

Linear Microbunching Gain Estimation Including CSR and LSC Impedances in Recirculation Machines

Cheng-Ying Tsai, Virginia Tech

David Douglas, Rui Li, and Chris Tennant, Jefferson Lab

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Stony Brook University, NY

Outline

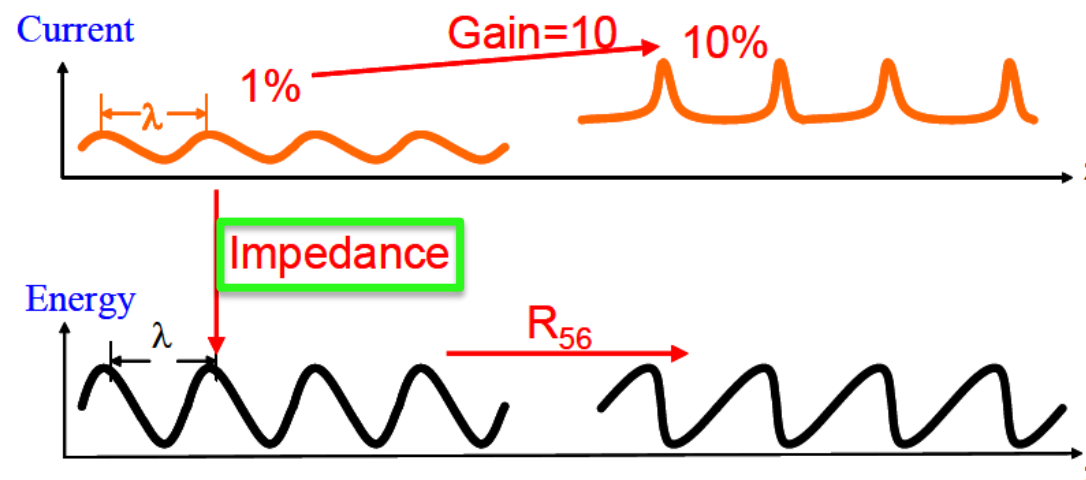
- Introduction and Motivation
 - Microbunching instability mechanism
 - What has been done and Why important in ERL
 - MEIC Circulator Cooling Ring (CCR) as an example
- Theoretical formulation
 - (Linear) Vlasov equation
 - Relevant collective effects: CSR and LSC
- Semi-analytical Vlasov solver
- Examples and Results
 - Two comparative high-energy transport/recirculation arcs
 - MEIC Circulator Cooling Ring
- Summary and Future work

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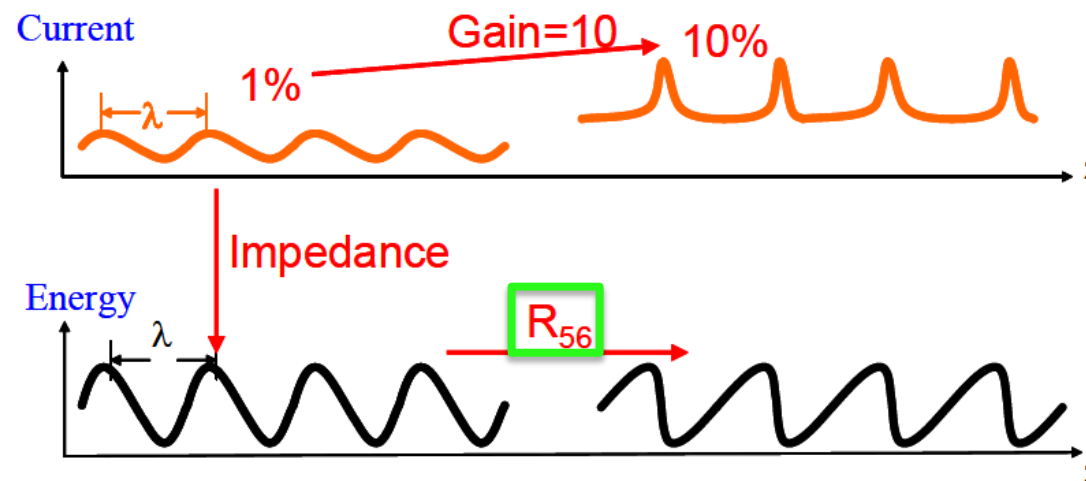
Introduction and Motivation: Microbunching instability mechanism

- An initial **density** modulation can induce **energy** modulation due to the presence of (high-frequency) impedance $Z(k)$, e.g. **LSC** or **CSR**.



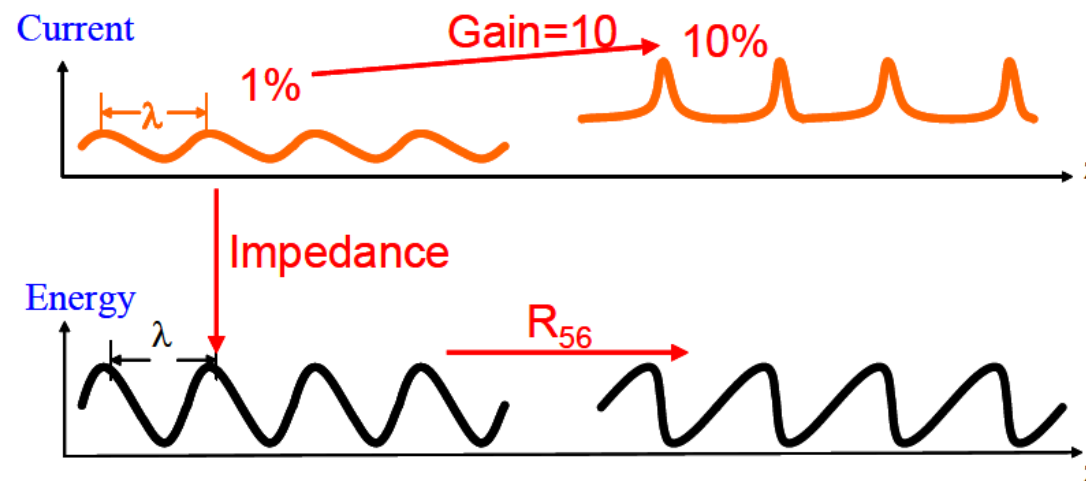
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- Such **energy** modulation can then convert to further **density** modulation via the momentum compaction R_{56} downstream and possibly induce emittance growth in the dispersive region.



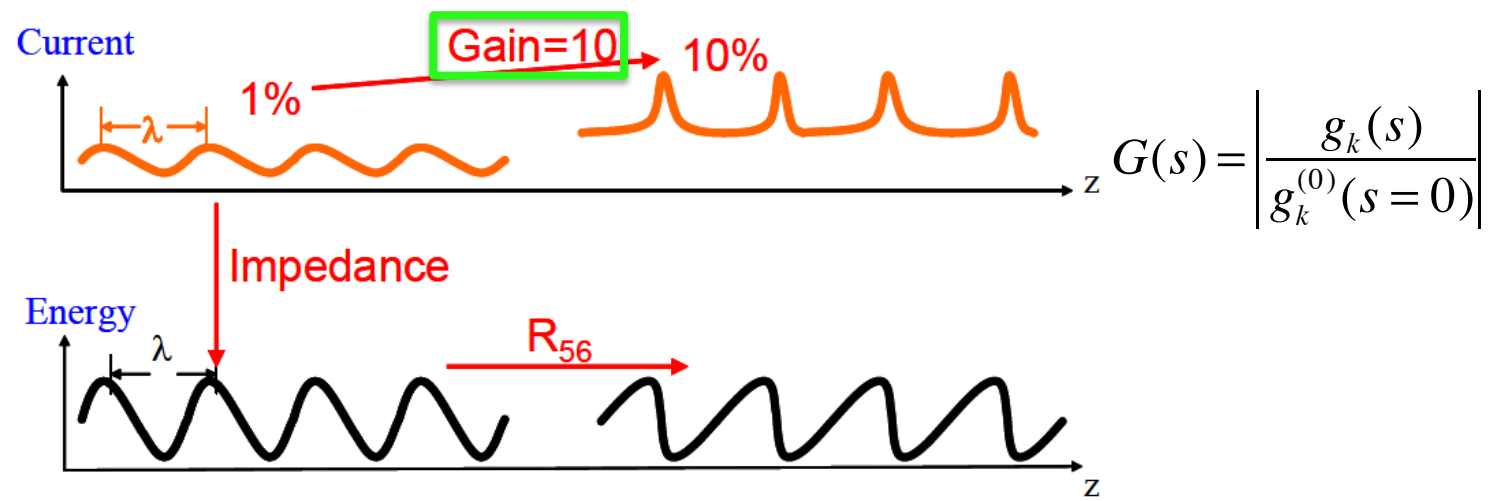
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What has been done

- Previous studies most focus on microbunching gain analysis for **bunch compressor chicanes**.

- Heifets, Stupakov, and Krinsky, PRST-AB 5, 064401 (2002)
- Huang and Kim, PRAT-AB 5, 074401 (2002)

- Vlasov-based analysis

- The **CSR-induced** microbunching gain is (relatively) low

- e.g. $G_f < 3$, for LCLS BC2

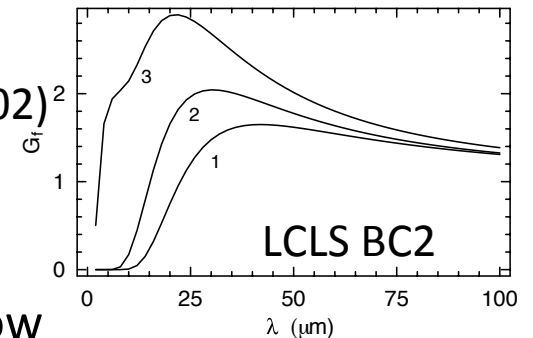
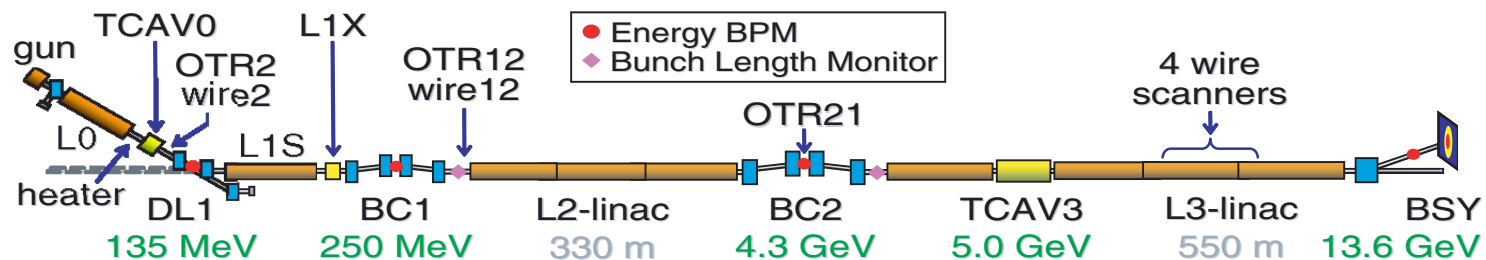
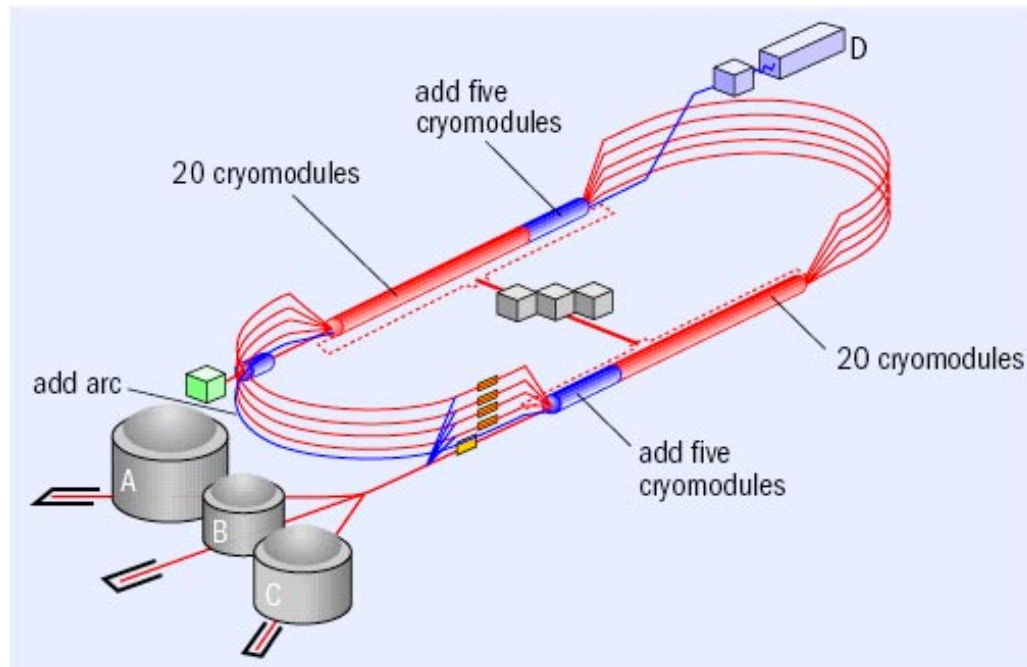


FIG. 3. Amplification factor G_f as a function of wavelength λ of the perturbation at the compressor entrance for various beam emittance and energy spread: (1) $\sigma_p = 3.0 \times 10^{-5}$, $\epsilon = 1 \mu\text{m}$; (2) $\sigma_p = 3.0 \times 10^{-5}$, $\epsilon = 0$; and (3) $\sigma_p = 3.0 \times 10^{-6}$, $\epsilon = 1 \mu\text{m}$.



PRST-AB 12, 030704 (2009)

Why important in ERLs



Even though CEBAF is not an ERL, but a recirculation linac, it is helpful to illustrate the important parts of ERL using this schematic layout.

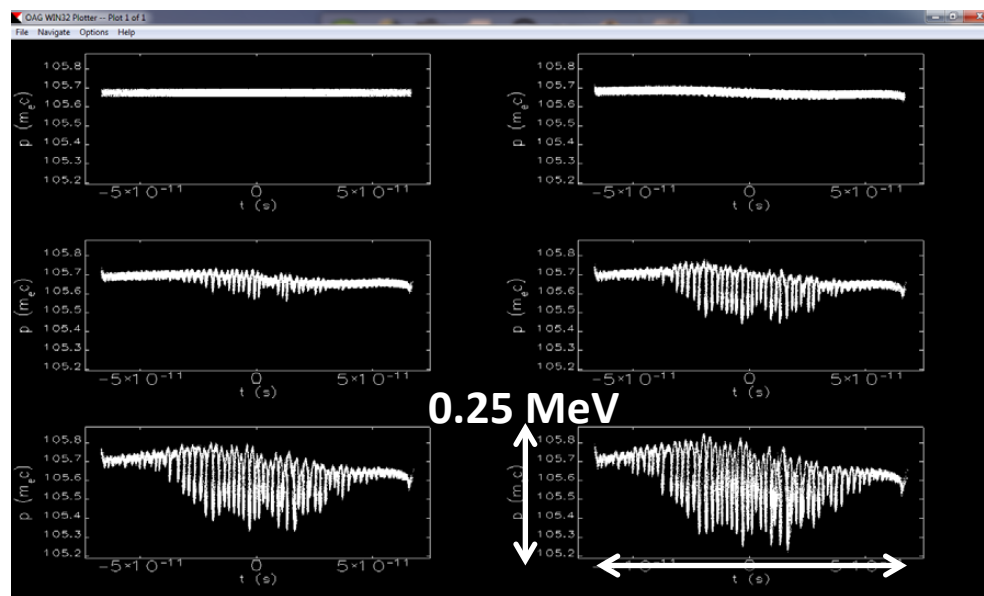
- However, in Energy Recovery Linac (ERL) or recirculation machines, microbunching instability can be a special concern, because:
 - **low energy merger (CSR & LSC)**
 - **spreaders/recombiners (CSR)**
 - **long transport/recirculation arcs (multiple bends) (CSR)**

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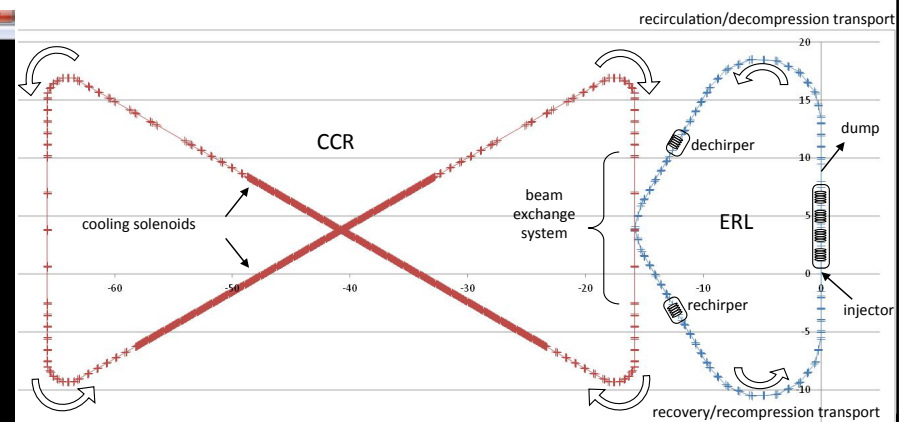
Introduction and Motivation: Microbunching instability in MEIC CCR

- Due to relatively **low energy** (~ 55 MeV) and **high bunch charge** (~ 2 nC), MEIC CCR is potentially subject to CSR-induced microbunching instability.



0.25 MeV

100 ps (~ 3 cm)

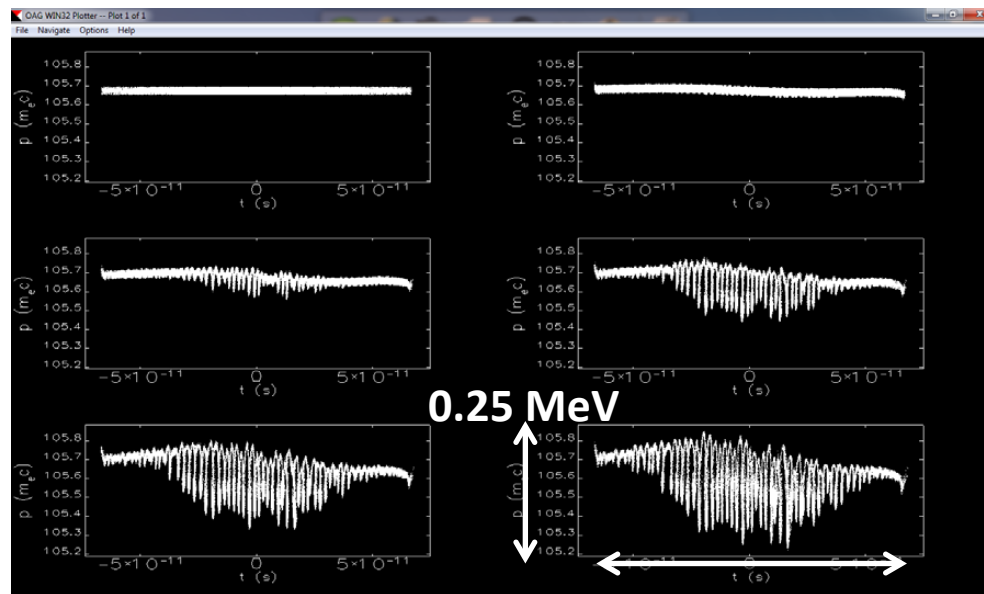


MEIC Design Report, 2012

	1 Turn	2 Turns	3 Turns	4 Turns	5 Turns
ϵ_x (mm-mrad)	2.9	3.1	3.8	4.5	5.1
ϵ_y (mm-mrad)	2.9	2.9	3.0	3.1	3.2
σ_t (ps)	29.33	29.31	29.28	29.24	29.19
$\sigma_{\Delta E/E}$ (%)	0.012	0.027	0.066	0.096	0.117

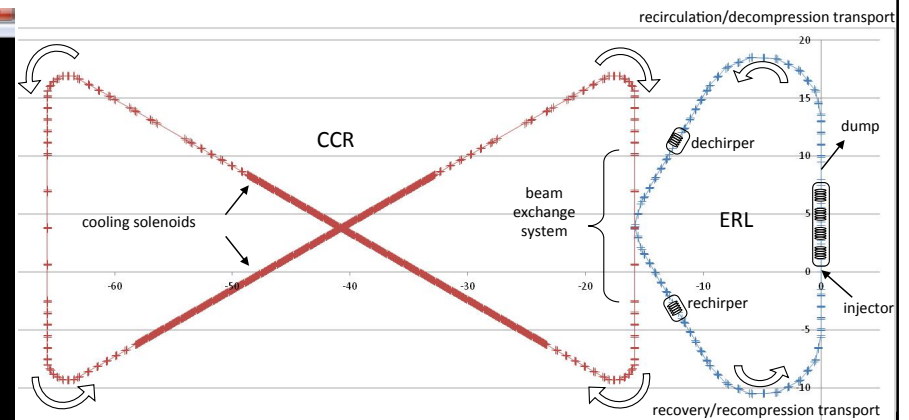
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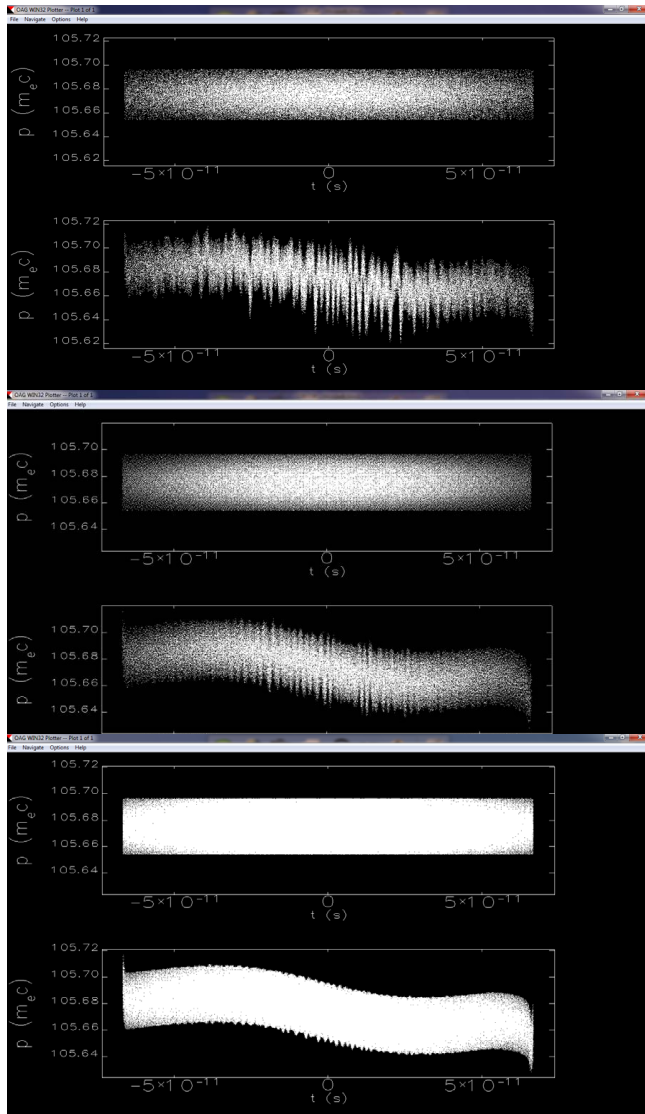
MEIC Design Report, 2012

Note:

- In the **ELEGANT** tracking simulation, **100,000** macroparticles with quiet start are used.

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Introduction and Motivation: Numerical challenges from particle tracking

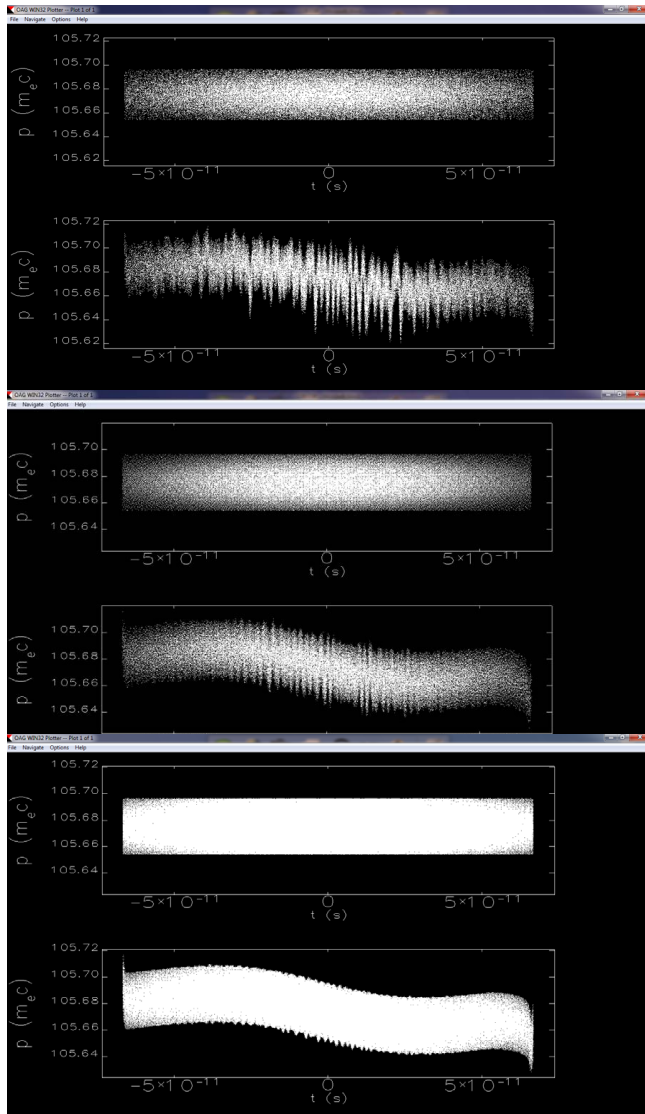


100K macroparticles, **without** quiet start

100K macroparticles, **with** quiet start

1000K macroparticles, **with** quiet start

Introduction and Motivation: Numerical challenges from particle tracking



- ◆ Particle tracking subject to initial **numerical noise**.
- ◆ **Increase** number of particles (with specialized initial quiet start algorithm)
 - ➔ **reduce** numerical noise
- ◆ How much of the microbunching structure contributed from **numerical** effect? How much from **physical** effect?
- ◆ To better design a machine, we would like to know the system **gain** to microbunching effects
- ◆ More and more macroparticles
 - ➔ more computation **time consuming**
 - ➔ difficult to do systematic study and/or lattice design optimization
- ◆ Necessary to develop an alternative model for gain analysis that is more robust and also serves to benchmark particle tracking results

Introduction and Motivation: Summary of our extended work

- Derived from **Vlasov** equation [Heifets et al. & Huang and Kim, 2002]
- **Extend** to include both **horizontal** and **vertical** bending
 - spreader/recombiners
- **Extend** to adopt a **general** linear lattice
 - for generic transport arc design (multiple bends)
- **Extend** to include more relevant collective effects, e.g.
 - non-ultrarelativistic **CSR** impedance for low energy beamline
 - transient **CSR** effects
 - 1-D **(L)SC** effect

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Vlasov formulation and model assumptions

- Linearization of Vlasov equation:
 - Heifets, Stupakov, and Krinsky (HSK), PRST-AB **5**, 064401 (2002); 129902 (2002).
 - Huang and Kim (HK), PRST-AB **5**, 074401 (2002); 129903 (2002).

$$\frac{\partial f}{\partial s} + \left(\frac{dz}{ds} \right) \frac{\partial f}{\partial z} + \left(\frac{d\delta}{ds} \right) \frac{\partial f}{\partial \delta} + \left(\frac{dx}{ds} \right) \frac{\partial f}{\partial x} + \left(\frac{d\theta}{ds} \right) \frac{\partial f}{\partial \theta} = 0$$

$$\frac{dz}{ds} = -\frac{x}{\rho}$$

$$\frac{d\delta}{ds} = -\frac{r_e}{\gamma} \int_{-\infty}^{\infty} dz' W_{\parallel}(z-z', s) n(z', s)$$

$$\frac{dx}{ds} = \theta$$

$$\frac{d\theta}{ds} = -k_{\beta}^2(s)x + \frac{\delta}{\rho}$$

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$$\frac{dy}{ds} = \theta_y$$

$$\frac{d\theta_x}{ds} = -k_{\beta x}^2(s)x + \frac{\delta}{\rho_x}$$

$$\frac{d\theta_y}{ds} = -k_{\beta y}^2(s)y + \frac{\delta}{\rho_y}$$

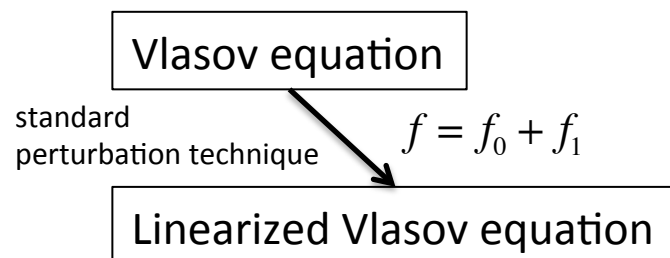
Including **vertical** bending is particularly useful for recirculation machines because such lattices usually contain **spreader** and **recombiner** parts.

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- Study this problem **in frequency domain**
 - the modulation of a bunch is characteristic of the (complex) bunching factor, i.e. Fourier spectral component of a bunch distribution
- Track the evolution of the **bunching factor**
- Take into account the relevant collective effects (**impedances**)

$$Z(k, s) = \int_0^\infty d\zeta W_{\parallel}(\zeta, s) e^{-ik\zeta}$$

$$g_k(s) = \int d\mathbf{X} f_1 e^{-ikz}$$

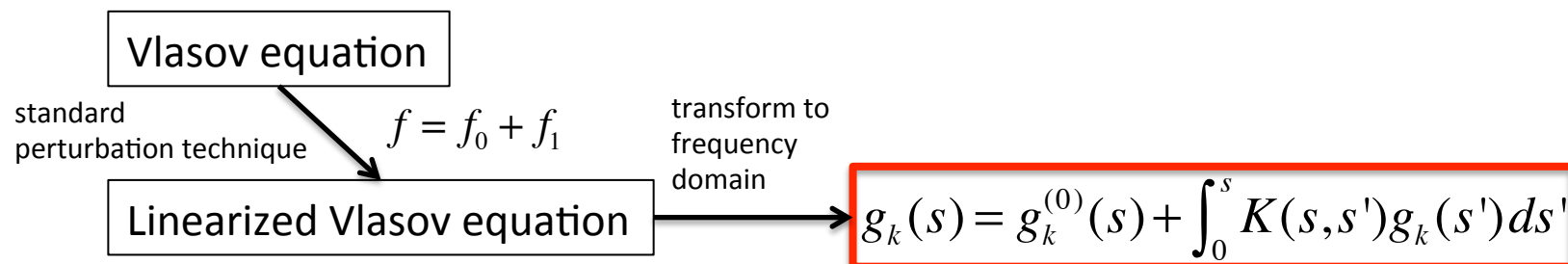


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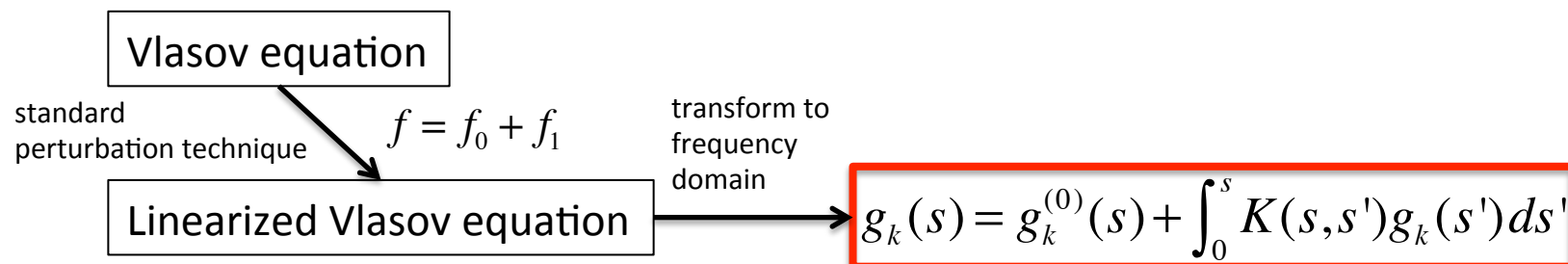
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$$G(s) = \left| \frac{g_k(s)}{g_k^{(0)}(s=0)} \right|$$



Summary of mathematical formulas

◆ Volterra integral equation:

$$g_k(s) = g_k^{(0)}(s) + \int_0^s K(s, s') g_k(s') ds'$$

$$K(s, s') = \frac{ik}{\gamma} \frac{I(s)}{I_A} C(s') R_{56}(s' \rightarrow s) Z(kC(s'), s') \times [\text{Landau damping}]$$

$$[\text{Landau damping}] = \exp \left\{ \frac{-k^2}{2} \left[\varepsilon_{x0} \left(\beta_{x0} R_{51}^2(s, s') + \frac{R_{52}^2(s, s')}{\beta_{x0}} \right) + \varepsilon_{y0} \left(\beta_{y0} R_{53}^2(s, s') + \frac{R_{54}^2(s, s')}{\beta_{y0}} \right) + \sigma_\delta^2 R_{56}^2(s, s') \right] \right\}$$

$$R_{56}(s' \rightarrow s) = R_{56}(s) - R_{56}(s') + R_{51}(s') R_{52}(s) - R_{51}(s) R_{52}(s') + R_{53}(s') R_{54}(s) - R_{53}(s) R_{54}(s')$$

$$R_{5i}(s, s') = C(s) R_{5i}(s) - C(s') R_{5i}(s') \text{ for } i = 1, 2, 3, 4, 6$$

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$$R_{5i}(s, s') = C(s) R_{5i}(s) - C(s') R_{5i}(s') \text{ for } i = 1, 2, 3, 4, 6$$

Physical meaning of $K(s, s')$:

initial **density** modulation at s' induces **energy** modulation by collective effects
induced **energy** modulation at s' converts to further **density** modulation via R_{56}

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Impedance models

- Coherent synchrotron radiation (CSR) effects:
 - **steady-state** CSR: ultrarelativistic and non-ultrarelativistic
 - **transient** CSR: entrance and exit
 - parallel-plate **shielded** CSR
- Longitudinal space charge (LSC) effects:
 - on-axis model
 - averaged (over radial dependence) model
 - transverse axisymmetric Gaussian model
- Summarized in [TUICLH2034](#) (this workshop)

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GUI: volterra_mat

- Read input information from **ELEGANT**
- Apply for **general** linear lattice
- Output **gain curves**
- **Fast** (compared with particle tracking)
- **GUI** (graphical user interface)

The screenshot displays the GUI for volterra_mat, divided into several sections:

- INPUT PARAMETERS - Beam (read from ELEGANT):** Includes fields for beam energy (GeV), initial beam current (A), compression factor, normalized horizontal and vertical emittance (um), rms energy spread, initial horizontal and vertical beta functions (m), initial horizontal and vertical alpha functions, and chirp parameter (m⁻¹).
- Lattice:** Fields for start position (m) and end position (m).
- Scan parameter:** Includes a small plot of G(s) vs s (m) and fields for lambda_start01, lambda_end01, scan_num01, lambda_start02, lambda_end02, scan_num02, and mesh_num.
- ADDITIONAL SETTINGS:** A series of checkboxes and dropdowns for advanced options like 'calculate iterative solutions?', 'include steady-state CSR in bends?', and 'include LSC in drifts?'.
- OUTPUT SETTING:** A section for selecting which plots to generate, such as 'Plot lattice function vs. s', 'Plot compression factor C(s)', 'Plot peak current evolution I_b(s)', 'Plot lattice quilt pattern R56(s->s)', 'plot density gain function G(s)', 'plot density gain function G(s) with lattice', 'plot density gain spectrum G(lambda)', 'Plot gain map G(s,lambda)', 'Plot energy modulation function', 'Plot energy modulation function with lattice', and 'Plot energy modulation spectrum'.
- Run:** A 'Run' button and a 'Note: to terminate, press Ctrl+C'.

The plot on the right shows the gain function G(s) versus position s (m). The y-axis ranges from 0 to 1.5, and the x-axis ranges from 0 to 20. The plot shows a step-like function that starts at approximately 0.8, drops to 0.4 at s ≈ 11, and then rises to 1.5 at s ≈ 21.

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- INPUT PARAMETERS - Beam (read from ELEGANT):** A red-bordered box containing fields for beam energy (GeV), initial beam current (A), compression factor, normalized horizontal and vertical emittance (um), rms energy spread, initial horizontal and vertical beta functions (m), initial horizontal and vertical alpha functions, and chirp parameter (m⁻¹).
- Lattice:** Fields for start position (m) and end position (m).
- Scan parameter:** A small plot showing G(s) vs s (m) with markers for lambda_start01, lambda_end01, lambda_start02, lambda_end02, scan_num01, scan_num02, scan_num03, and scan_num04.
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- Run:** A 'GO HOKIES!!!' button and a note to terminate with Ctrl+C.

The plot on the right shows the gain G(s) as a function of position s (m). The gain starts at approximately 0.8, remains constant until s ≈ 11 m, then drops to about 0.4, and remains constant until s ≈ 21 m, where it rises sharply to 1.5.

GUI: volterra_mat

- Read input information from **ELEGANT**
- Apply for general
- Output gain curve
- Fast (compared with)
- GUI (graphical user interface)

<Student Version> : GUI_volterra

INPUT PARAMETERS

Beam (read from ELEGANT)

beam energy (GeV)	4.54
initial beam current (A)	480
compression factor	8.3187
normalized horizontal emittance (um)	1
normalized vertical emittance (um)	1
rms energy spread	3e-06
initial horizontal beta function (m)	105
initial vertical beta function (m)	22
initial horizontal alpha function	5
initial vertical alpha function	0
chirp parameter (m ⁻¹) (z < 0 for bunch head)	39.83

Lattice

start position (m)	end position (m)
0	22.099

Scan parameter

ADDITIONAL SETTING

on

if yes a

if y

OUTPUT SETTING

Plot

GUI: volterra_mat

- Set up additional numerical parameters
- Apply for general linear lattice
- Output gain curves
- Fast (compared with particle tracking)
- GUI (graphical user interface)

The screenshot displays the GUI for the volterra_mat software, divided into several sections:

- INPUT PARAMETERS - Beam (read from ELEGANT):** Includes fields for beam energy (GeV), initial beam current (A), compression factor, normalized horizontal/vertical emittance (um), rms energy spread, initial horizontal/vertical beta function (m), initial horizontal/vertical alpha function, and chirp parameter (m⁻¹).
- Lattice:** Fields for start position (m) and end position (m).
- Scan parameter:** A sub-section with a small plot of G(s) vs s(m) and fields for lambda_start01, lambda_end01, scan_num01, lambda_start02, lambda_end02, scan_num02, and mesh_num.
- ADDITIONAL SETTINGS:** A series of checkboxes and dropdowns for advanced options like 'calculate iterative solutions?', 'include CSR in bends?', and 'include LSC in drifts?'.
- OUTPUT SETTING:** A section with checkboxes for various plots such as 'Plot lattice function vs. s', 'Plot compression factor C(s)', 'Plot peak current evolution I_b(s)', 'Plot lattice quilt pattern R56(s->s)', 'plot density gain function G(s)', 'plot density gain function G(s) with lattice', 'plot density gain spectrum G(lambda)', 'Plot gain map G(s,lambda)', 'Plot energy modulation function', 'Plot energy modulation function with lattice', and 'Plot energy modulation spectrum'.
- Run:** A 'Run' button and a 'Note: to terminate, press Ctrl+C'.

The plot on the right shows the gain function G(s) versus position s (m). The y-axis ranges from 0 to 1.5, and the x-axis ranges from 0 to 20. The plot shows a step-like function that starts at approximately 0.8, drops to 0.4 at s ≈ 11, and then rises back to 1.5 at s ≈ 21.

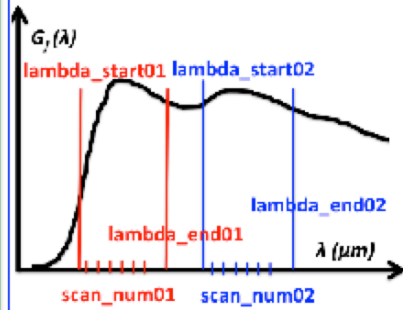
- Set up additional numerical parameters
- Apply for general linear lattice
- Output gain curves
- Fast (compared with particle tracking)
- GUI (graphical user interface)

initial beam current (A)	480
compression factor	8.3187
normalized horizontal emittance (um)	1
normalized vertical emittance (um)	1
rms energy spread	3e-06
initial horizontal beta function (m)	105
initial vertical beta function (m)	22
initial horizontal alpha function	5
initial vertical alpha function	0
chirp parameter (m ⁻¹) (z < 0 for bunch head)	39.83

Lattice

start position (m)	end position (m)
0	22.099

Scan parameter



lambda_start01 (um)	1
lambda_end01 (um)	100
scan_num01	50
lambda_start02 (um)	1
lambda_end02 (um)	1
scan_num02	0
mesh_num	400

GUI: volterra_mat

- Apply relevant impedance models
- Apply for general linear lattice
- Output gain curves
- Fast (compared with particle tracking)
- GUI (graphical user interface)

The screenshot displays the GUI for the volterra_mat software, divided into several panels:

- INPUT PARAMETERS - Beam (read from ELEGANT):** Contains fields for beam energy (4.54 GeV), initial beam current (480 A), compression factor (8.3187), normalized horizontal and vertical emittances (1 μm), rms energy spread (3e-08), and initial horizontal and vertical beta functions (105 m, 22 m).
- Lattice:** Fields for start position (0 m) and end position (22.099 m).
- Scan parameter:** Includes a plot of G(s) vs s(m) and fields for lambda_start01, lambda_end01, scan_num01, lambda_start02, lambda_end02, scan_num02, and mesh_num (400).
- ADDITIONAL SETTINGS:** A central panel with various checkboxes and dropdowns for advanced options like "calculate iterative solutions?", "include CSR in bends?", and "include LSC in drifts?".
- OUTPUT SETTING:** A panel with checkboxes for "Plot lattice function vs. s", "Plot compression factor C(s)", "Plot peak current evolution I_b(s)", "Plot lattice quilt pattern R56(s->s)", "plot density gain function G(s)", "plot density gain function G(s) with lattice", "plot density gain spectrum G(lambda)", "Plot gain map G(s,lambda)", "Plot energy modulation function", "Plot energy modulation function with lattice", and "Plot energy modulation spectrum".
- Plot:** A graph showing G(s) on the y-axis (ranging from 0 to 1.5) versus s (m) on the x-axis (ranging from 0 to 20). The plot shows a step function that starts at approximately 0.8, drops to 0.4 at s ≈ 11, and then rises back to 1.5 at s ≈ 21.

At the bottom right, there are logos for VirginiaTech and Jefferson Lab, and a note: "Note: if want to edit/save plots, use 'OUTPUT SETTING-Plot' in GUI_volterra".

GUI: volterra_mat

- Apply relevant impedance models

<Student Version> : GUI_volterra

Apply for general linear lattice

Output gain curves

Fast (compared with particle tracking)

GUI (graphical user interface)

Energy (GeV)	4.54	calculate iterative solutions? (1-Yes, 0-No)	0
Current (A)	480	if yes above, calculate stage gain coefficient d_m? (1-Yes, 0-No)	0
Dispersion factor	8.3187	only calculate stage gain spectrum? (can speed up calculation) (1-Yes, 0-No)	0
Distance (um)	1	include steady-state CSR in bends? (1-Yes, 0-No)	1
Distance (um)	1	if yes above, specify ultrarelativistic or non-ultrarelativistic model? (UR:1, NUR:2)	1
Energy spread	3e-06	want to include possible CSR shielding effect? (1-Yes, 0-No)	0
Function (m)	105	if yes above, specify the full pipe height in cm	1e+50
Function (m)	22	include transient CSR in bends? (1-Yes, 0-No)	0
Sha function	5	include CSR in drifts? (1-Yes, 0-No)	0
Sha function	0	include LSC in drifts? (1-Yes, 0-No)	0
unch head)	39.83	if yes above, specify a model? (1: on-axis, 2: ave, 3: axisymmetric Gaussian)	-1
end position (m)	22.099	include any RF element in the lattice? (1-Yes, 0-No)	0
		if yes above, include linac geometric impedance? (1-Yes, 0-No)	0
		longitudinal z distribution? (1-coasting, 2-Gaussian)	1
		calculate energy modulation function? (1-Yes, 0-No)	0
		calculate energy modulation spectrum? (1-Yes, 0-No)	0 36

GUI: volterra_mat

- Read input information from **ELEGANT**
- Apply for **general** linear lattice (not shown here)
- Output gain curves
- Fast (compared with particle tracking)
- **GUI** (graphical user interface)

The screenshot displays the GUI for the volterra_mat software, divided into several sections:

- INPUT PARAMETERS - Beam (read from ELEGANT):** Includes fields for beam energy (GeV), initial beam current (A), compression factor, normalized horizontal and vertical emittance (um), rms energy spread, initial horizontal and vertical beta functions (m), initial horizontal and vertical alpha functions, and chirp parameter (m⁻¹).
- Lattice:** Fields for start position (m) and end position (m).
- Scan parameter:** Includes a plot of G(s) vs s (m) and fields for lambda_start01, lambda_end01, scan_num01, lambda_start02, lambda_end02, scan_num02, and mesh_num.
- ADDITIONAL SETTINGS:** A series of checkboxes for various simulation options such as "calculate iterative solutions?", "include CSR in bends?", and "include LSC in drifts?".
- OUTPUT SETTING:** A section for selecting which plots to generate, including "Plot lattice function vs. s", "Plot compression factor C(s)", "Plot peak current evolution I_b(s)", "Plot lattice quilt pattern R56(s->s)", "plot density gain function G(s)", "plot density gain function G(s) with lattice", "plot density gain spectrum G(lambda)", "Plot gain map G(s,lambda)", "Plot energy modulation function", "Plot energy modulation function with lattice", and "Plot energy modulation spectrum".
- Run:** A "GO HOKIES!!!" button and a note to terminate with Ctrl+C.

The plot on the right shows the gain function G(s) versus position s (m). The y-axis ranges from 0 to 1.5, and the x-axis ranges from 0 to 20. The plot shows a step-like function that starts at approximately 0.8, drops to 0.4 at s ≈ 11, and then rises back to 1.5 at s ≈ 21.

GUI: volterra_mat

- Read input information from **ELEGANT**
- Apply for **general linear lattice**
- **Output capabilities**
- **Fast** (compared with particle tracking)
- **GUI** (graphical user interface)

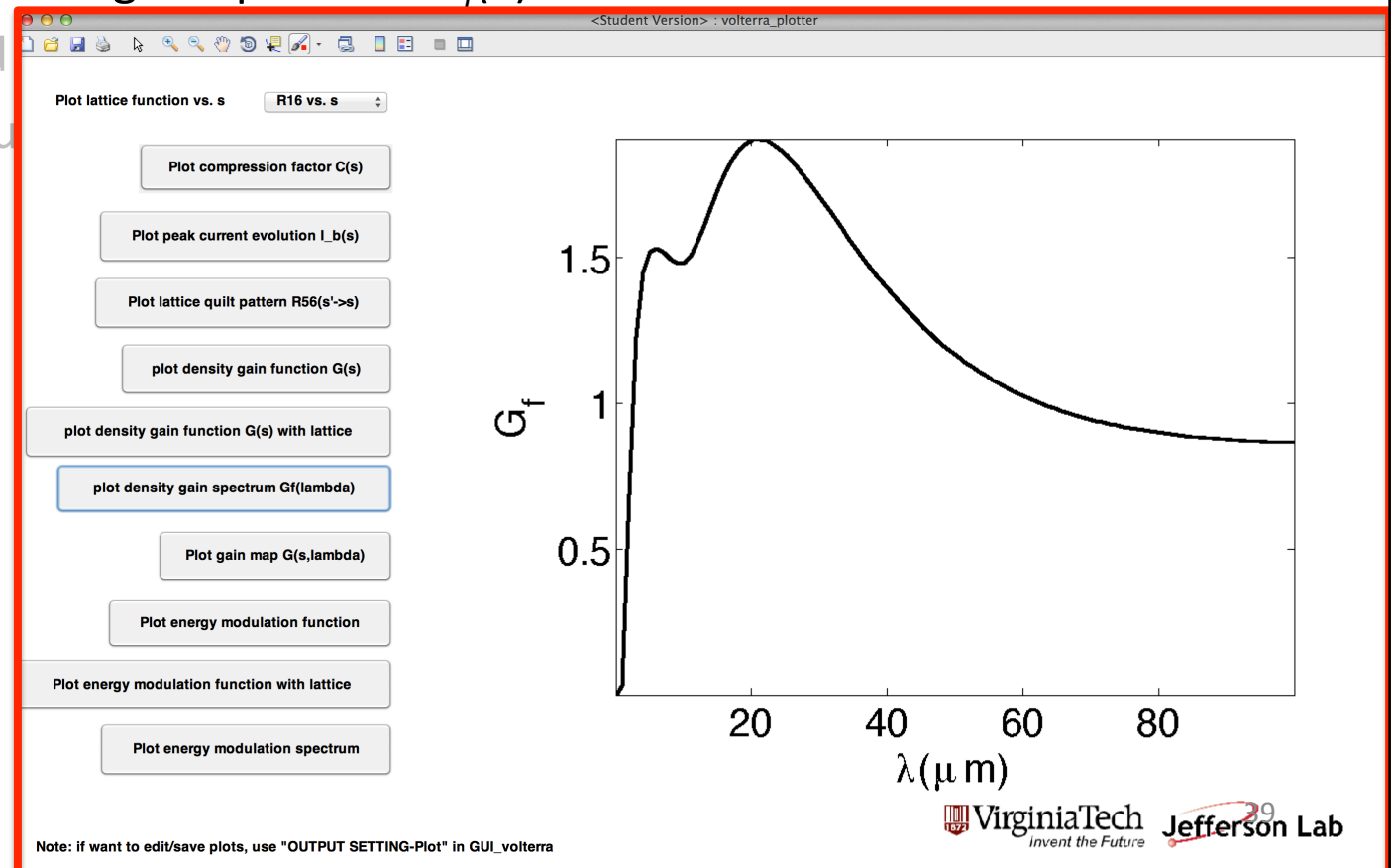
The screenshot displays the GUI for 'volterra_mat' in two windows. The left window, titled '<Student Version> - GUI_volterra', contains several sections:

- INPUT PARAMETERS - Beam (read from ELEGANT):** Includes fields for beam energy (GeV), initial beam current (A), compression factor, normalized horizontal and vertical emittance (um), rms energy spread, initial horizontal and vertical beta functions (m), initial horizontal and vertical alpha functions, and chirp parameter (m^-1).
- Lattice:** Fields for start position (m) and end position (m).
- Scan parameter:** A plot of G(s) vs s (m) with markers for lambda_start01, lambda_end01, lambda_start02, lambda_end02, scan_num01, scan_num02, and mesh_num.
- ADDITIONAL SETTINGS:** A list of checkboxes for various simulation options like 'calculate iterative solutions?', 'include CSR in bends?', and 'include LSC in drifts?'.
- OUTPUT SETTING:** A list of checkboxes for output options like 'plot lattice functions', 'plot beam current evolution', and 'plot energy modulation function'.
- Run:** A 'GO HOKIES!!!' button and a 'Note: to terminate, press Ctrl+C'.

The right window, titled '<Student Version> - volterra_plotter', shows a plot of G(s) vs s (m). The plot has a y-axis from 0 to 1.5 and an x-axis from 0 to 20. The curve starts at G ≈ 0.8, drops to G ≈ 0.4 at s ≈ 11, and then rises to G ≈ 1.5 at s ≈ 21. Below the plot are several buttons for different plot types, such as 'Plot compression factor C(s)', 'Plot peak current evolution I_b(s)', and 'Plot energy modulation function'.

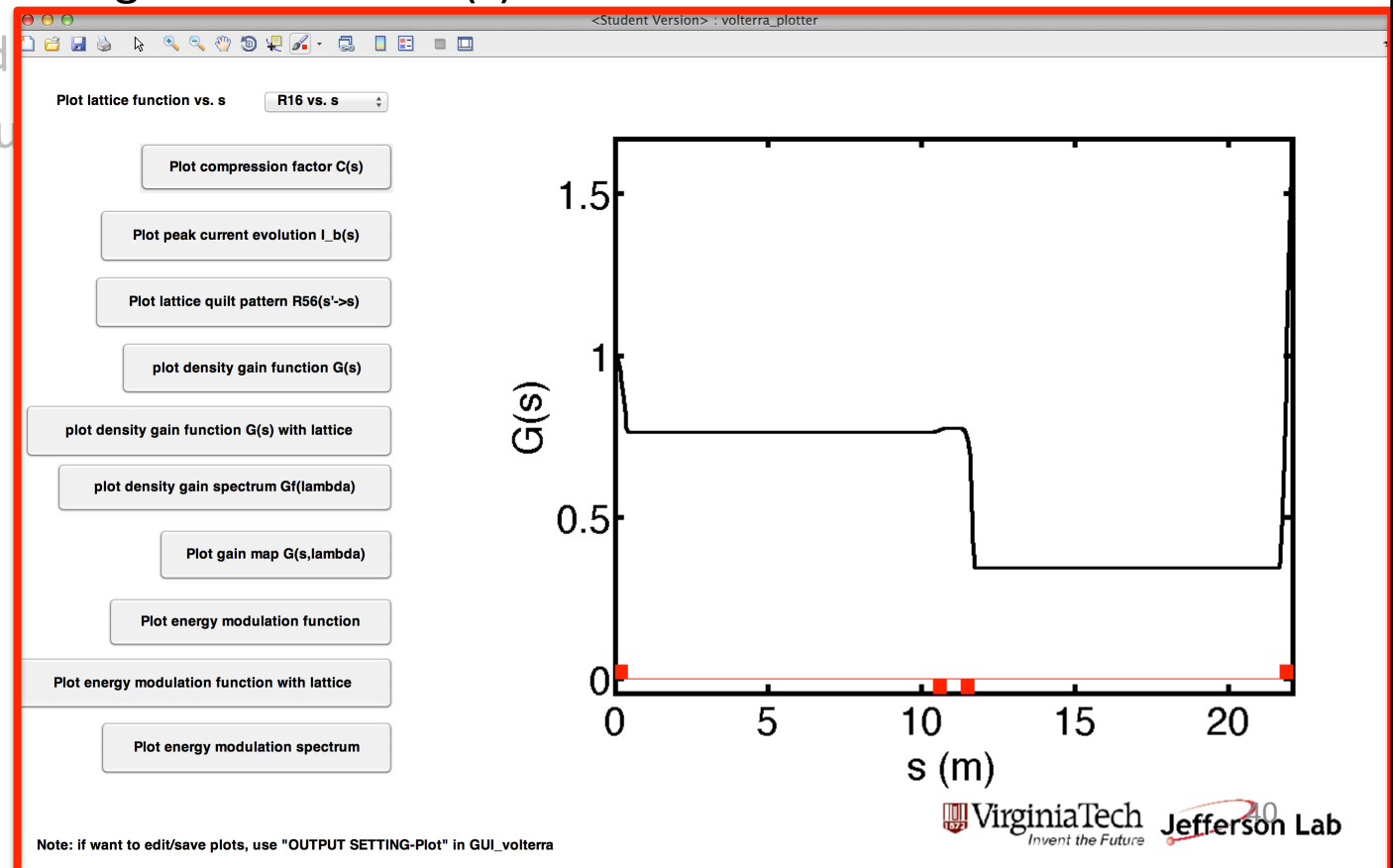
GUI: volterra_mat

- Read input information from **ELEGANT**
- Apply for **general** linear lattice
- Output **capabilities**: gain spectrum $G_f(\lambda)$
- **Fast** (compared to other tools)
- **GUI** (graphical user interface)



GUI: volterra_mat

- Read input information from **ELEGANT**
- Apply for **general** linear lattice
- Output **capabilities**: gain functions $G(s)$
- **Fast** (compared to other tools)
- **GUI** (graphical user interface)



GUI: volterra_mat

- Read input information from **ELEGANT**
- Apply for **general** linear lattice
- Output **capabilities**
- **Fast** (compared with particle tracking)
- **GUI** (graphical user interface)

The screenshot displays the GUI for volterra_mat, divided into several sections:

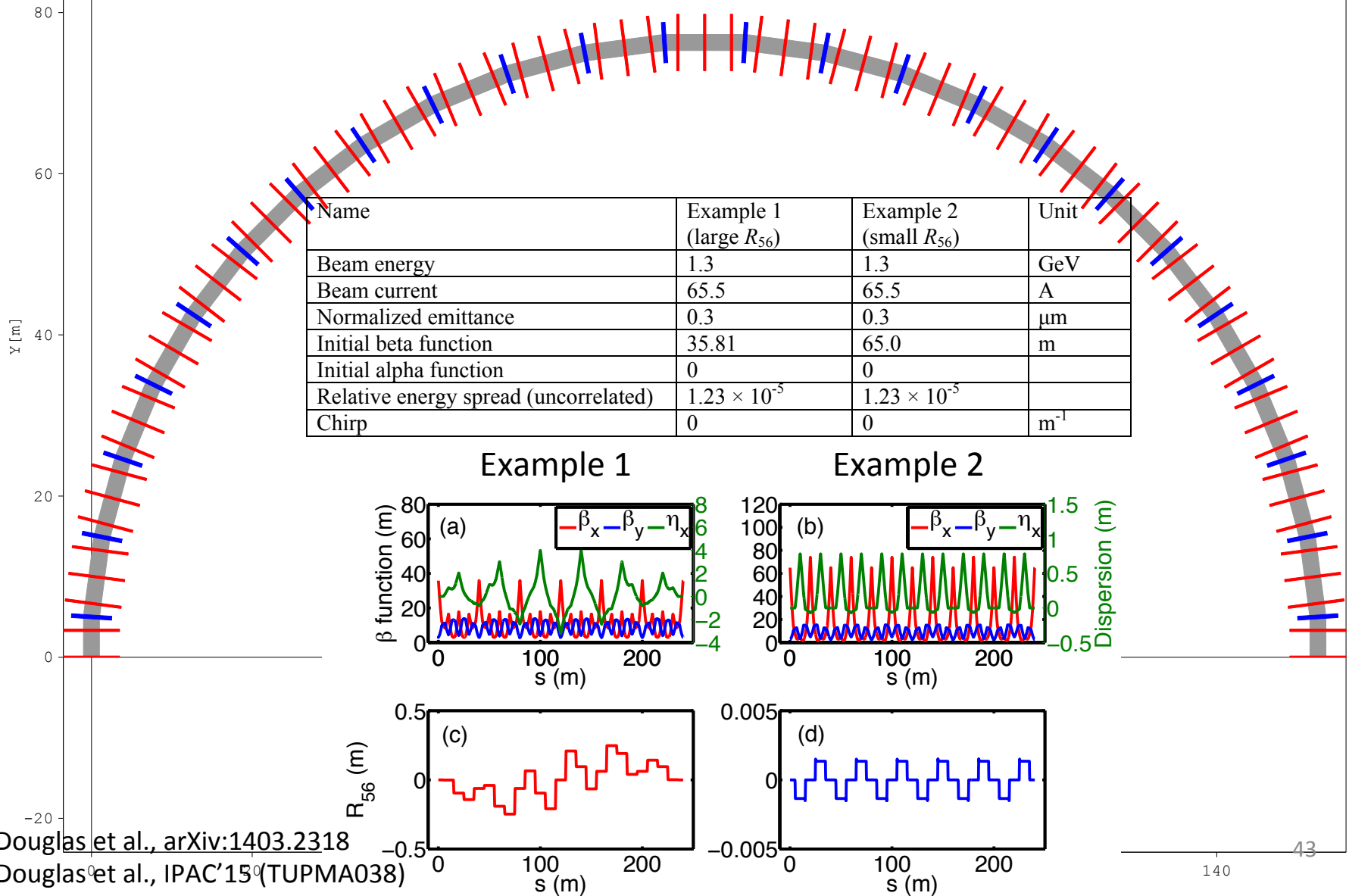
- INPUT PARAMETERS - Beam (read from ELEGANT):** Includes fields for beam energy (GeV), initial beam current (A), compression factor, normalized horizontal and vertical emittance (um), rms energy spread, initial horizontal and vertical beta functions (m), initial horizontal and vertical alpha functions, and chirp parameter (m⁻¹).
- Lattice:** Fields for start position (m) and end position (m).
- Scan parameter:** Includes a small plot of G(s) vs s(m) and fields for lambda_start01, lambda_end01, scan_num01, lambda_start02, lambda_end02, scan_num02, and mesh_num.
- ADDITIONAL SETTINGS:** A series of checkboxes for various simulation options like 'calculate iterative solutions?', 'include CSR in bends?', and 'include LSC in drifts?'.
- OUTPUT SETTING:** A section with checkboxes for plotting various parameters such as 'Plot lattice function vs. s', 'Plot compression factor C(s)', 'Plot peak current evolution I_b(s)', 'Plot lattice quilt pattern R56(s->s)', 'plot density gain function G(s)', 'plot density gain function G(s) with lattice', 'plot density gain spectrum G(lambda)', 'Plot gain map G(s,lambda)', 'Plot energy modulation function', 'Plot energy modulation function with lattice', and 'Plot energy modulation spectrum'.
- Run:** A 'Run' button and a 'GO HOKIES!!!' button.

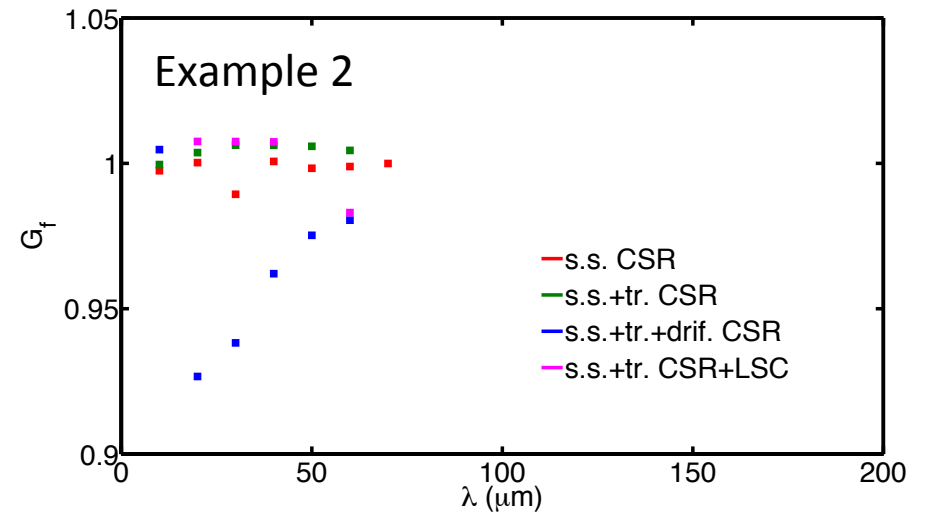
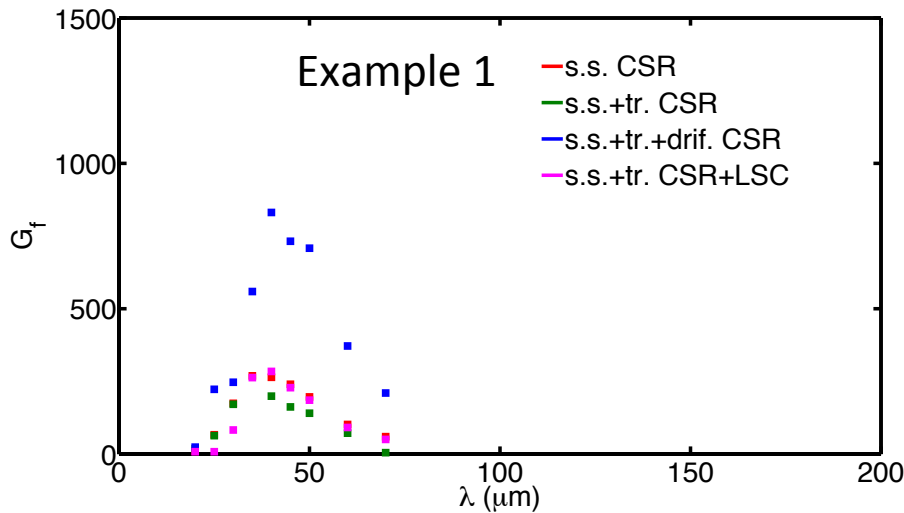
The plot on the right shows G(s) on the y-axis (ranging from 0 to 1.5) versus s (m) on the x-axis (ranging from 0 to 20). The plot shows a step function that starts at approximately 0.8, drops to 0.4 at s ≈ 11, and then rises back to 1.5 at s ≈ 21.

Outline

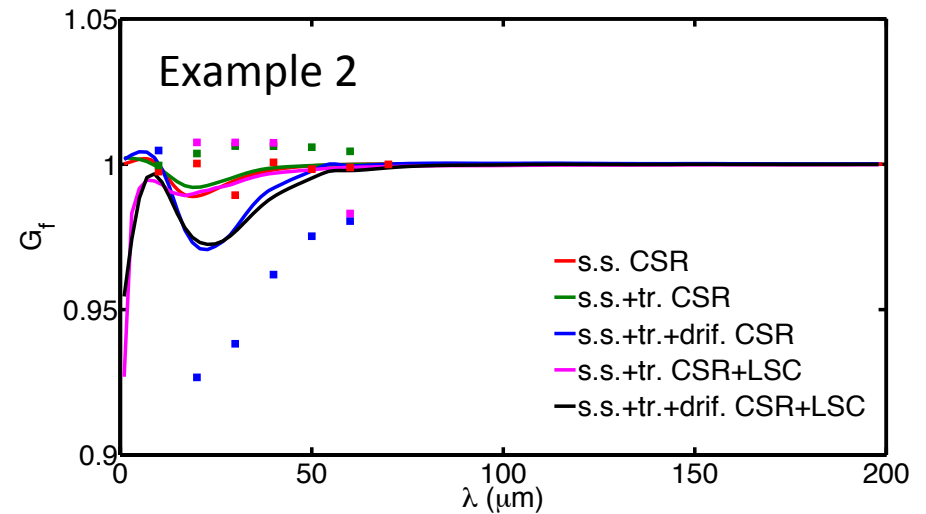
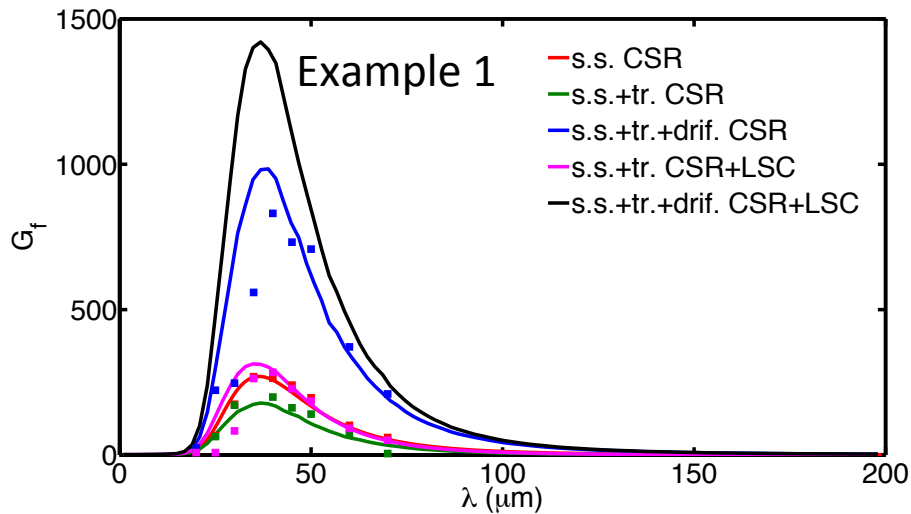
- Introduction and Motivation
 - Microbunching instability mechanism
 - What has been done and Why important in ERL
 - MEIC Circulator Cooling Ring (CCR) as an example
- Theoretical formulation
 - (Linear) Vlasov equation
 - Relevant collective effects: CSR and LSC
- Semi-analytical Vlasov solver
- **Examples and Results [$G_f(\lambda)$, $G(s)$]**
 - Two comparative high-energy transport/recirculation arcs
 - MEIC Circulator Cooling Ring
- Summary and Future work

Application: high-energy transport arcs

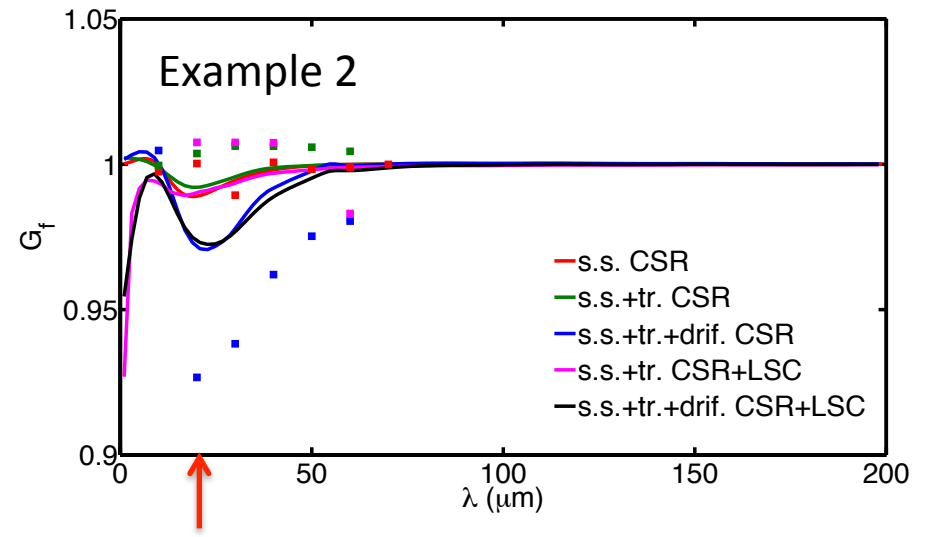
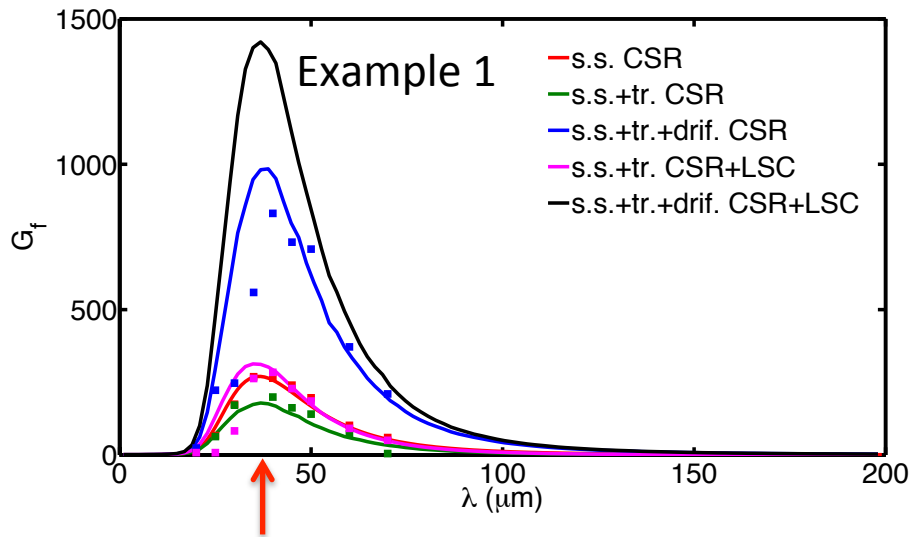




- ❑ For each “dot”, it takes several hours to run ELEGANT tracking, after careful treatment to ensure numerical convergence in linear amplification regime. (particularly for Example 1)
- ❑ Example 1 is subject to microbunching instability; Example 2 not.
- ❑ (Blue) adding “CSR drift” increases the overall gain up to 200 %
- ❑ (Black) include all CSR and LSC effects

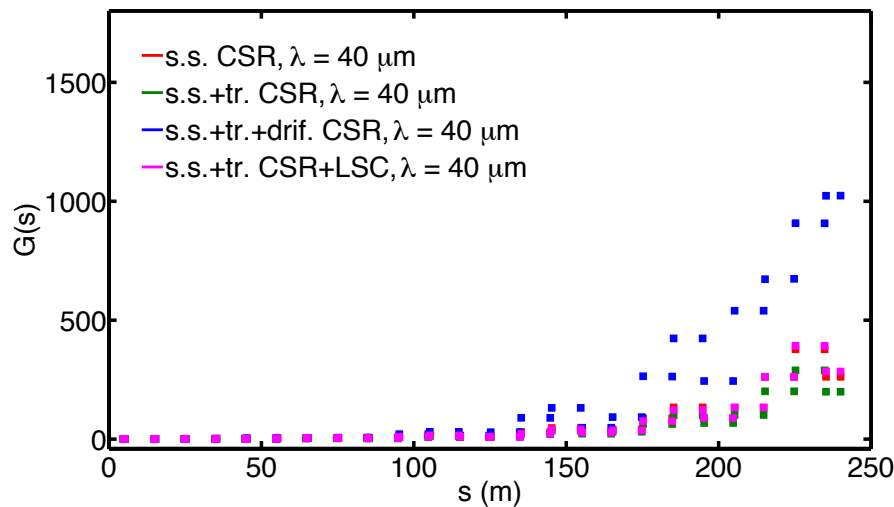


- For each “dot”, it takes several hours to run ELEGANT tracking, after careful treatment to ensure numerical convergence in linear amplification regime. (particularly for Example 1)
- Example 1 is subject to microbunching instability; Example 2 not.
- (Blue)** adding “CSR drift” increases the overall gain up to 200 %
- (Black)** include all CSR and LSC effects

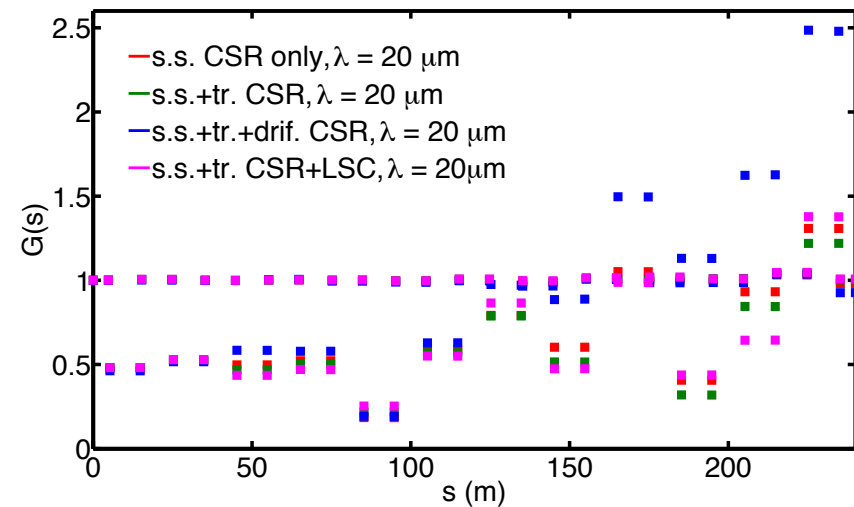


- ❑ For each “dot”, it takes several hours to run ELEGANT tracking, after careful treatment to ensure numerical convergence in linear amplification regime. (particularly for Example 1)
- ❑ Example 1 is subject to microbunching instability; Example 2 not.
- ❑ **(Blue)** adding “CSR drift” increases the overall gain up to 200 %
- ❑ **(Black)** include all CSR and LSC effects

Example 1

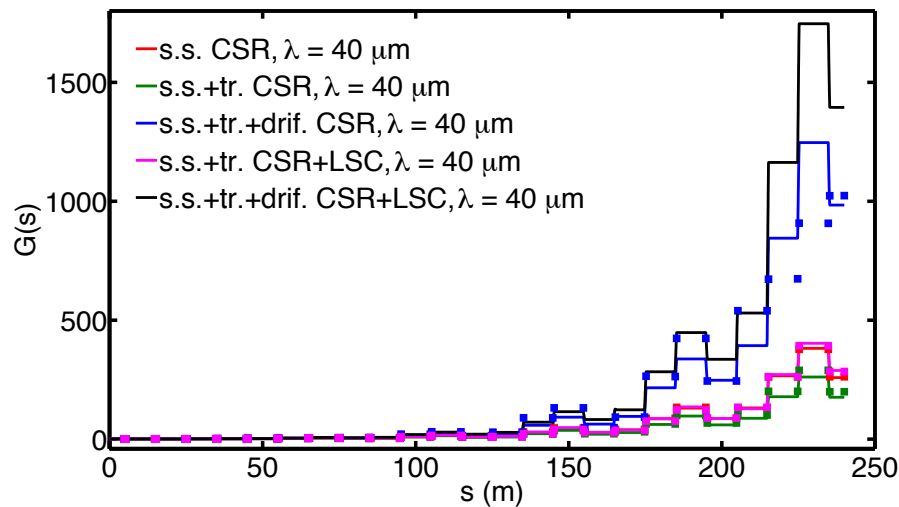


Example 2

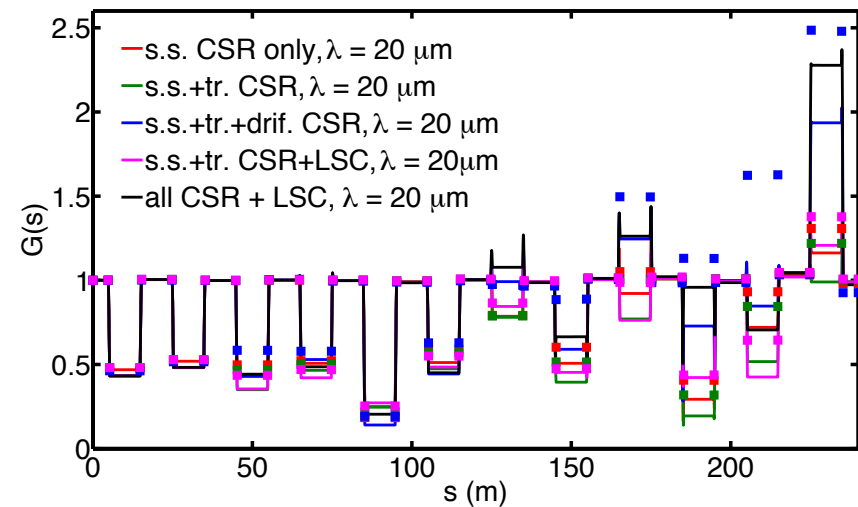


- ❑ Because of local achromaticity ($\eta = 0$) and isochronicity ($R_{56} = 0$), CSR effect is much reduced for Example 2 lattice.
- ❑ Example 1: **global** isochronous, **large** R_{56} modulation
Example 2: **local** isochronous, **small** R_{56} modulation
- ❑ Overall, microbunching instability can result in a significant effect on the beam quality, depending on lattice design itself.
- ❑ For the two cases, the incoming beams have (almost) the same properties.

Example 1



Example 2



- ❑ Because of local achromaticity ($\eta = 0$) and isochronicity ($R_{56} = 0$), CSR effect is much reduced for Example 2 lattice.
- ❑ Example 1: **global** isochronous, **large** R_{56} modulation
Example 2: **local** isochronous, **small** R_{56} modulation
- ❑ Overall, microbunching instability can result in a significant effect on the beam quality, depending on lattice design itself.
- ❑ For the two cases, the incoming beams have (almost) the same properties.

ELEGANT numerical parameter setting

Name	Example 1	Example 2	MEIC CCR	Note
Number of macroparticles	50-70 M	50 M	50 M	for initial beam preparation
N_KICKS	400	400	200 (H), 300 (V)	for CSRCSBEND
NONLINEAR	0	0	0	for CSRCSBEND
LINEARIZE	1	1	1	for CSRCSBEND
BINS	12000	20000	10000	for CSRCSBEND
STEADY_STATE	0	0	1*	for CSRCSBEND
HIGH_FREQUENCY_CUTOFF0	0.08	0.1	0.144	for CSRCSBEND
HIGH_FREQUENCY_CUTOFF1	0.08	0.1	0.144	for CSRCSBEND
DZ	0.01	0.01	N.A.	for CSRDRIFT
USE_STUPAKOV	1	1	N.A.	for CSRDRIFT
BINS	12000	20000	N.A.	for LSCDRIFT

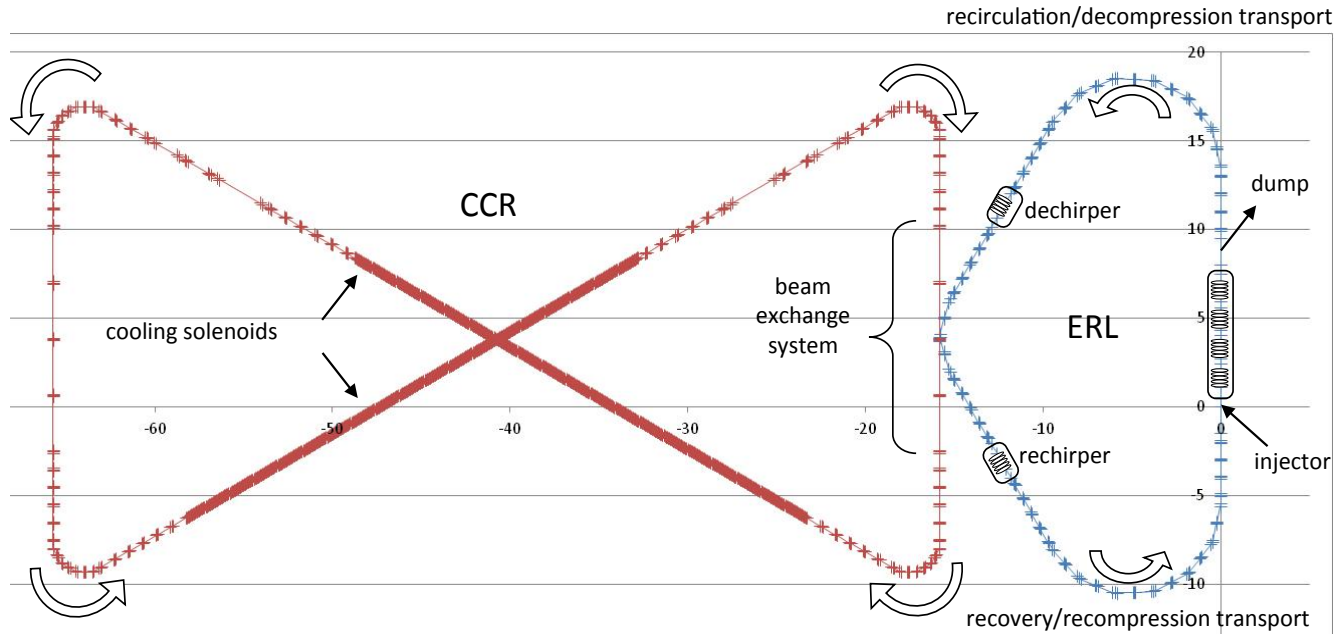
Note:

1. Those (any) sextupole fields are turned off to match the theoretical formulation.
2. Details can be found in JLAB-TN-14-016.

Outline

- Introduction and Motivation
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 - MEIC Circulator Cooling Ring
- Summary and Future work

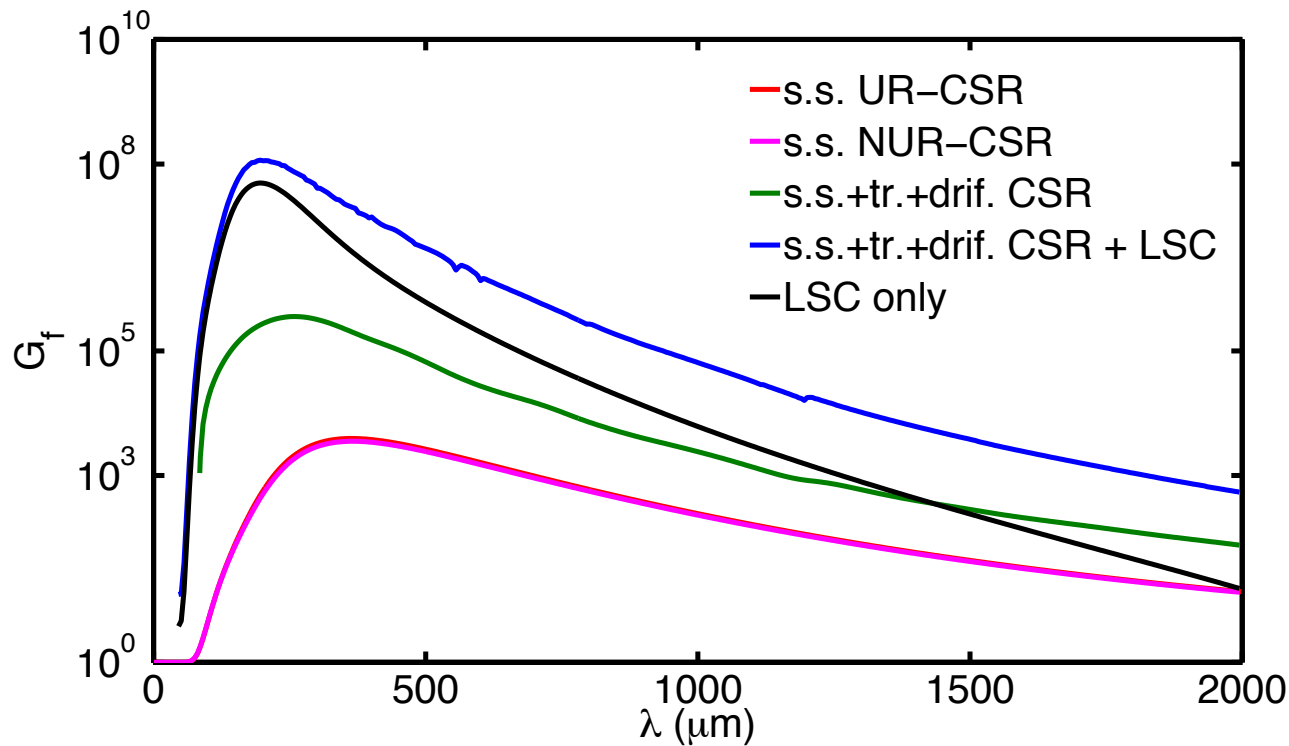
Application: MEIC Circulator Cooling Ring (CCR)



Name	Value	Unit
Beam energy	54	MeV
Beam current	60	A
Normalized emittances	3 (in both planes)	μm
Initial horizontal beta function	10.695	m
Initial vertical beta function	1.867	m
Initial alpha functions	0 (in both planes)	
Relative energy spread (uncorrelated)	1.0×10^{-4}	
Chirp	0	m^{-1}

Note: cooling solenoids have been removed in our simulation.

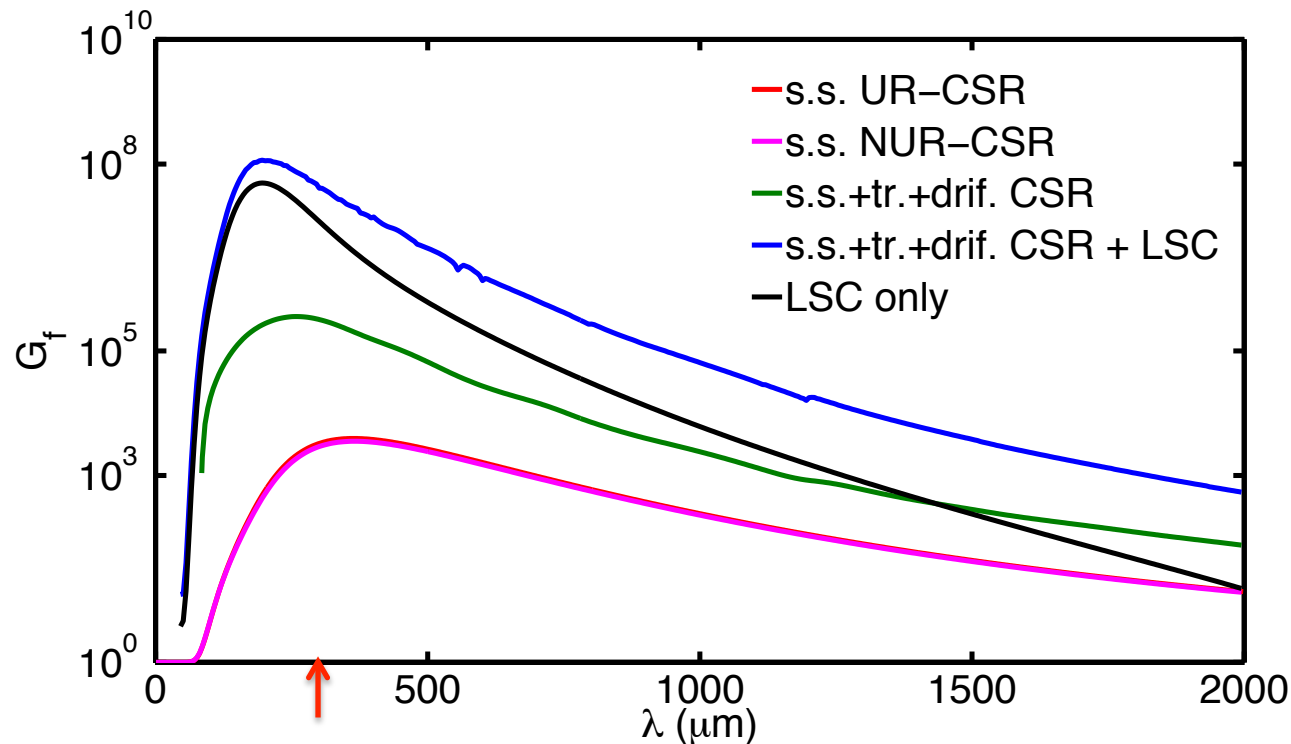
Application: MEIC Circulator Cooling Ring (CCR)



Note:

1. LSC effect can be more severe than CSR on microbunching instability.
2. LSC can be underestimated (because of clipping of solenoid sections).

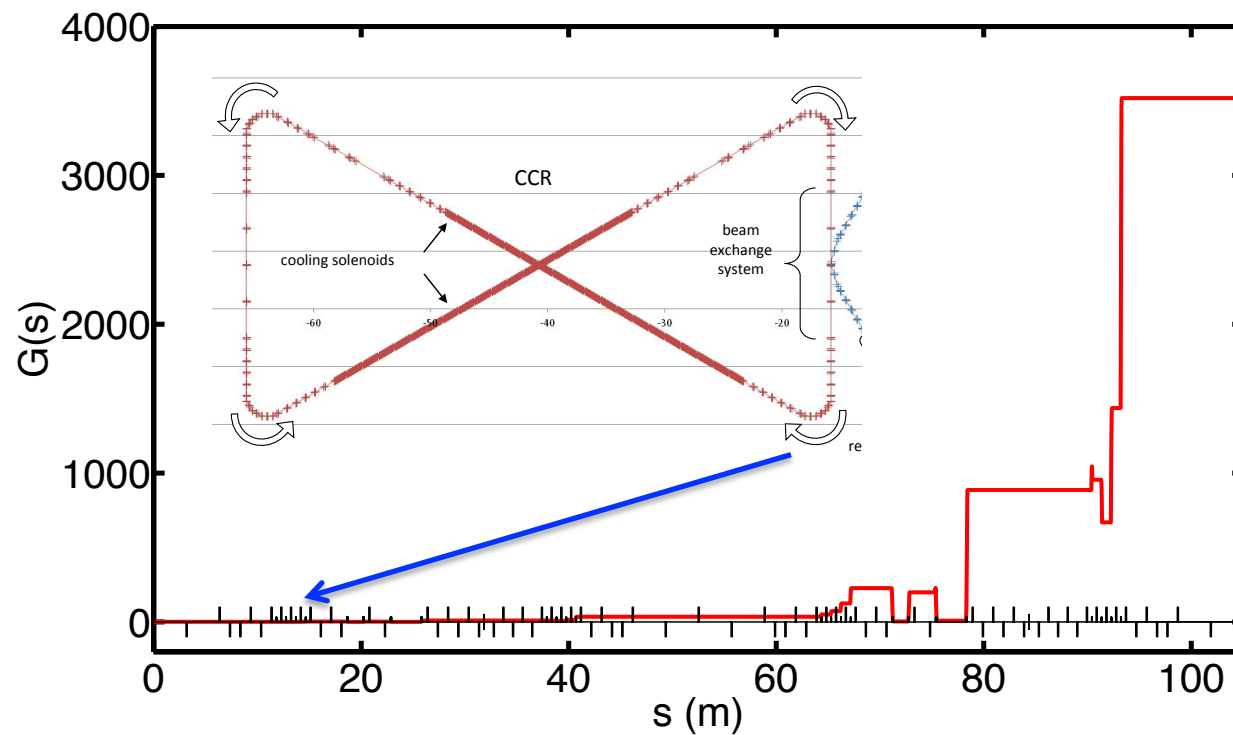
Application: MEIC Circulator Cooling Ring (CCR)



Note:

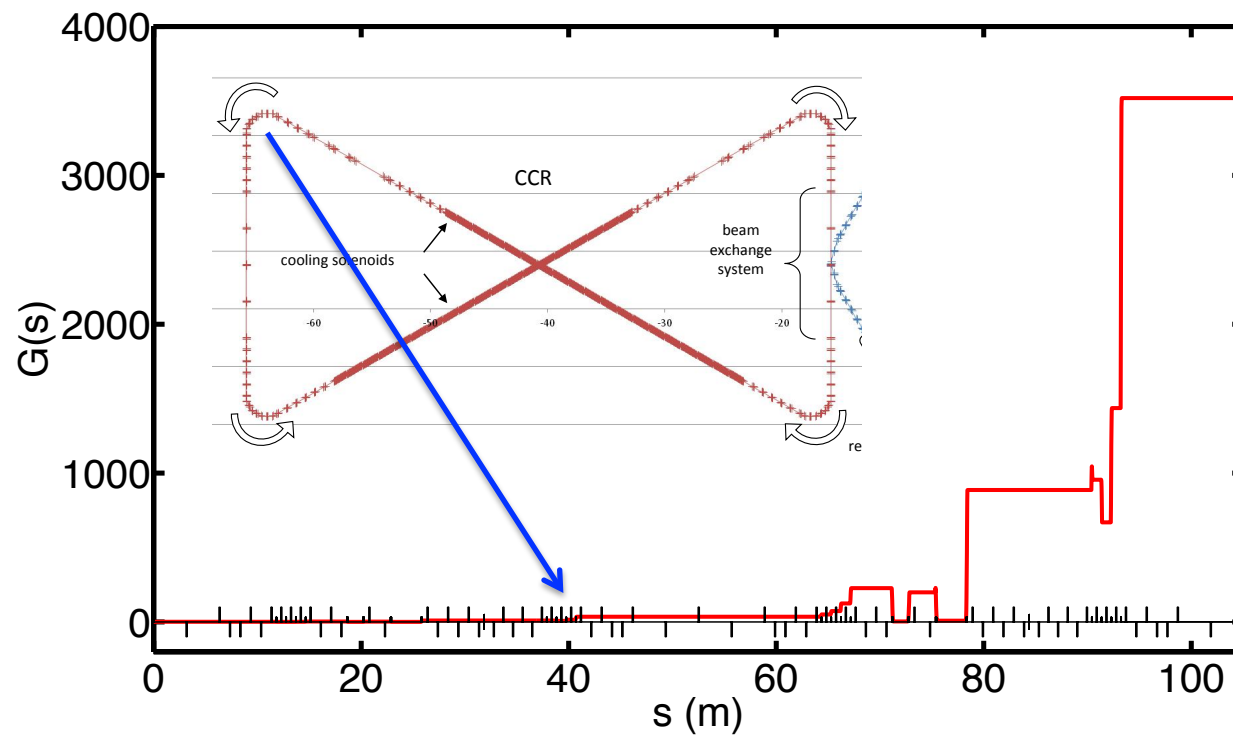
1. LSC effect can be more severe than CSR on microbunching instability.
2. LSC can be underestimated (because of clipping of solenoid sections).

Application: MEIC Circulator Cooling Ring (CCR)



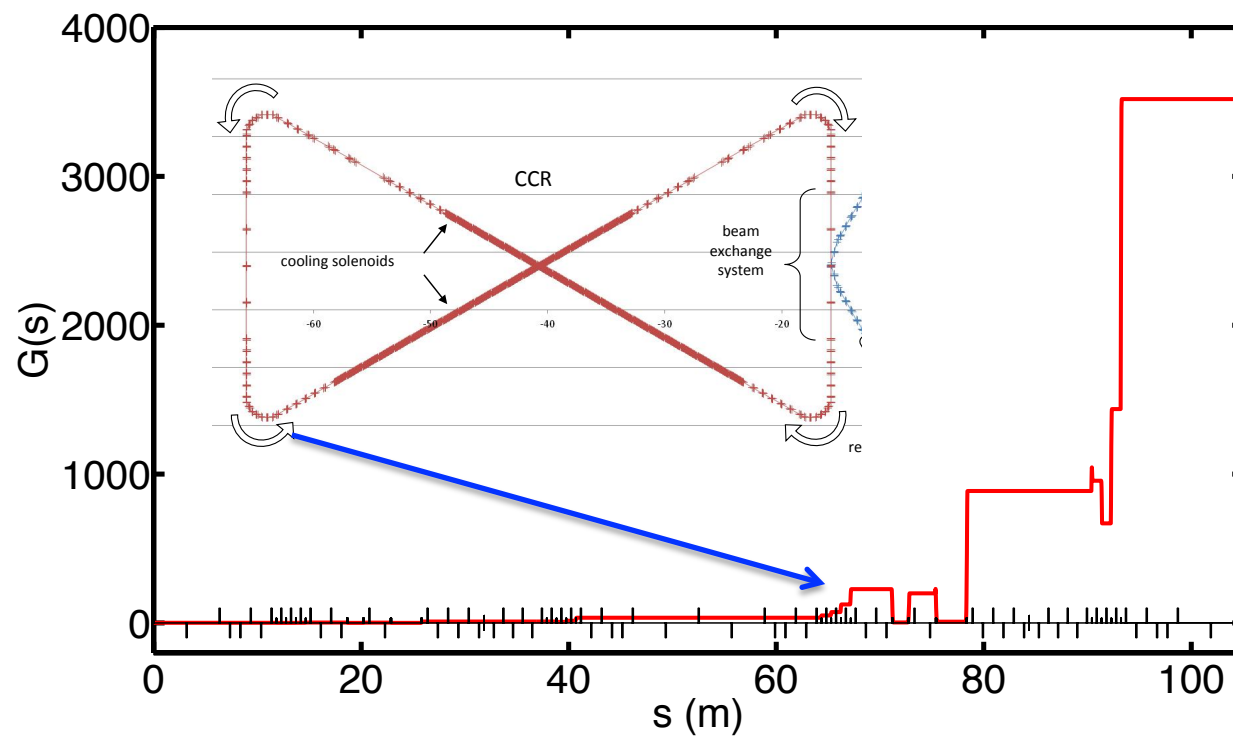
A specific example shows how a “resonant” wavelength ($\lambda = 350 \mu\text{m}$) causes microbunching amplification along CCR.

Application: MEIC Circulator Cooling Ring (CCR)



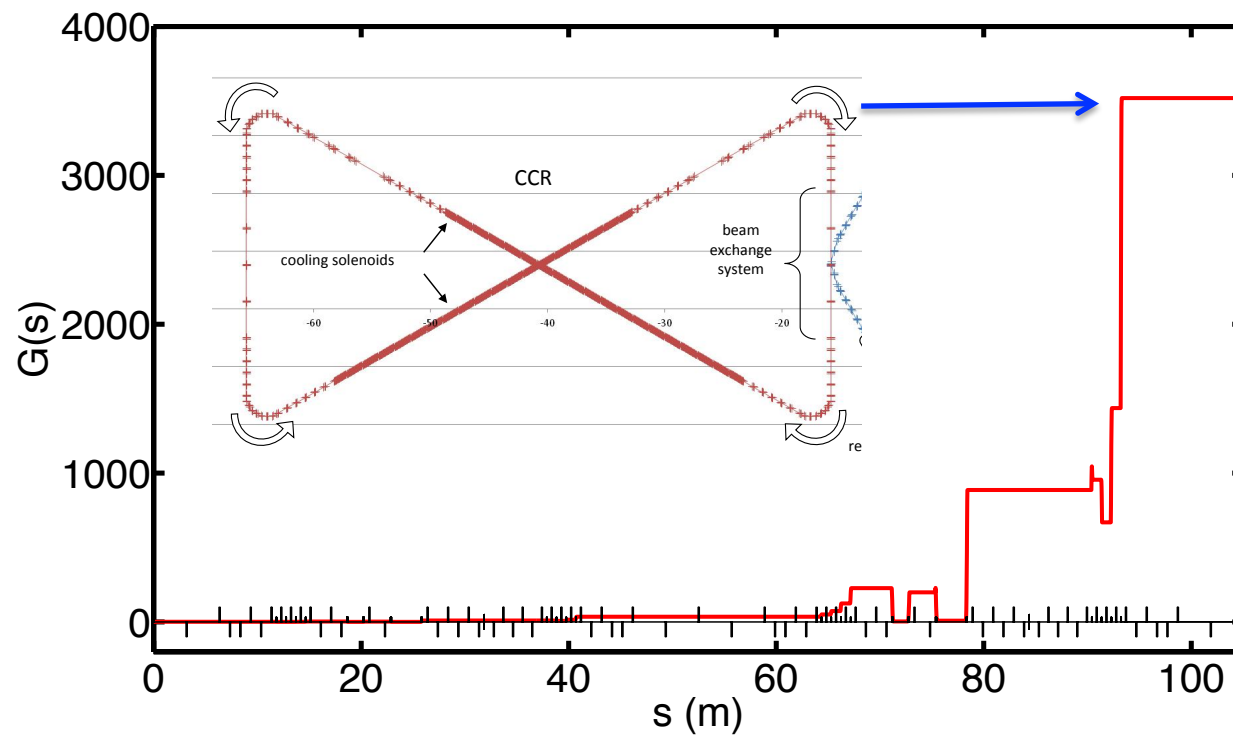
A specific example shows how a “resonant” wavelength ($\lambda = 350 \mu\text{m}$) causes microbunching amplification along CCR.

Application: MEIC Circulator Cooling Ring (CCR)



A specific example shows how a “resonant” wavelength ($\lambda = 350 \mu\text{m}$) causes microbunching amplification along CCR.

Application: MEIC Circulator Cooling Ring (CCR)



A specific example shows how a “resonant” wavelength ($\lambda = 350 \mu\text{m}$) causes microbunching amplification along CCR.

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- Summary and Future work

Summary

- Microbunching instability can be a special concern for ERL or recirculation machines.
- **CSR** and **LSC** can cause severe microbunching instability (e.g. Example 1 and MEIC CCR).
- Impact of **lattice optics** can be significant for ERL-related lattice design (Example 1 vs. Example 2).
- **Quick** estimation of microbunching gain by our developed code.
- Example 2 demonstrates that it is possible to preserve emittance and also suppress microbunching gain at the same time.

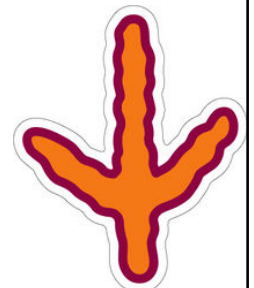
Future work

- **Add** more relevant impedance models, including both analytical and numerical impedance models.
- **Extend** the existing (constant-energy) formulation to include a more general case, with beam **acceleration** or deceleration. This is particularly important in recirculation machines.
- Investigate the physical connection between a single-pass or few-pass system and storage-ring system (∞ -pass).
- **Experimental** benchmarking of our microbunching studies [JLab LDRDs]

Thank you for your attention

Acknowledgements

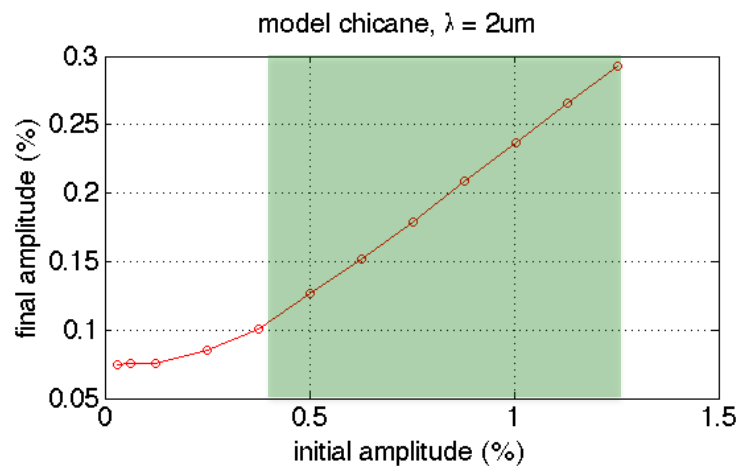
- Thanks to my advisors, co-authors for their kind support, insights, discussion and stimulation:
 - Rui Li and Mark Pitt (advisors)
 - Steve Benson, Dave Douglas, Chris Tennant
- Thanks to JSA Graduate Fellowship Program for travel support
- This material is based on work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under contract DE-AC05-06OR23177.



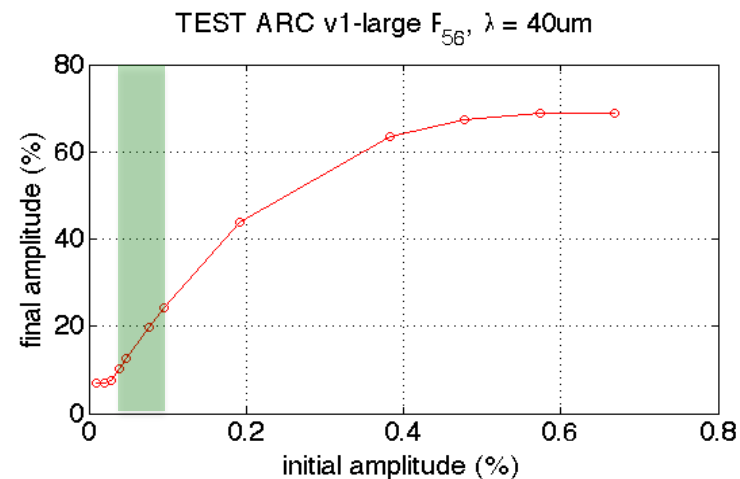
BACKUP SLIDES

Introduction and Motivation: Numerical challenges from particle tracking

- To investigate microbunching gain, we usually impose an initial density modulation, and study how such modulation evolves along the beamline.
- The level of modulation depth should be **small enough** to keep microbunching amplification in linear regime, while **large enough** to surpass (residual) numerical noise.
- For higher gain case, the number of simulation particles becomes more demanding, in order to avoid exaggerated (numerical) fluctuation between the (integration) bins at specific wavelength scale.



(relatively) low-gain case



(relatively) high-gain case

Introduction and Motivation: Numerical challenges from particle tracking

- It has been known that microbunching instability is **sensitive** to fluctuation/noise in beam phase space density.
- Even if several **specialized** algorithms had been developed to reduce the numerical noise (while to keep the level of physical noise) during particle beam transport subject to microbunching effect, tracking a bunch of several tens of millions of particles (or more) is indeed **time consuming**.
- Instead of doing particle tracking, we can formulate this problem using fluid model, i.e. **Vlasov** equation.
- In this presentation, we shall focus on **linear** regime in particular, i.e. linearized Vlasov equation.

Impedance models: CSR

- **Steady-state ultrarelativistic CSR impedance:** [J. Murphy et al., Part. Accel. 1997, Vol. 57, pp. 9-64]

$$Z_{CSR}^{ss,UR}(k(s); s) = \frac{-ik(s)^{1/3} A}{|\rho(s)|^{2/3}}, \quad k = \frac{2\pi}{\lambda} : \text{wave number, } \rho : \text{bending radius} \quad A = 3^{-1/3} \Gamma\left(\frac{2}{3}\right)(\sqrt{3}i - 1)$$

- **Steady-state non-ultrarelativistic CSR impedance:** [R. Li and C. -Y. Tsai, IPAC'15 (MOPMN004)]

$$\begin{aligned} \text{Re}[Z_{CSR}^{s.s.NUR}(k(s); s)] &= \frac{-2\pi k(s)^{1/3}}{|\rho(s)|^{2/3}} \text{Ai}'\left(\frac{(k(s)|\rho(s)|)^{2/3}}{\gamma^2}\right) + \frac{k(s)\pi}{\gamma^2} \left(\int_0^{(k(s)|\rho(s)|)^{2/3}/\gamma^2} \text{Ai}(\zeta) d\zeta - \frac{1}{3} \right) \\ \text{Im}[Z_{CSR}^{s.s.NUR}(k(s); s)] &\approx \frac{2\pi k(s)^{1/3}}{|\rho(s)|^{2/3}} \left\{ \frac{1}{3} \text{Bi}'(x) + \int_0^x [\text{Ai}'(x)\text{Bi}(t) - \text{Ai}(t)\text{Bi}'(x)] dt \right\}, \quad x = \frac{(k(s)|\rho(s)|)^{2/3}}{\gamma^2} \end{aligned}$$

- **Entrance transient CSR impedance:** [D. Zhou, IPAC'12 (MOOBB03)]

$$Z_{CSR}^{ent}(k(s); s) = \frac{-4}{s^*} e^{-4i\mu(s)} + \frac{4}{3s^*} (i\mu(s))^{1/3} \Gamma\left(\frac{-1}{3}, i\mu(s)\right) \quad \begin{aligned} \mu(s) &= k(s)z_L(s) \\ z_L &= (s^*)^3 / 24\rho^2 \end{aligned}$$

s^* is the longitudinal coordinate measured from dipole entrance

- **Exit transient CSR impedance:**

$$Z_{CSR}^{exit}(k(s); s) = \frac{-4}{L_b + 2s^*} e^{\frac{-ik(s)L_b^2}{6|\rho(s)|^2}(L_b + 3s^*)} \quad Z_{CSR}^{drif}(k(s); s) \approx \begin{cases} \frac{2}{s^*}, & \text{if } \rho^{2/3} \lambda^{1/3} \leq s^* \leq \lambda\gamma^2 / 2\pi \\ \frac{2k(s)}{\gamma^2}, & \text{if } s^* \geq \lambda\gamma^2 / 2\pi \\ 0, & \text{if } s^* < \rho^{2/3} \lambda^{1/3} \end{cases}$$

s^* is the longitudinal coordinate measured from dipole exit

Impedance models: LSC

- M. Venturini, PRST-AB **11**, 034401 (2008)

- **On-axis LSC model:**

$$Z_{LSC}^{on-axis}(k(s); s) = \frac{4i}{\gamma r_b(s)} \frac{1 - \xi K_1(\xi)}{\xi}, \quad \xi = \frac{k(s)r_b(s)}{\gamma}$$

- **Average LSC model:**

$$Z_{LSC}^{ave}(k(s); s) = \frac{4i}{\gamma r_b(s)} \frac{1 - 2I_1(\xi)K_1(\xi)}{\xi}, \quad \xi = \frac{k(s)r_b(s)}{\gamma}$$

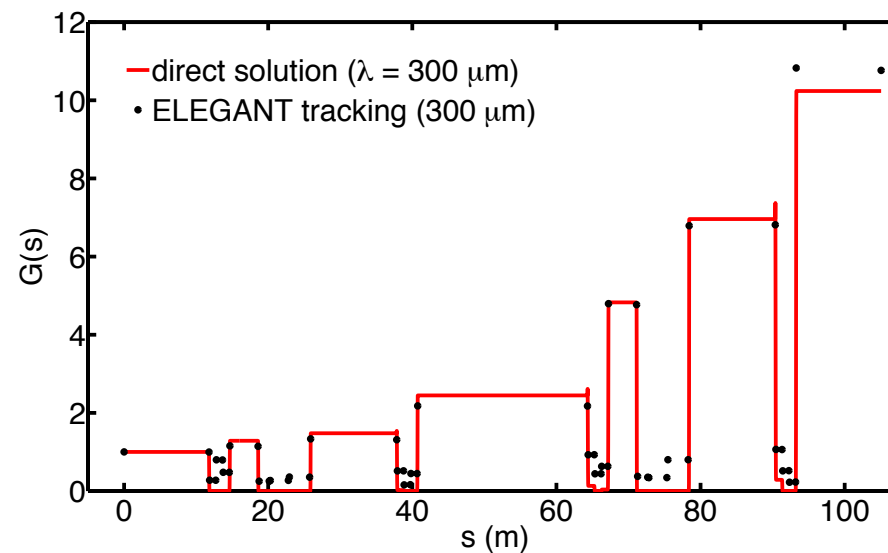
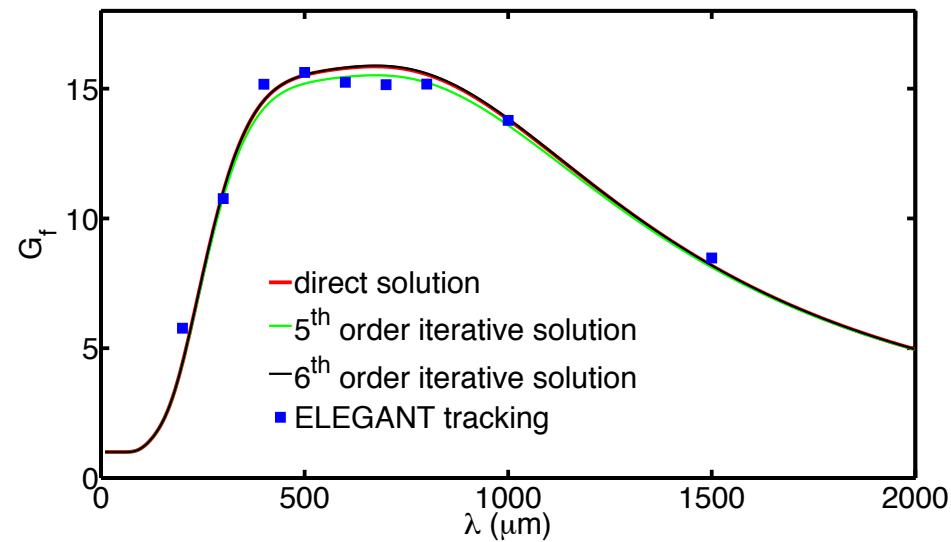
- **Transverse axisymmetric LSC model:**

$$Z_{LSC}^{ave.Gaussian}(k) = -i \frac{\xi_\sigma}{\sigma \gamma} e^{\xi_\sigma^2/2} \text{Ei}\left(\frac{-\xi_\sigma^2}{2}\right), \quad \xi_\sigma = \frac{k(s)\sigma}{\gamma}$$

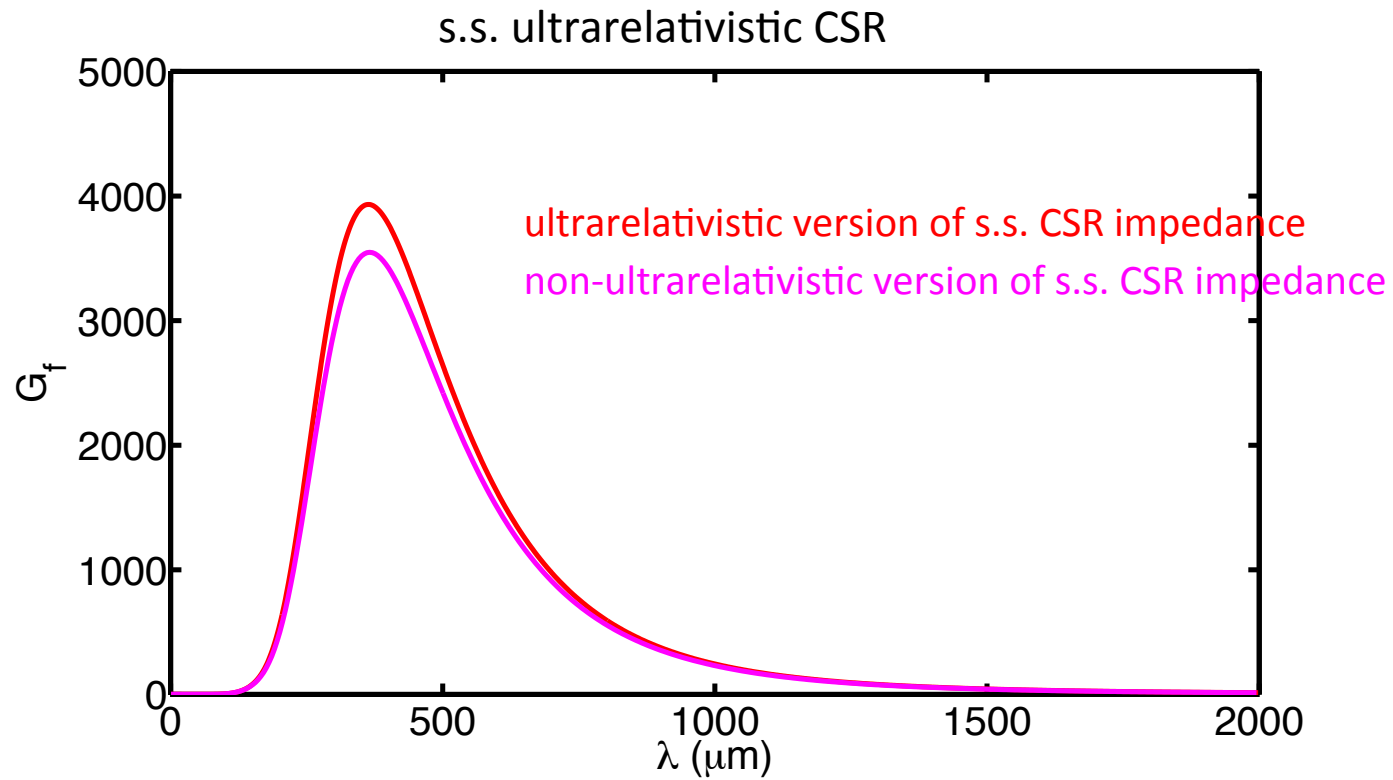
- These LSC models are implemented in our code by adopting transverse rms beam sizes σ_x and σ_y from ELEGANT and applying weighted average over them:

$$r_b(s) = \frac{1.747}{2} (\sigma_x(s) + \sigma_y(s))$$

Microbunching gains for MEIC CCR with 10x emittance

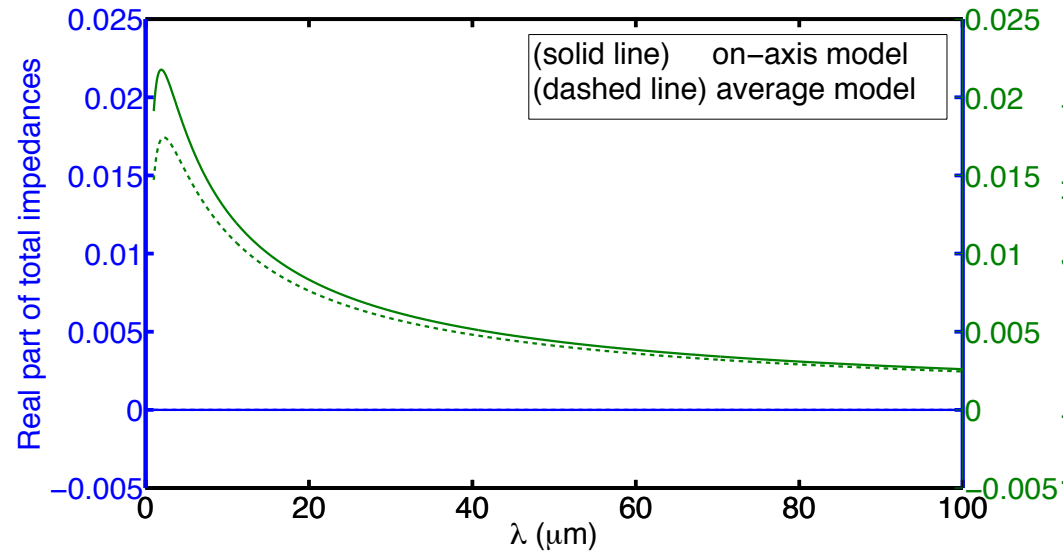


Application: MEIC Circulator Cooling Ring (CCR)



Impedance comparisons

LSC
(on-axis vs. ave.)



CSR
NUR vs. UR

