THE OPTICS OF THE LOW ENERGY FFAG CELL OF THE eRHIC COLLIDER USING REALISTIC FIELDS*

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Abstract

The proposed electron accelerator of the eRHIC complex [1] will use a 1.32 GeV Energy Recovery Linac (ERL) to accelerate electron bunches to a top energy of 21.2 GeV, and subsequently will collide with hadron bunches. The electron bunches will attain the 21.2 GeV energy after passing through the ERL 16 times while they recirculate in two FFAG rings [1] which are placed alongside the RHIC hadron accelerator. The two rings are made of periodic cells and each cell is made of one focusing and one defocusing permanent magnet quadrupoles. In this paper we present the electromagnetic calculations of the 2D and 3D models of a cell which is comprised of two modified Halbach quadrupoles [2], and the optical properties of the cell. The magnetic measurements of a modified Halbach quadrupole will also be presented and compared with the model calculations.

INTRODUCTION

The mission and the design of the electron-ion collider (eRHIC) to be used in the electron-Ion-Collisions (EIC) physics program are described in a published report [1]. The eRHIC which is shown schematically in Figure 1 will make use of the existing Relativistic Heavy Ion Collider (RHIC) hadron accelerator which provides various hadronic species having maximum energies ranging from 250 GeV polarized protons to 100 GeV/u Uranium ions. The electron accelerator of the eRHIC complex will be built alongside the hadron accelerator and will provide electrons with maximum energy of 21.2 GeV. The electron accelerator shown as two red rings in Figure 1 will be built alongside the existing hadron accelerator which is shown as a blue ring in Figure 1. Polarized electron bunches will be injected at an energy of 12 MeV and will receive an energy increase of 1.32 GeV of each time they pass through the ERL a cryomodule of which is shown as an insert at the right-top of Figure 1. The electron bunches will recirculate in the two FFAG rings sixteen times to achieve the maximum energy of 21.2 GeV. The "low energy" electron ring will circulate the bunches with five energies, 1.32, 2.64, 3.96, 5.29, and 6.61 GeV and the "high energy" ring will circulate 11 beams with energies ranging from 7.93 GeV to 21.2 GeV. Each ring is based on the Fixed Field Alternating Gradient (FFAG) principle and consists of FODO cells. Each FODO cell is comprised of one focusing and one defocusing quadrupole.

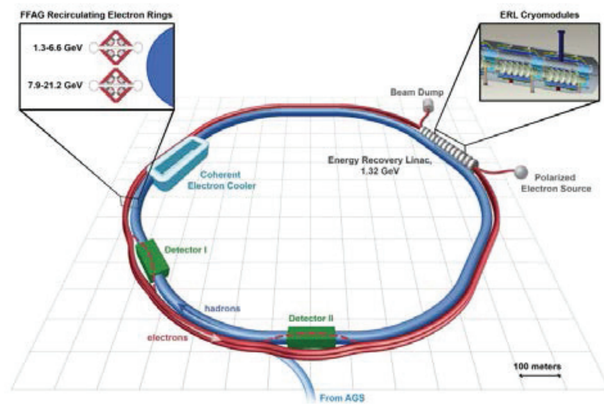

Figure 1: Schematic diagram of the eRHIC.

One of the proposed designs of the FODO cell's quadrupoles is a Halbach type permanent magnet quadrupole which is modified to prevent the interaction of the permanent magnet material with the synchrotron radiation emitted by the circulating electron bunches. This paper will present some information on the cell's beam optics which is based on computed field maps of the cell's quadrupoles.

THE FFAG CELL

One of the possible cell designs to be used for the low energy arcs of the electron accelerator is shown in Figure 2. The cell quadrupoles have aperture of radius 2 cm and are separated transversely by 5.4 mm. The low energy ring consists of 6x212 cells therefore each low energy cell focuses and bends the electron bunches by 4.9396 mrad (0.2830º). The remarkable property of the FFAG cell is indicated in Figure 2 which shows the trajectories of the electron bunches in the large energy range from 1.334 GeV to 6.622 GeV to be contained within a transverse space of 16.9 mm.

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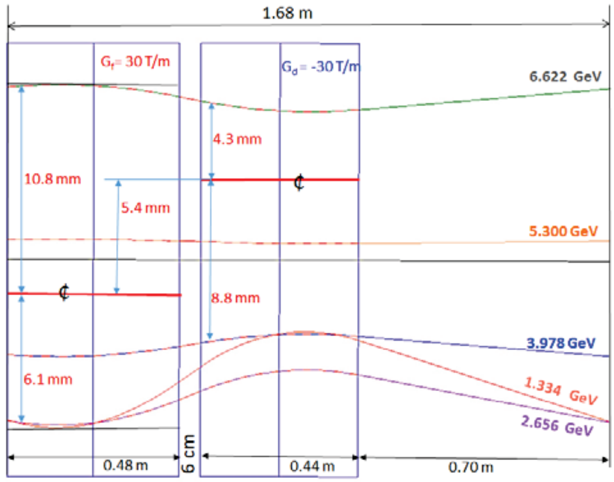

Figure 2: Schematic diagram of the 1.68 m long low energy FFAC cell.

MODELING AND ELECTROMAGNETIC DESIGN OF THE CELL

The electromagnetic design of the cell is done in two steps, first the 2D design followed by the 3D design. Both cell quadrupoles have the same cross section which is shown in Figure 3. The modification of the Halbach design consists in removing two permanent magnet wedges from left and right (see Figure 3) to prevent their damage from the synchrotron radiation, and also remove the two wedges from top and bottom to keep the four fold symmetry. A SmCo-R26SH BH curve has been used in both the 2D and 3D calculations [3].

The 2D Design

The 2D design establishes the cross section of each quadrupole to achieve the required gradient, and also minimize the 12pole multipole by varying the direction of the easy axis of blue-shaded wedges shown in Figure 3.

The 3D Design

The 6 cm longitudinal separation of the quadrupoles in the cell and the proximity between the cells justifies the 3D electromagnetic modeling. To obtain the field map of a cell we modelled three sequential cells using the OPERA computer code [4]. Details on the modelling as well as on other aspects of this paper appear in Ref [3].

An isometric view of the cell's two quadrupoles is shown in Figure 4. The 3D calculations provided the field map of the cell on a rectangular grid. The field map was used to calculate the optical properties of the cell.

MAGNETIC MEASUREMENTS OF A SINGLE QUADRUPOLE

To establish confidence on the calculated field maps we built a 6 cm long model of the quadrupole shown in Figure 5. The housing of the permanent magnet wedges is made by a 3D printer. The cross section of the built quadrupole is identical to that shown in Figure 3, the direction of the easy axis of the wedges of the built model was different from that which minimized the 12pole multipole [3].

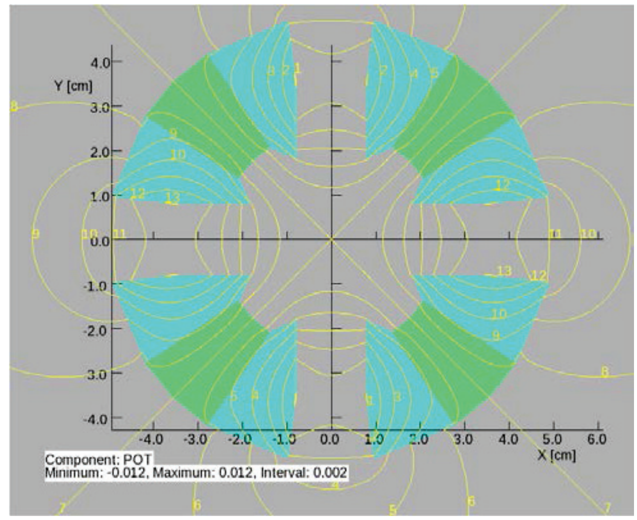

Figure 3: The cross section of the modified Halbach quadrupole.

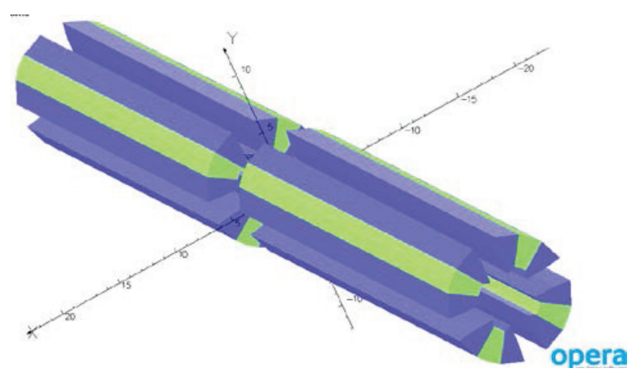

Figure 4: Isometric view of the cell's two quadrupoles.

Table 1 shows the integrated strengths of the quadrupole and 12pole at a radius of 1 cm. From Table 1 we can calculate that the difference between the calculated and measured 12pole is ~9×10^{-4} cm^{-4} at a radius of r=1 cm relative to the quadrupole strength. This discrepancy is within our tolerances as the optics calculations show.

Table 1: Measured and Calculated Strengths of the Quad

Quad	Int. Quad [Gauss]	Int. 12pole [Gauss.cm^{-4}]
Calculations	18730.5	337.4
Measurements	18650.0	350.6

Quadrupole correctors, will be included with the quad.

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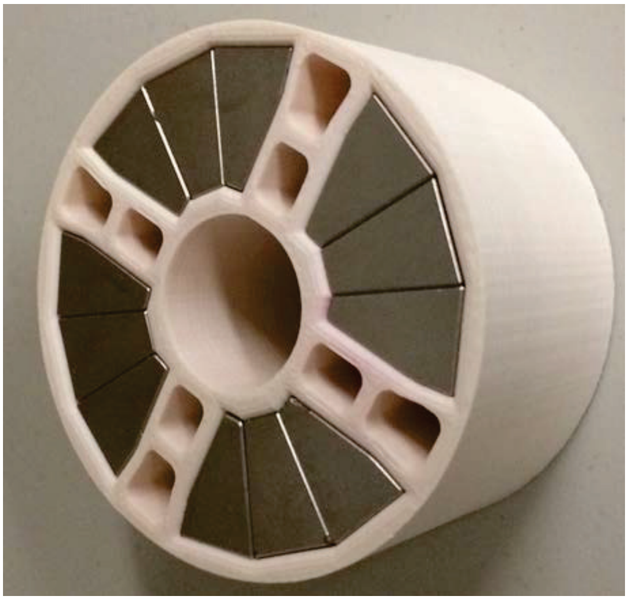

Figure 5: A picture of the modified Halbach quadrupole. The housing of the permanent magnet wedges is made by a 3D printer.

OPTICAL PROPERTIES OF THE CELL

The beam optics of the cell consists in studying the following properties of the low energy ring:

- The closed orbits of the ring for the electron bunches within the energy range 1.3 to 6.6 GeV. This study proves that the magnetic field map of a single cell allows the formation of closed orbits in the ring.
- The horizontal and vertical tunes Q_x and Q_y and the chromaticities ξ_x, ξ_y, which correspond to the closed orbits. This study provides information on the stability of the closed orbits.
- The acceptance of the ring, which is the maximum beam emittances ε_x, and ε_y, of each beam bunch that corresponds to a closed orbit of a given energy.
- The dynamic aperture for each close orbit.
- The beta functions in the cell.

The zgoubi computer code [5] has been used to calculate the optical properties.

The Closed Orbits

Figure 6 shows 42 closed orbits of electron bunches in a cell in the energy range of 1.3 GeV to 6.6 GeV. In the zgoubi code x-axis is along the beam direction and y is radial.

Tunes and Chromaticities

Figure 7 is a plot of the tunes Q_x, a n d Q $_y$, and the chromaticities ξ_x, ξ_y. The betatron tunes in combination with the chromaticities as plotted in Figure 7 indicate that the electron bunches are away from beam resonances.

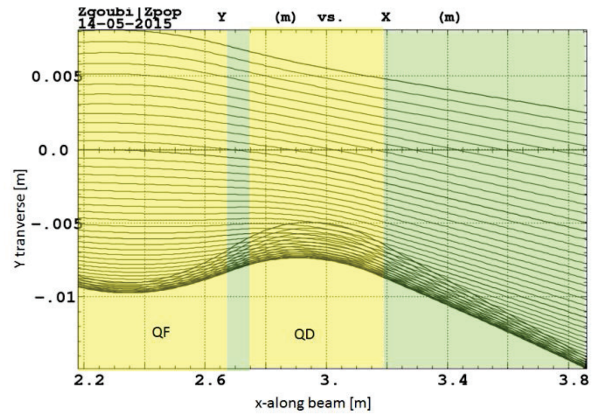

Figure 6: Closed orbits in the low energy cell of eRHIC.

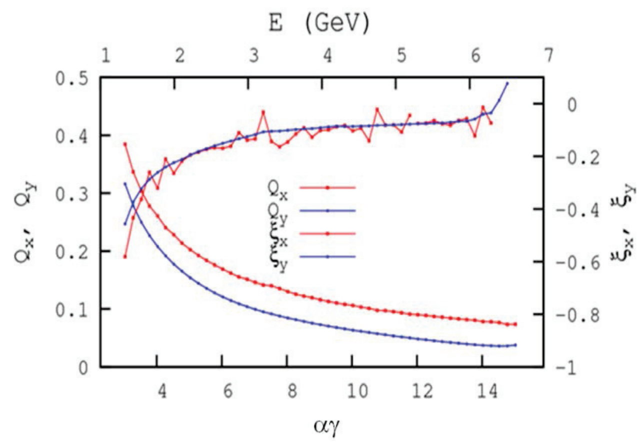

Figure 7: Plot of tunes and the chromaticities in a cell as a function of electron's kinetic energy (top scale). The symbol α=g/2-1=1.16x10^{-3} in the label of the horizontal scale and g is the g-factor of the electron.

The Acceptance of the Ring

Figures 8 and 9 show the horizontal and vertical rms acceptances of the low energy ring for five of the electron bunches.

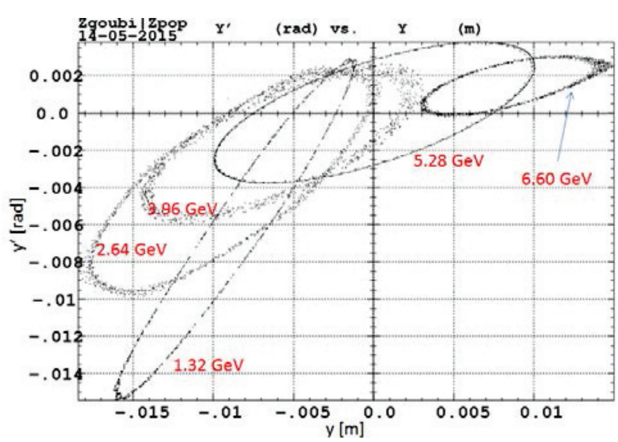

Figure 8: The horizontal rms beam acceptance for each of the five energy-bunches being transported in the ring.

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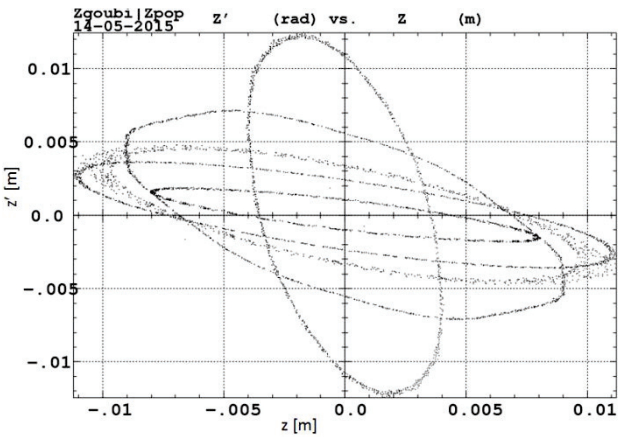

Figure 9: The vertical rms beam acceptance for each of the five energy-bunches being transported in the ring.

The minimum horizontal or vertical calculated rms value of the ring's acceptance is ~0.02 [m.rad]. This value is much larger than the required horizontal and vertical beam emittance of 30×10^{-6} [m.rad] to be transported in the ring.

The Dynamic Acceptance of the Ring

Starting with the maximum y and z coordinates of each of the five acceptance phase space ellipses as we calculated in the previous section, the "zgoubi" computer code calculated the maximum y and z particle-coordinates at the entrance of the ring that can be transported at the exit. Figure 10 is a plot of the dynamic aperture for the energy-bunches mentioned in this paper in the range 1.3 to 6.6 GeV.

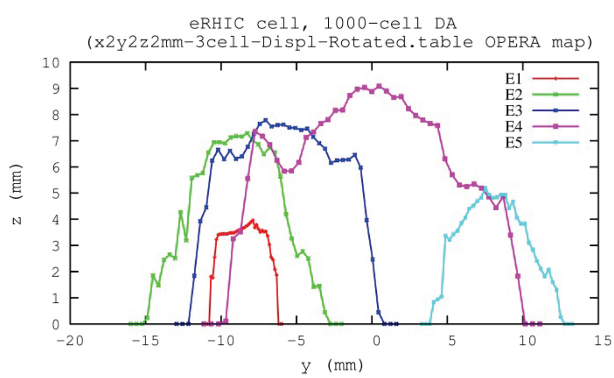

Figure 10: The dynamic acceptance of the low energy ring for the five energy bunches mentioned in the paper.

The Beta Functions of the Cell

Figure 11 plots the horizontal and vertical beta functions of the low energy FFAG eRHIC cell for three energies. The focusing and defocusing quadrupoles of the cell are highlighted.

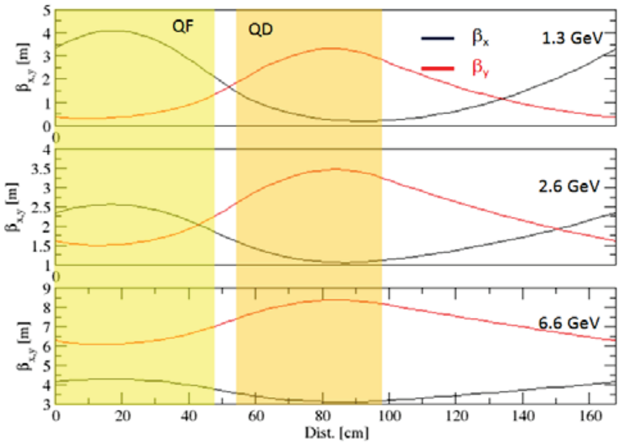

Figure 11: The horizontal and vertical beta functions of the low energy FFAG eRHIC cell for three energies. The shaded areas are the focusing and defocusing quads.

CONCLUSIONS

Using realistic field maps computed with the OPERA computer code, we calculated the optical properties of a cell to be used in the FFAG lattice of the eRHIC's recirculating electron ring. The calculated horizontal/vertical tunes, and chromaticities, indicate stability for the transported beam bunches in the required energy range. The calculated phase space for the beam acceptances and the dynamic apertures are large enough to accept the required beam emittances. The 3D calculations performed on a modified Halbach quadrupole are in agreement with the measurements thus these magnets can provide the desired magnetic fields for the cell of the FFAG lattice of the eRHIC.

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