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Trieste



# ***Transverse Emittance Preserving Arc Compressor: Sensitivity to Beam Optics, Charge and Energy***

S. Di Mitri

**Elettra Sincrotrone Trieste**

# Where I come from...



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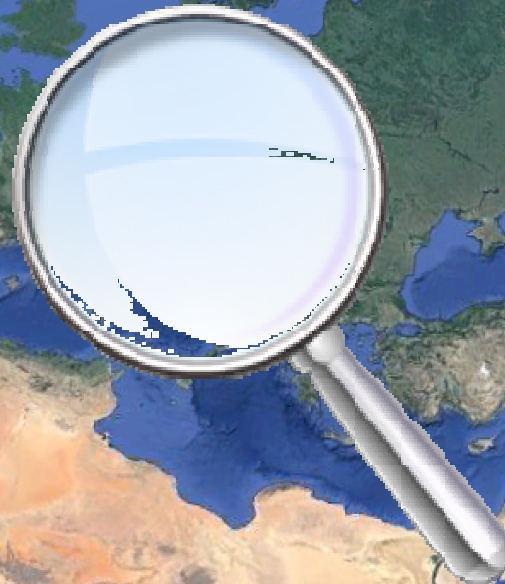


Image Landsat  
Image IBCAO  
Image U.S. Geological Survey  
Data SIO, NOAA, U.S. Navy, NGA, GEBCO

GOO



is a nonprofit shareholder company of **national interest**, established in **Trieste, Italy** in 1987 to **construct and manage synchrotron light sources** as **international facilities**.

ELETRA (Synchrotron Light Source);  
up to 2.4 GeV, top-up mode,  
~800 proposals from 40 countries every year

FERMI, externally seeded FEL:  
Running User Facility  
Two FEL lines, covering 100 – 4 nm

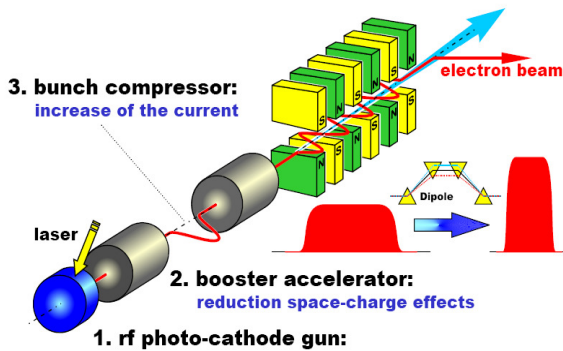
# Outline

- Prologue
  - Motivations and Challenges
  - Arc Compressors in Literature
- Optics Balance in a Transfer Line ( $C=1$ )
  - 1-D CSR model & Experimental Proof
- Periodic Arc Compressor ( $C=45$ )
  - Optics Considerations
  - Analysis and Simulations: Emittance vs. Bunch Charge, Energy and Optics
- Conclusions & Outlook

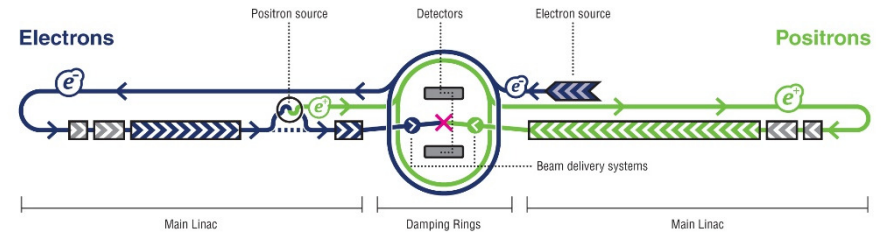
# Motivations & Challenges

□ Why magnetic bunch length compression?

• FELs:  
 $P_{out} \propto I$

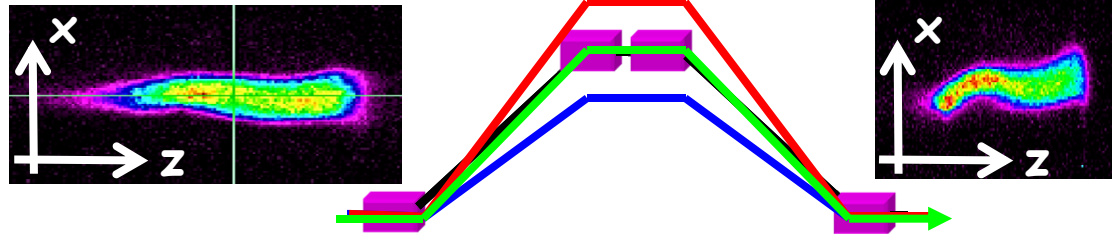


• Linear Colliders:  $\Delta\epsilon_{x,y}(w_{\perp}) \propto I^{-1}$



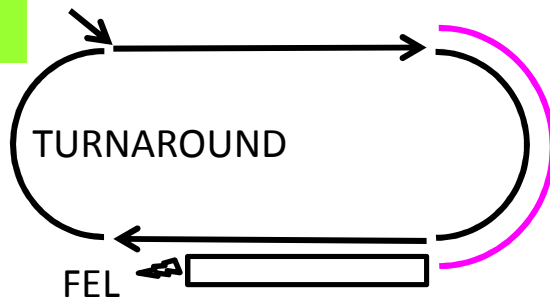
□ What are the challenges of  $\sigma_z$ -compression?

• CSR:  
 $\Delta\epsilon_x \propto (\sigma_{z,CSR})^{-8/3}$



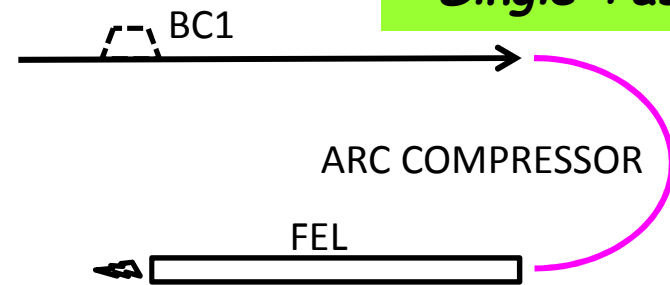
□ Why an arc compressor?

• ERLs:



Recirculation  
AND  
Compression

• Single-Pass:



# Arc Compressors in (Recent) Literature

□ **Past ERLs design studies:** *BNL (2001), KEK (2007), ANL (2008), JLAB (2011), Cornell (2013).*

$E > 0.6 \text{ GeV}$

$Q = 50\text{--}150 \text{ pC}$

$C < 30 \text{ (77pC, 3.0GeV)}$

$\Delta\epsilon_{nx} \sim 0.1 \text{ }\mu\text{m}$

- **Minimize the CSR-dispersion function.**  
[R. Hajima, 528 (2004) 335].
- **CSR primarily suppressed with a low charge.**

□ **Our proposal:**

$E > 0.5 \text{ GeV}$

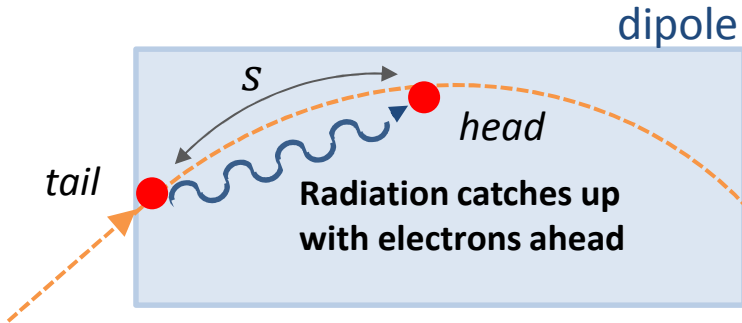
$Q = 100\text{--}500 \text{ pC}$

$C = 45 \text{ (500pC, 2.4GeV)}$

$\Delta\epsilon_{nx} \sim 0.1 \text{ }\mu\text{m}$

- **Optics balance to cancel successive CSR kicks...**  
[Di Mitri, Cornacchia, Spampinati, PRL 110 014801 (2013)].
- **...extended to a varying bunch length.**  
[Di Mitri, Cornacchia, EPL 109 (2015) 62002].
- **Background:** D.Douglas, JLAB-TN-98-012 (1998);  
Y.Jiao et al., PRTSAB 17, 060701 (2014).

# CSR Picture



- Consider **1-D steady-state CSR emission**, and **linear optics**.
- Transient CSR effects and nonlinear dynamics will be included in the simulations.

**RELATIVE ENERGY SPREAD of GAUSSIAN bunch, per DIPOLE:**

$$\sigma_{\delta, CSR} = 0.2459 \cdot r_e^2 \frac{N_e \theta R^{1/3}}{\gamma \sigma_z^{4/3}}$$

**For a SINGLE PARTICLE:**

Photon emission/absorption,  
 $\Delta x = \Delta x_{\beta} + \Delta x_{\eta} = 0$

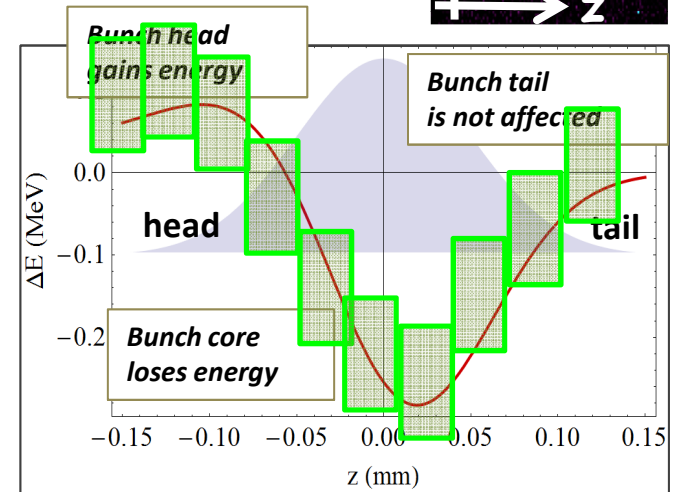
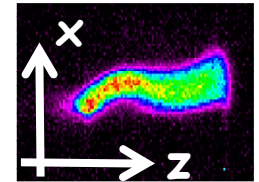
**New betatron amplitude,**  
 $\Delta x_{\beta, phot} = -\eta \delta_{phot}$

**New dispersive trajectory,**  
 $x_{\eta, phot} = x_{\eta} + \eta \delta_{phot}$

**Initial betatron oscillation,  $x_{\beta}$**

**Initial reference (dispersive) trajectory,  $x_{\eta}$**

**For a BUNCH:**



**Note: distortion is both in  $x$  and  $x'$**

# Projected Emittance Growth, $\sigma_z = \text{const.}$

➤ **Multiple and identical CSR kicks** (this applies to an isochronous transfer line):

A. Use the **Courant-Snyder formalism** for the particle coordinates, linear transport matrices,  $\mathbf{x}_1 = M\mathbf{x}_0$ , and  $J(0)=0$ .

B. When traversing a dipole, add the CSR induced  $\eta$ -terms. This leads to an **increase** of the particle's **C-S invariant**:

C. Repeat until the end of the line. All kicks are identical in module, and we can write  $J_f = J_f(J_1)$ . After averaging we find:

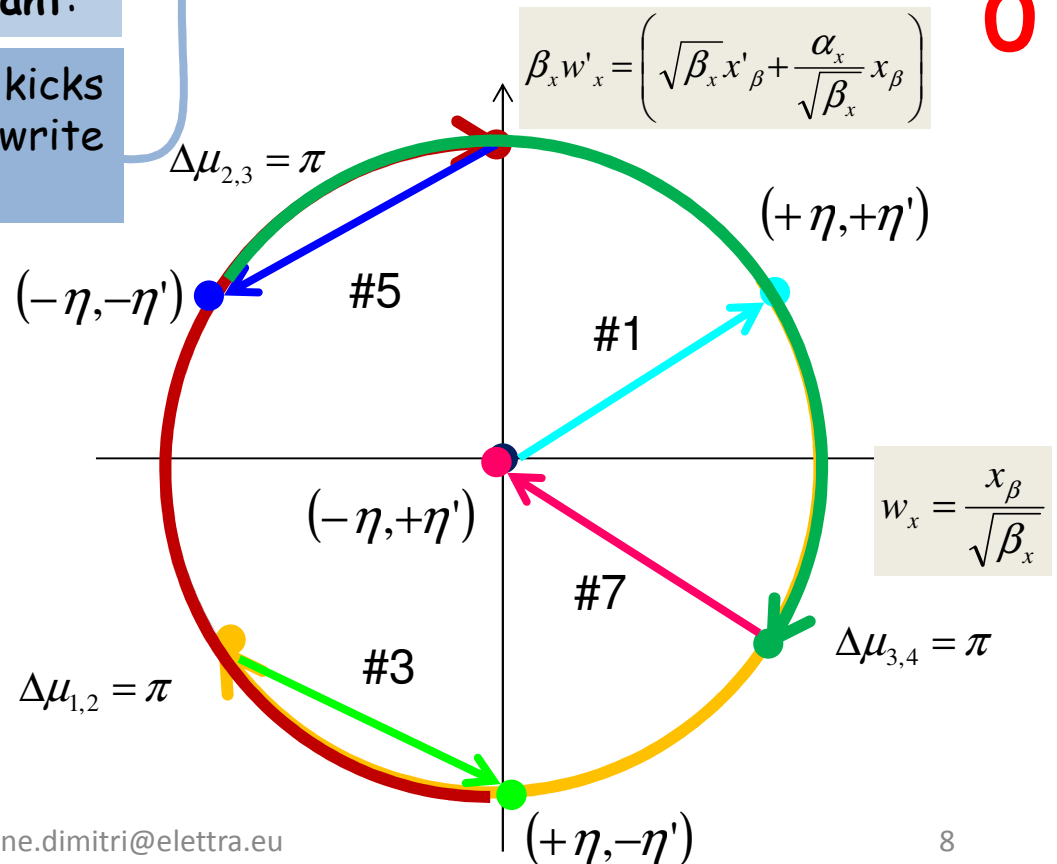
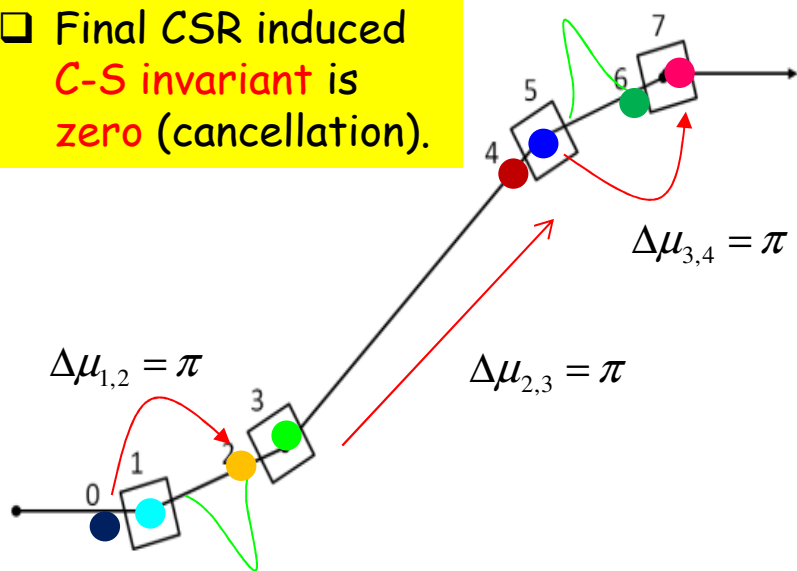
□ Final CSR induced **C-S invariant** is **zero** (cancellation).

$$J_1 = \gamma_1 x_1^2 + 2\alpha_1 x_1 x_1' + \beta_1 x_1'^2 = (\gamma_1 \eta_1^2 + 2\alpha_1 \eta_1 \eta_1' + \beta_1 \eta_1'^2) \delta_{CSR}^2 \equiv H_1 \delta_{CSR}^2$$

$$\varepsilon_{x,f}^2 = \varepsilon_{x,0}^2 + \varepsilon_{x,0} J_1 (\sigma_{\delta,CSR}^2) X(\beta_x, \alpha_x, \mu_x)$$

$$\beta_x w'_x = \left( \sqrt{\beta_x} x'_\beta + \frac{\alpha_x}{\sqrt{\beta_x}} x_\beta \right)$$

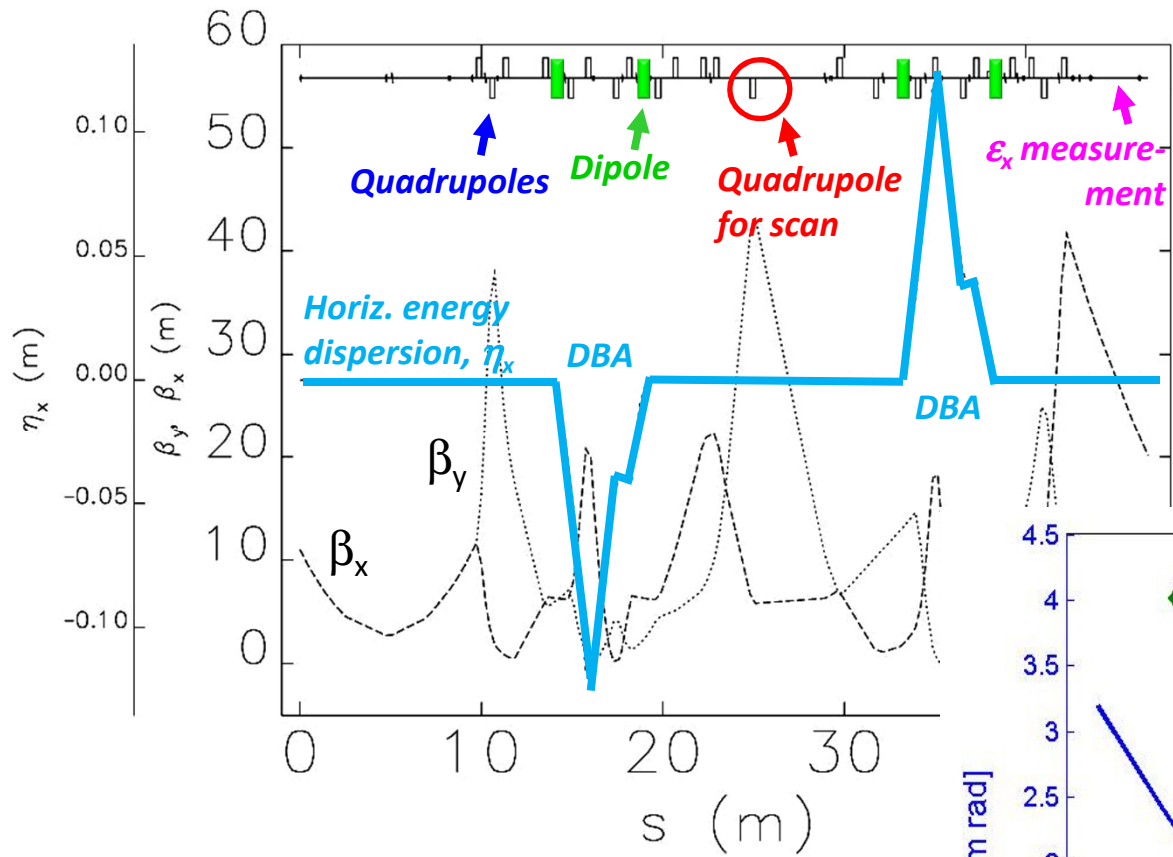
$$w_x = \frac{x_\beta}{\sqrt{\beta_x}}$$





# Experimental Proof at FERMI

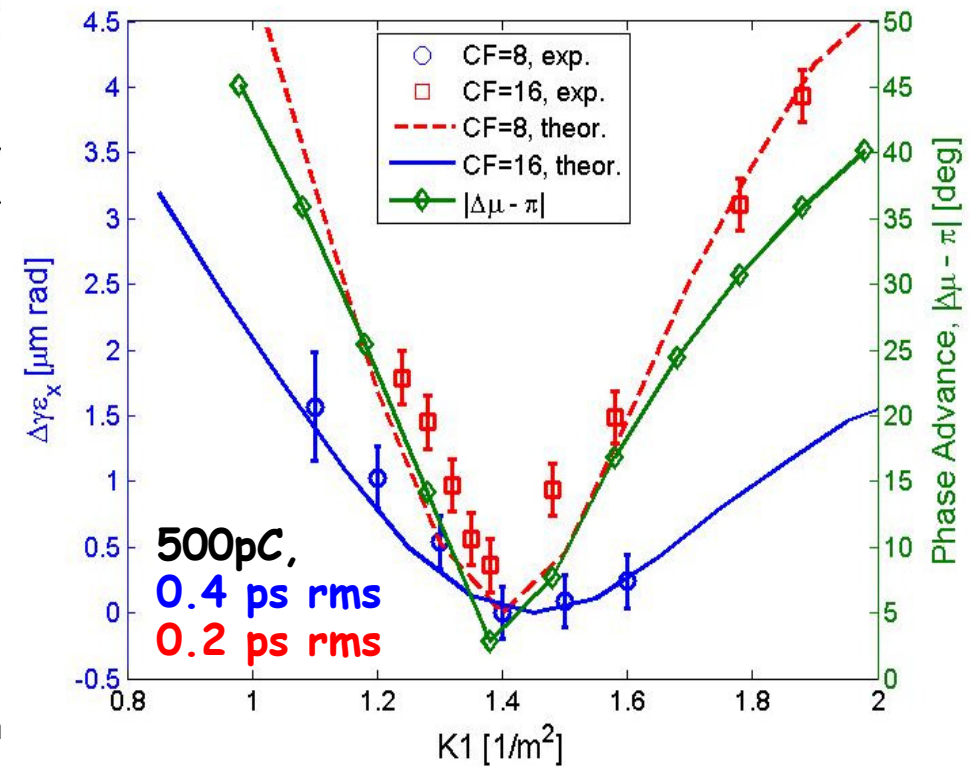
PRL 109, 014801 (2013)  
Phys. Reports 539, 1 (2014)



Quadrupoles ensure  $\pi$ -phase advance between dipoles and proper values of  $\beta_x$ ,  $\alpha_x$  to cancel the CSR-emittance.

One quadrupole's strength is scanned to vary the phase advance between the DBAs.

- Results:**
- Minimum  $\epsilon_{n,x}$  for nominal optics ( $\pi$ -phase advance and optimum Twiss parameters).
  - Larger  $\epsilon_{n,x}$  for shorter beam.



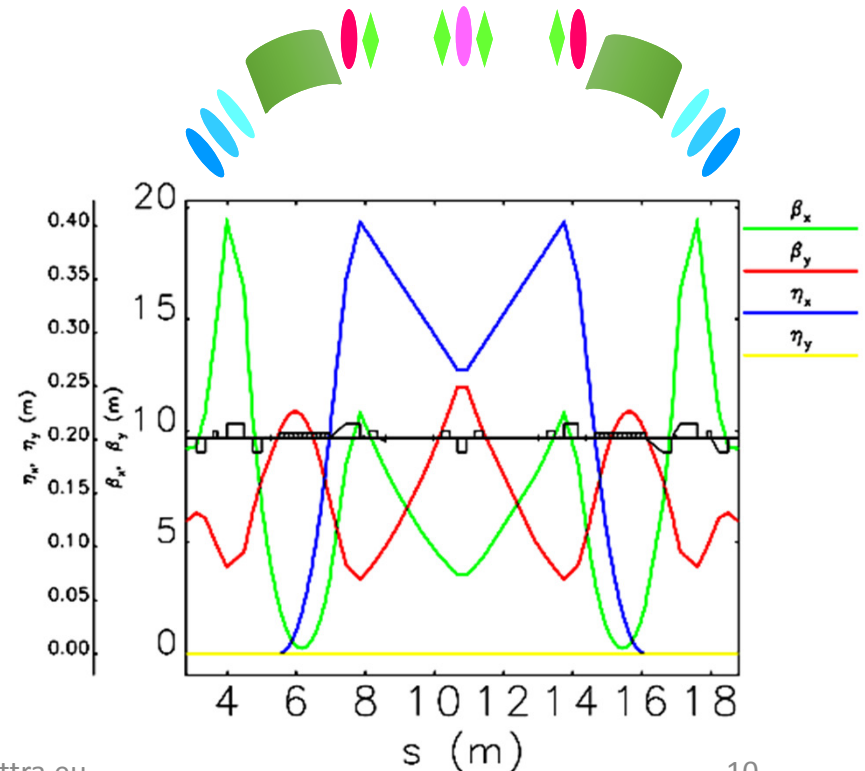
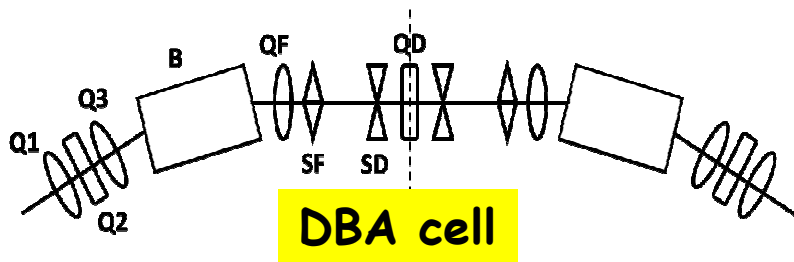
# Periodic Arc Compressor

- $H_x$  has to be small at the dipoles  $\Rightarrow \theta$  and  $\beta_x$  small
- $R_{56}$  has to be large enough to cumulate a  $C > 30 \Rightarrow \theta$  not too small
- Suitable  $\beta_x, \alpha_x, \mu_x$  along the line for CSR cancellation  $\Rightarrow$  many quadrupoles
- We want to linearize the longitudinal phase during compression  $\Rightarrow$  sextupoles
- Possibly simple, robust and compact lattice



- 6 DBA cells (Elettra-like lattice, ECG).
- 180°, 125 m long at 2.4 GeV
- $R_{56} = +35$  mm per cell

Due to symmetry and short bends,  $\Delta\mu_x \approx \pi$ .



# Optimum Optics in a Single DBA:

For a single DBA, at the exit of the 2<sup>nd</sup> dipole we have:

$$\begin{cases} x_3 = -\rho^{4/3}k_1(\theta C_\theta - 2S_\theta) + \rho^{4/3}k_2(\theta C_\theta - 2S_\theta) \\ x_3' = -\rho^{1/3}k_1\theta S_\theta - \rho^{1/3}k_2\theta S_\theta - \frac{2\alpha_2}{\beta_2}\rho^{4/3}k_1(\theta C_\theta - 2S_\theta), \\ \delta_3 = \rho^{1/3}k_1\theta + \rho^{1/3}k_2\theta \end{cases}$$

where  $C_\theta = \cos(\theta/2)$ ,  $S_\theta = \sin(\theta/2)$

$$k_i = 0.2459r_e Q / (e \gamma \sigma_{z,i}^{4/3})$$

$k_{i+1} = C_{i+1}^{4/3} k_i$  CSR kick scales with  $\sigma_z$



$$J_3 \cong \left( \frac{k_1 \rho^{1/3} \theta^2}{2} \right)^2 \left[ \beta_2 (C^{4/3} + 1)^2 + \frac{1}{\beta_2} \left( \frac{l_b}{6} \right)^2 \left[ (C^{4/3} - 1)^2 + \alpha_2^2 (C^{4/3} - 3)^2 \right] + 2\alpha_2 \left( \frac{l_b}{6} \right) (C^{4/3} + 1)(C^{4/3} - 3) \right]$$

Look for the optimum Twiss parameters at the dipoles:

$$\left( \frac{dJ_3}{d\alpha_2} \right)_{\beta_2} \equiv 0 \quad \Rightarrow \quad \alpha_{2,opt} = -\frac{\beta_2}{\left( \frac{l_b}{6} \right)} \frac{(C^{4/3} + 1)}{|C^{4/3} - 3|}$$

$$\left( \frac{dJ_3}{d\beta_2} \right)_{\alpha_2} \equiv 0 \quad \Rightarrow \quad \beta_{2,opt} = \left( \frac{l_b}{6} \right) \frac{\sqrt{(C^{4/3} - 1)^2 + \alpha_2^2 (C^{4/3} - 3)^2}}{(C^{4/3} + 1)}$$

$$J_3 = 0 \Leftrightarrow C = 1$$

$$\alpha_{2,opt}(C = 1) = -\frac{6\beta_{2,opt}}{l_b},$$

as already in [Y. Jiao et al. PRTSAB 17, 060701 (2014)].

# Optimum Optics *along* the Arc Compressor:

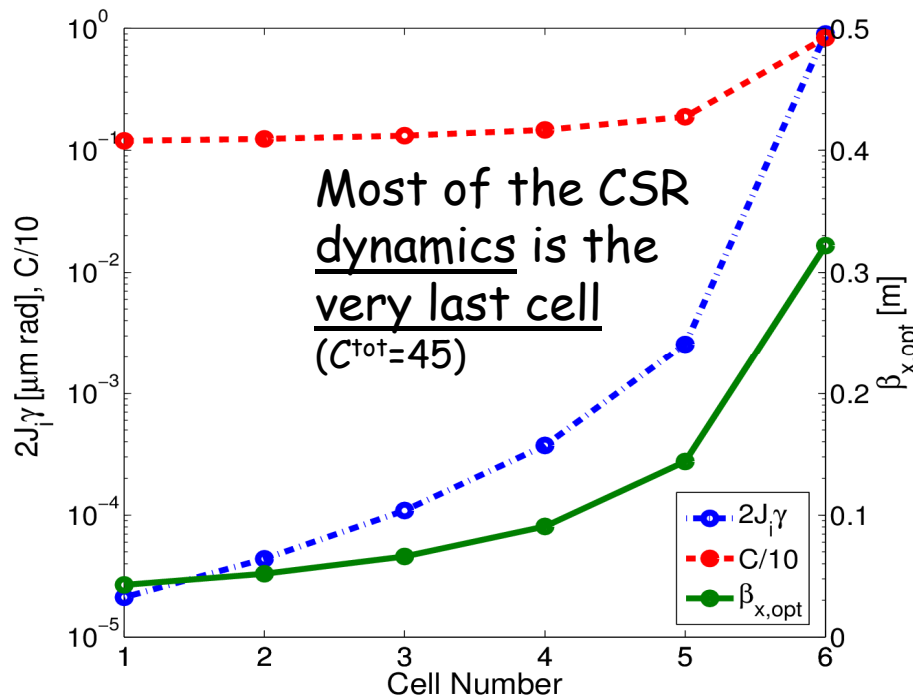
1. The local  $C_i$  depends on the upstream E-chirp, which varies along the arc:
2. The optimum  $\beta_{x,dip}$  depends on  $C_i$ , thereby it varies along the arc.

$$h_i = \frac{1}{E_0} \left( \frac{dE}{dz} \right)_i \approx \frac{\sigma_{\delta,0}}{\sigma_{z,i}}$$

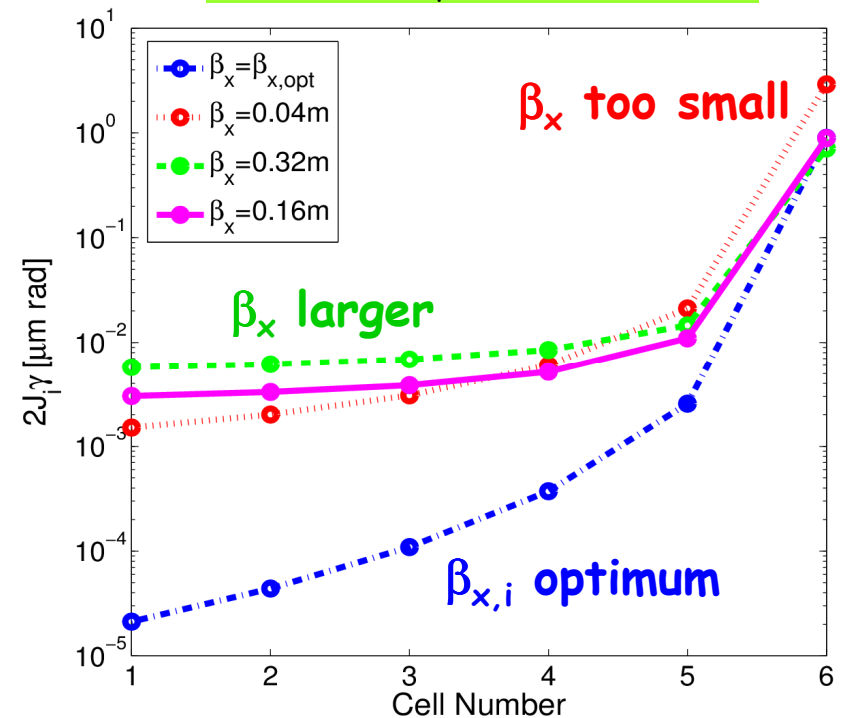
$$C_i^{loc} = \frac{1}{|1 + C_{i-1} h_{i-1} R_{56}|},$$

$$C_j^{tot} = \prod_{i=1}^j C_i^{loc} \quad i, j = 1, \dots, 6$$

Optimum  $\beta_{x,dip}$ ,  $C_i^{loc}$  and  $J_i$ , for  $\alpha_{x,dip}=0$ :



$J_i$  vs.  $\beta_{x,dip}$ , for  $\alpha_{x,dip}=0$ :

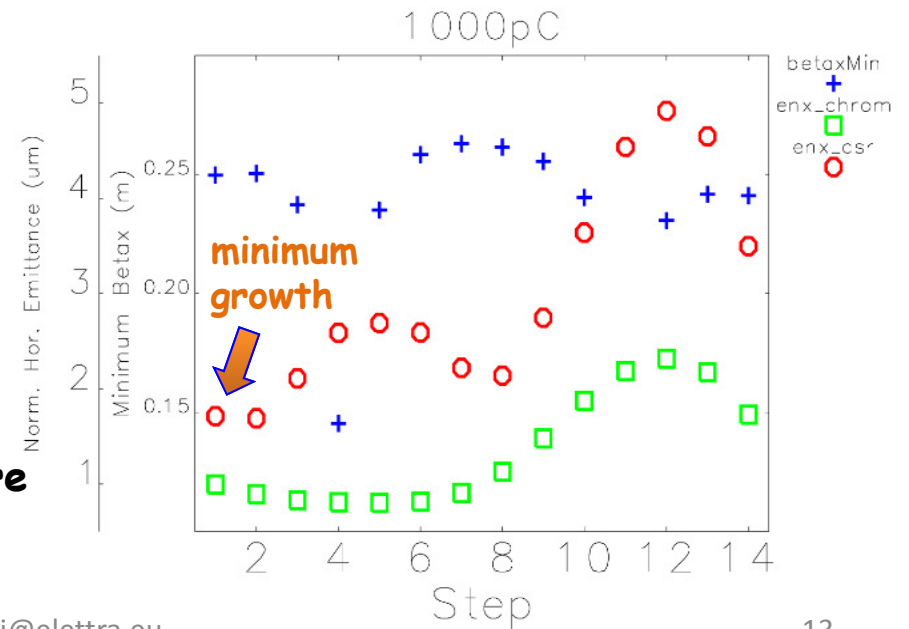
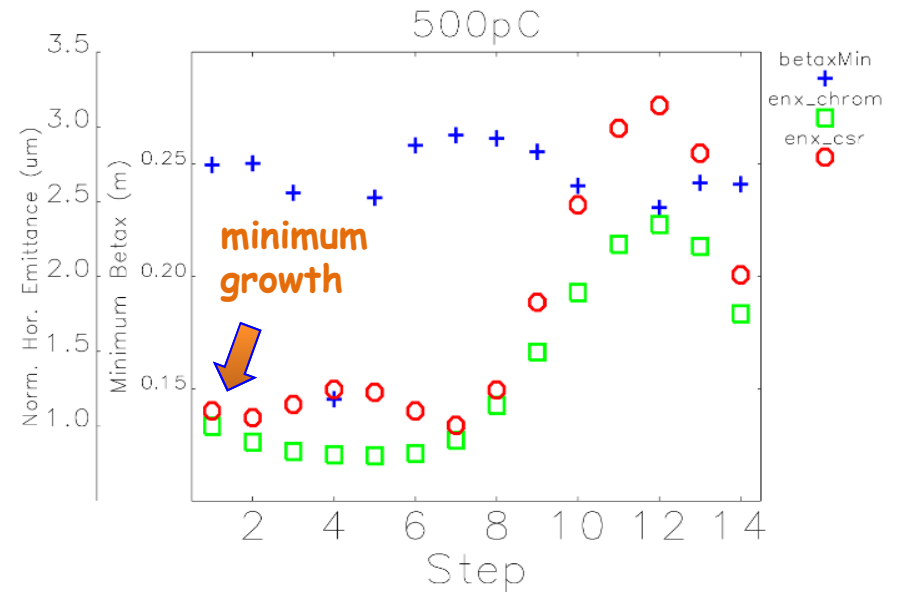
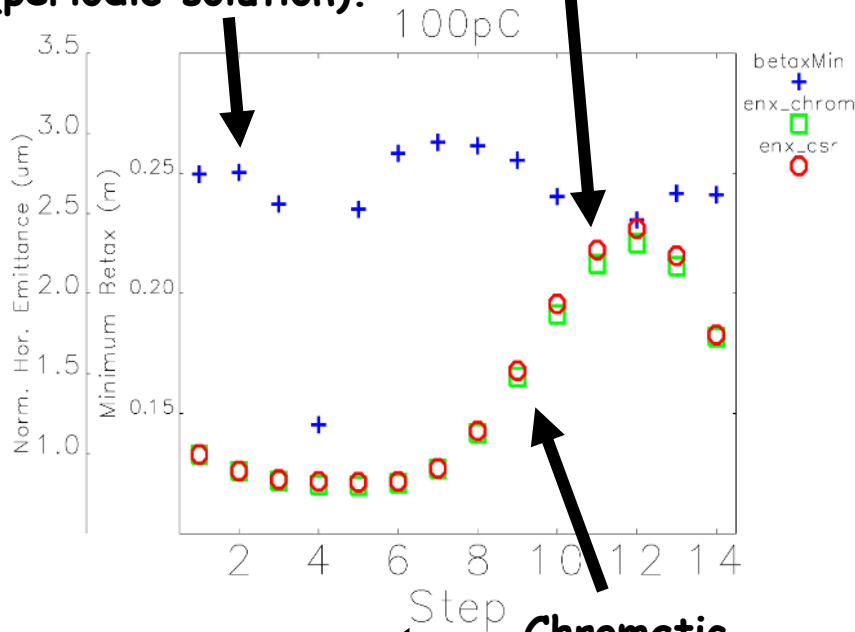


# Final Emittance vs. Charge and Optics

$E = 2.4 \text{ GeV}$	$C_{\text{tot}} = 45$
$\sigma_{z,0} = 3 \text{ mm}$	$\epsilon_{nx,0} = 0.8 \text{ } \mu\text{m}$
$h_0 = -4.7 \text{ m}^{-1}$	

$\beta_{x,\text{dip}}$  is the same for all the dipoles (periodic solution).

$$\Delta\epsilon_{x,\text{CSR}} = \text{red circle} - \text{green square}$$



Each step corresponds to a different  $\beta_{x,\text{dip}}$ . Chromatic aberrations (green square) are NOT corrected at each step.

# Final Emittance vs. Charge and Energy

Ansatz: the emittance sums in quadrature after each DBA:

$$\varepsilon_{x,i}^2 = \varepsilon_{x,i-1}^2 + \varepsilon_{x,i-1} J_{i-1}$$

$$\varepsilon_{x,f}^2 \approx \varepsilon_{x,0}^2 + \varepsilon_{x,0} \sum_1^j J_{i-1} \Leftrightarrow \sum_1^j J_{i-1} \ll \varepsilon_{x,0}$$

- Theory:  $J_i$  as above.
- Steady: 1-D CSR in Elegant.
- Total: Steady + Edges + Drifts.

This is:

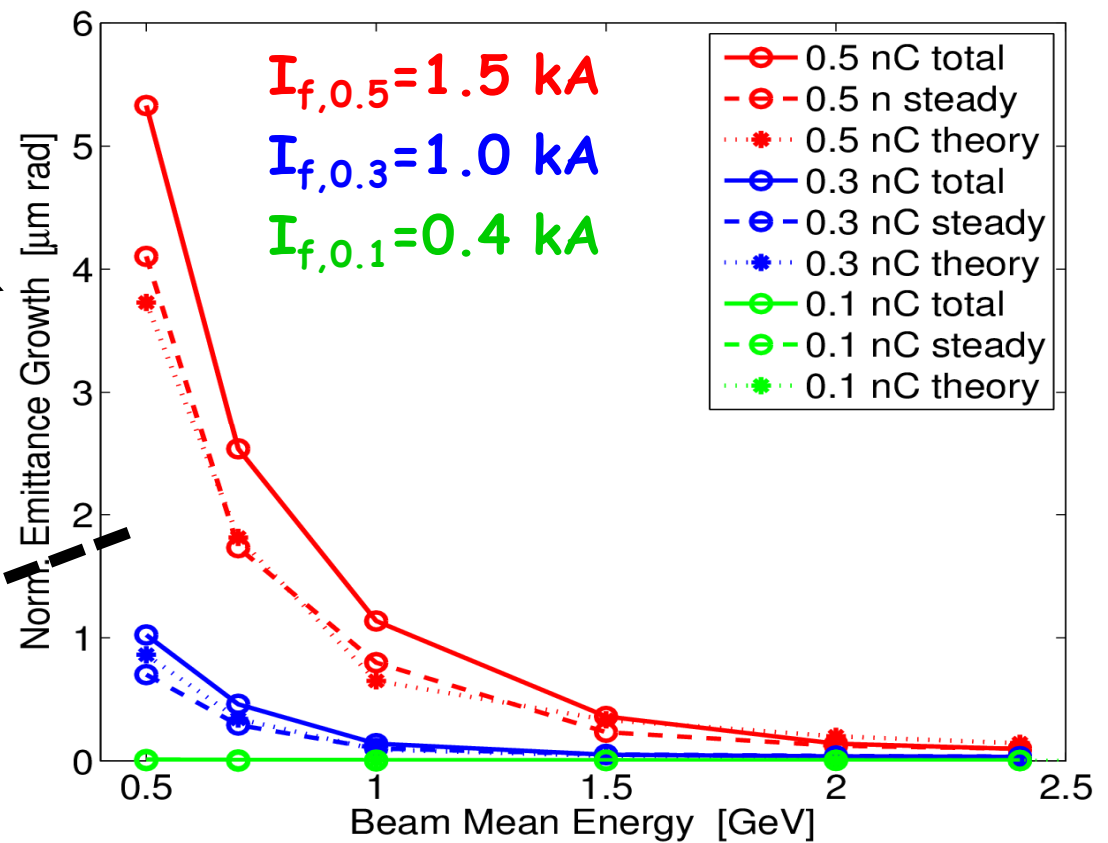
$$\text{sqrt}[(\varepsilon_{nx,f})^2 - (\varepsilon_{nx,0})^2],$$

and  $\varepsilon_{nx,0} = 0.8 \mu\text{m}$ .

We may achieve

$\Delta\varepsilon_{nx} \leq 0.1 \mu\text{m}$  for, e.g.:

- 100pC,  $E > 0.5 \text{ GeV}$
- 300pC,  $E > 1.0 \text{ GeV}$
- 500pC,  $E > 2.0 \text{ GeV}$



# Full Particle Tracking (Elegant)

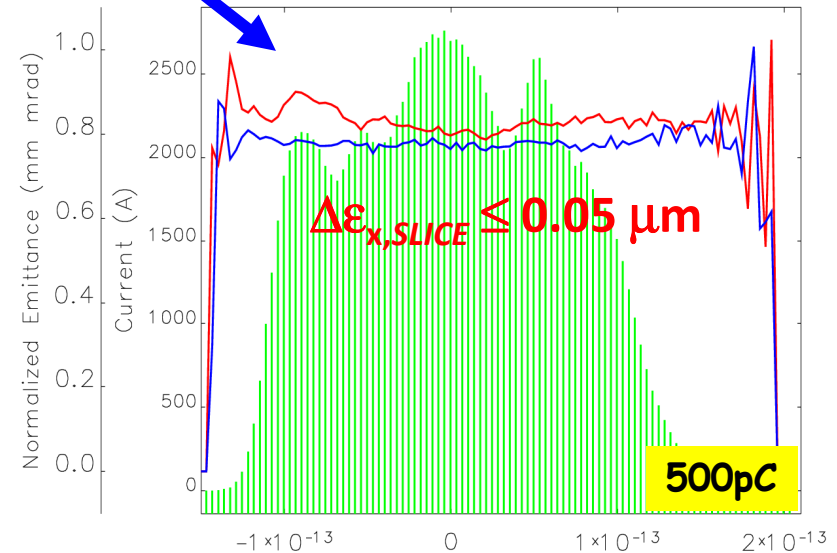
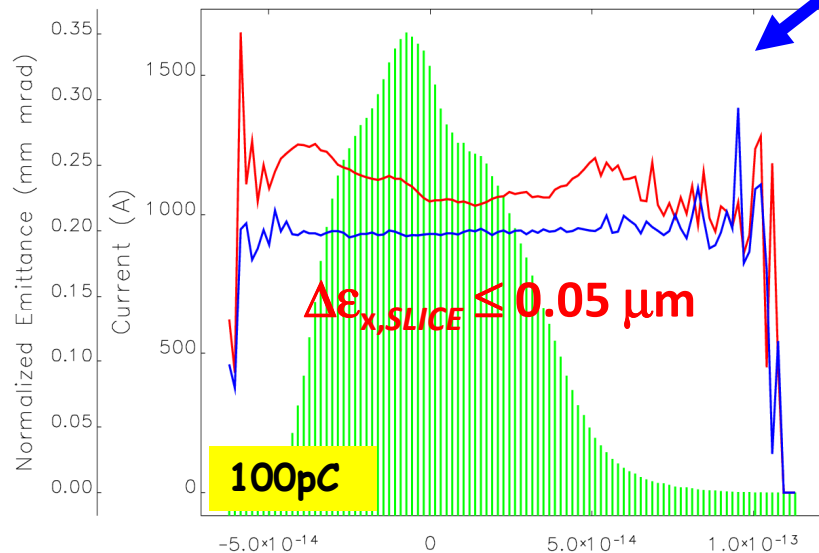
## □ Particle tracking now including:

- 3<sup>rd</sup> order transport matrices, ISR and CSR transient effects,
- CSR-induced microbunching from a quiet start and 5 M-particles
- Enlarged uncorrelated energy spread, as from a laser heater (30 keV rms)

## □ Two sets of beam parameters at 2.4 GeV, C=45:

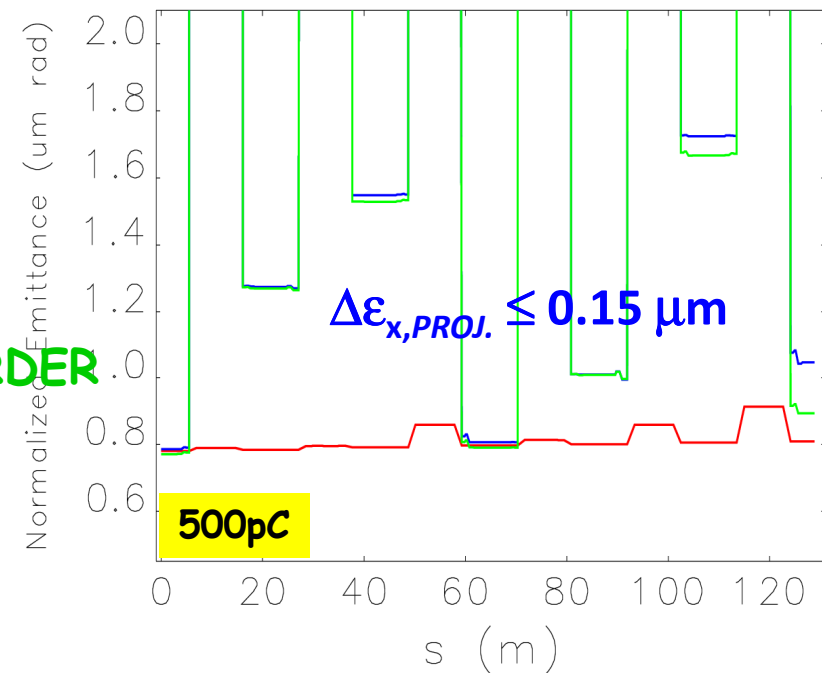
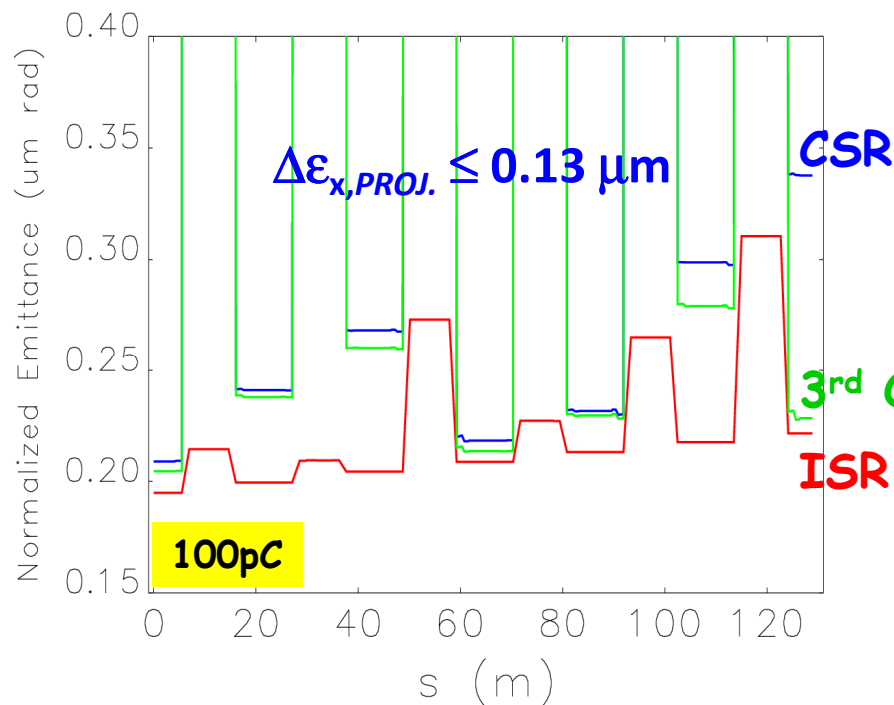
- $Q = 100 / 500 \text{ pC}$
- $I_f = 1.3 / 2.2 \text{ kA}$
- $\sigma_{\delta,0} = 0.1 / 0.4 \%$
- $\epsilon_{nx,0} = 0.2 / 0.8 \text{ } \mu\text{m rad}$

*Slice emittances and peak current*



# Nonlinear Dynamics

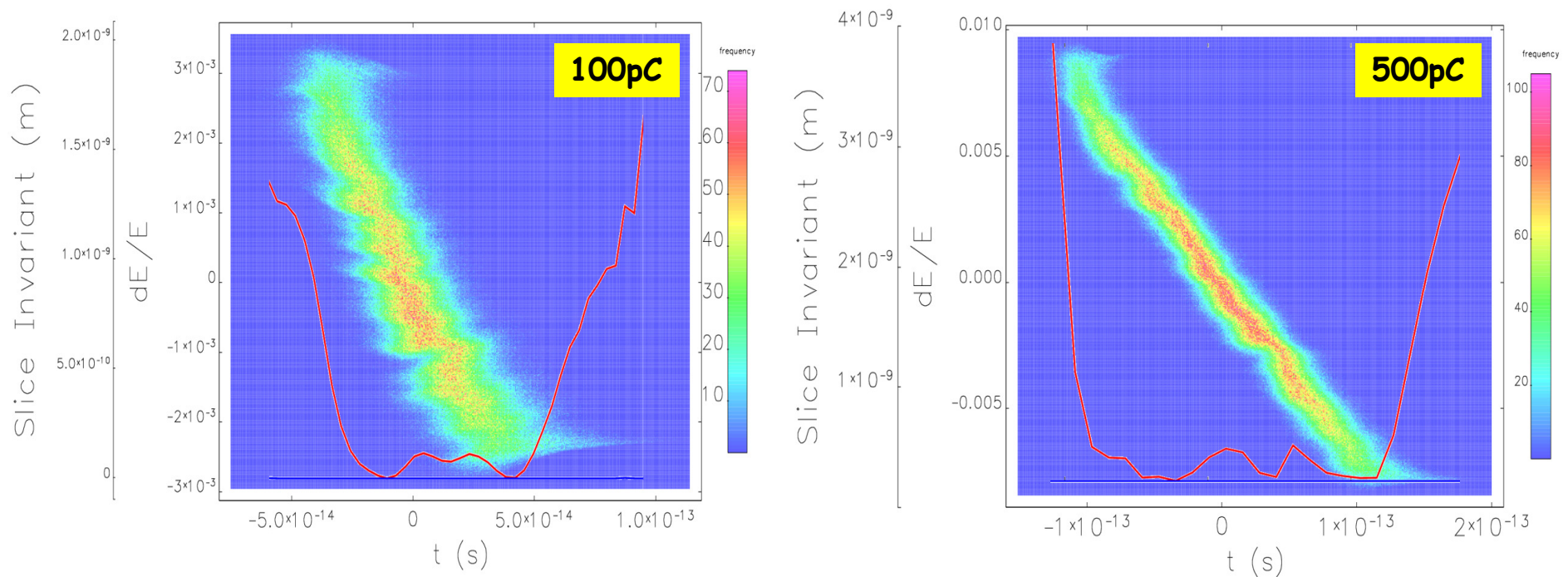
- Nonlinearities in the longitudinal phase space evolve during compression due to:
  - Incoming RF curvature,
  - $T_{566}$  of the DBA cells,
  - Nonlinear CSR-induced energy chirp.
- **24 sextupole magnets linearize the compression.** Strengths and positions optimized for minimizing chromatic aberrations (these are responsible for the emittance modulation along the line, see below).





# Microbunching Instability

- ❑ The effect of CSR-induced microbunching (MB) is damped by the initial beam heating. The strength of MB dynamics sounds over-estimated because:
  - E/I-modulations tend to smear as the # of particles increases from 0.1 to 5 M;
  - filtering suppresses  $\lambda_f \leq 1 \mu\text{m}$ , while MB appears at  $\lambda_f > 10 \mu\text{m}$ ;
  - transverse emittance smearing effect *not* included.
  
- ❑ *Accurate MB analysis is pending. See, e.g., C.-S. Tsai's talk (today, this session)*



# Conclusions & Outlook

- ✓ The extension of CSR-driven liner optics balance to the case of varying bunch length leads to a simple formula for a **periodic** system.
- ✓ The final **emittance estimate** is in reasonable agreement with 1-D tracking results (see also C.Hall's talk, this session).
- ✓ For a DBA-based 180° arc compressor, we expect a **"gain" ~ 10** in " $Q \times C / E$ ", w.r.t. the existing literature.
- Working plan:
  - more accurate **microbunching** instability analysis;
  - massive **numerical optimization** of the emittance vs. linear and nonlinear dynamics;
  - scaling to a (low energy) **compact ERL**, and to a (high energy) **single-pass** beamline.
- A proof-of-principle **experiment** may be possible at the FERMI Main e-Beam Dump line (two big dipoles and quads available at 1.5 GeV).

# Acknowledgements

A special thank to my good friends, collaborators, and often co-authors, M. Cornacchia and S. Spampinati:



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## *Thank You for Your attention*