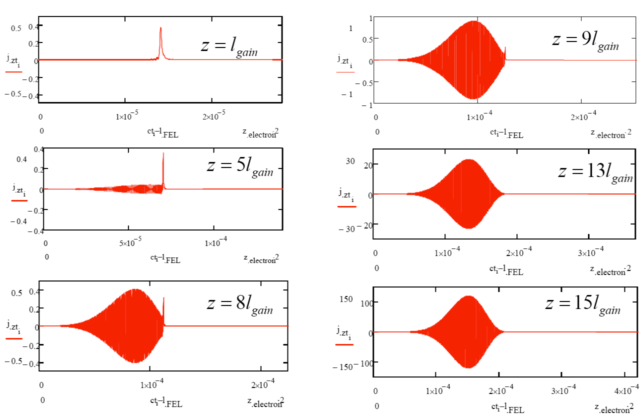


Figure 4: Evolution of initial distortion of electron beam density caused by a proton in the CeC modulator in FEL using 1D FEL approach for eRHIC CeC (see table below). The clips show the location along z in the FEL-power gain-lengths. Note that horizontal axis is in meters and direction is reversed – the origin is located at the right side of the wave-packet.

Similar features are observed in analytical evaluation of the evolution (in 1D FEL) of the initial distortion of electron beam density caused by a proton (see Fig. 4). After the process of initial formation of the wave-packet, it scales accordingly in expected way. Inclusion of the energy spread, see Fig.5, into the equation does not change the overall qualitative picture, but affect the FEL gain through its gain-length and a slight shift of the peak of the envelope.

Addition of the space charge also affects the gain length and can cause shift of the envelope and, naturally, its phase. Group velocity can increase by 20-25% for a space charge dominated case ($\hat{\Lambda}_p \sim 1$).

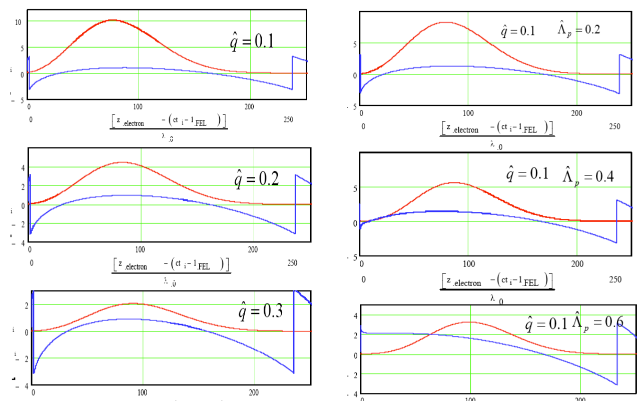


Figure 5: Dependence of the Green-function envelope´s amplitude (red line) and its phase (blue line) on energy spread $\hat{q}$ parameter and on space charge $\hat{\Lambda}_p$ parameter (see definitions in [8]). Horizontal axis is in units of optical wavelengths. Total slippage is 240 wavelengths.

GREEN FUNCTION OF 3D FEL

As the first test, we selected parameters of an FEL amplifier we considered to be suitable to cool 250 GeV protons in RHIC [2]. These parameters give amplitude gain-length of 2 meters. They had not been optimized and length of the FEL wiggler was used as parameter. Table 1 lists main parameters of the FEL system.

Table 1: Main FEL parameters

Energy, MeV	136.2	γ	266.45
Peak current, A	100	λ_o, nm	700
Bunchlength, psec	50	λ_w, cm	5
Emittance, norm	5 mm mrad	a_w	0.994
Energy spread	0.03%	Wiggler	Helical

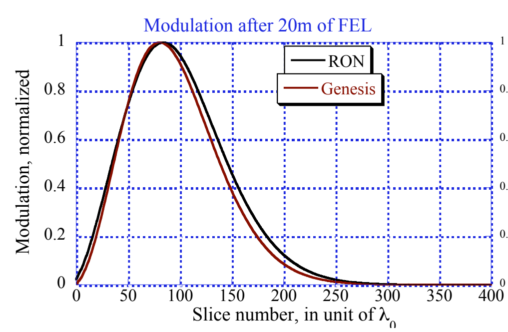


Figure 6: Comparison of the normalized envelopes the Green functions, i.e. the electron beam density modulation caused calculated by RON and GENESIS FEL codes. Data evaluated after 20meter-long wiggler.

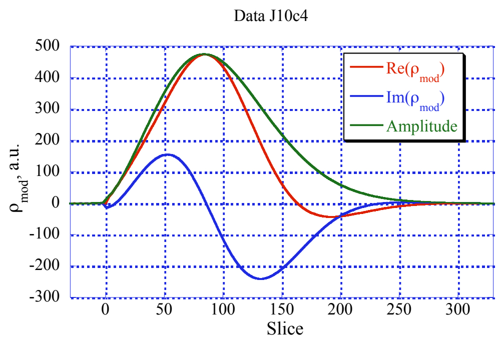


Figure 7: The Green-function envelope calculated by RON at z=20m, i.e. after 10 gain-length FEL, resembles all the features seen in the 1D case (Fig.3).

We used 3D FEL codes, RON [3] and Genesis 1.3 [4], based on completely different approaches. RON used multiple frequencies and FFT to find the time response on a short spike of the modulation in electron beam. The Genesis 1.3 was ran in the time-domain mode with only one of 1000 slices (slice had length of λ_o) had been modulated with 1e-4 bunching value, while the rest of slides have quiet loading. Results of both 3D simulations compare very well with each other (see Fig.6).

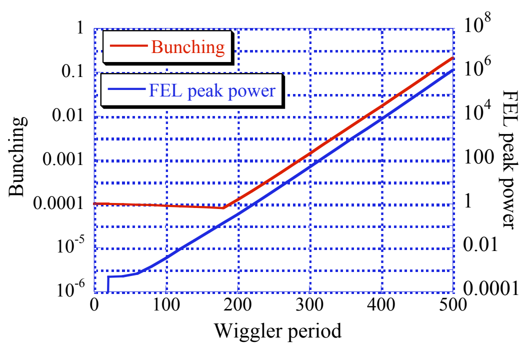


Figure 8: Evolution of the maximum bunching in the e-beam and the FEL power simulated by Genesis.

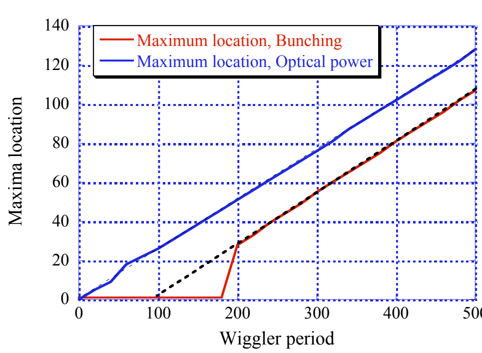


Figure 9: Evolution of the maxima locations in the e-beam bunching and the FEL power simulated by Genesis.

Evolution of the wave-packets of the bunch modulation and optical power, shown in Figs. 8 and 9, can be described by the their peak value (maxima) and by the location of the maxima. After a period of establishment of the exponential regime both the modulation and optical power grow with correct gain-length (40 period or 2 m for amplitude and 20 periods or 1 m for the power).

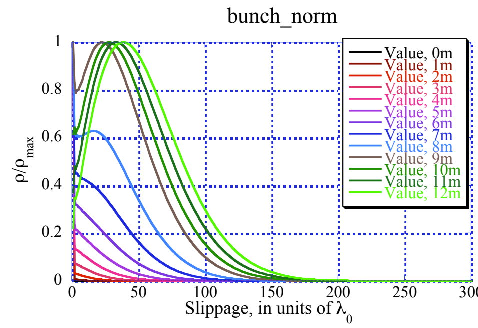


Figure 10. Evolution of the bunching envelope.

Nevertheless, one should notice that it takes four and a half gain-lengths for bunching to grow above the initial 10^{-4} level. The location of the maxima, both for the optical power and the bunching progresses with a lower speed compared with prediction by 1D theory. It corresponds to a lower group velocity compared with eq. (7):

$$v_g \cong \frac{c + 3\langle v_z \rangle}{4} = c\left(1 - \frac{3}{8}\frac{1 + a_w^2}{\gamma_o^2}\right),$$

i.e. electron beam plays 75% role in its value. It is also noticeable that e-beam modulation lags behind the optical power – and fig. 10 illustrates the initial stages of this development - and effectively "misses" about two and a half gain-length of the propagation from the origin. The Green function after 10 gain-lengths (which oscillates) had also smaller effective RMS length [1] of 0.96 slippage units (i.e. about 38 optical wavelengths, or 27 microns).

CONCLUSIONS

These initial studies did not find any phenomena, which is significantly deviates from that used for initial CeC estimations [1,2]. At the same time, we found a number of new and interesting details to pursue further.

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