Investigation of the effect of space charge in compact-Energy Recovery Linac

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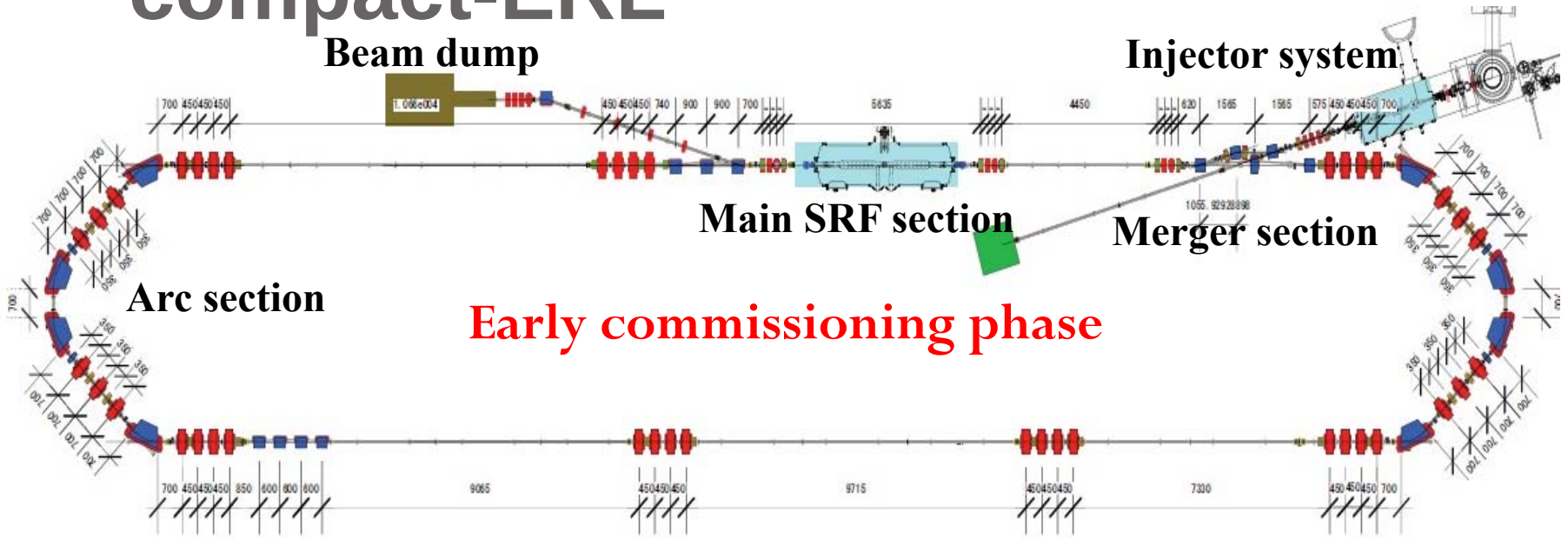
compact-ERL



The generation of ultra-low emittance beams is need to demonstrate before constructing 3GeV ERL .

→ *Compact-ERL*

cERL at the end of FY2012 (minimum mode)

- 35 MeV, 10 mA, 1 mm-mrad

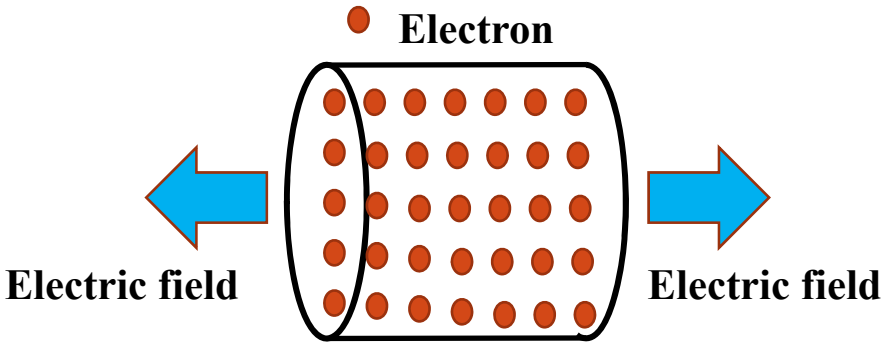
Final Feature of cERL (final mode)

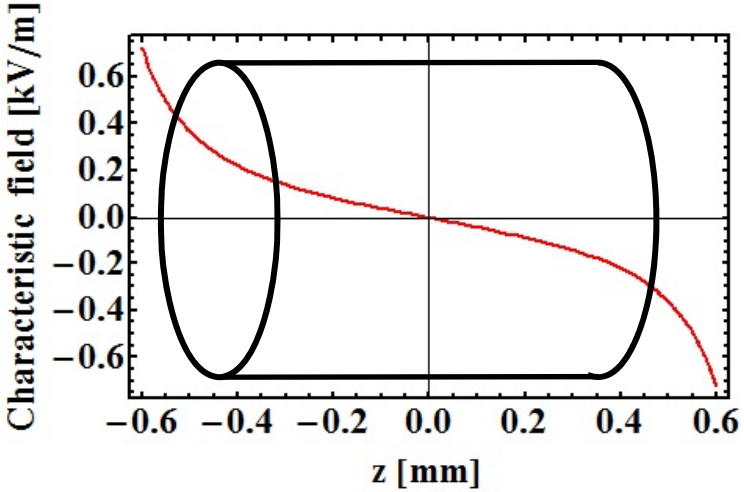
2 loop, 125 - 245 MeV,

- 100 mA, 77 pC, 1 mm-mrad
- 10 mA, 7.7 pC, 0.1 mm-mrad

Longitudinal Space Charge Effect

Initial beam distribution (Beer-can shape)





$$E_z(z) = \frac{Q}{2\pi\varepsilon_0 a^2}\left[-\left|\frac{1}{2}-\frac{z}{L}\right|+\left|\frac{1}{2}+\frac{z}{L}\right|+\sqrt{\left(\frac{z}{L}-\frac{1}{2}\right)^2+A^2}-\sqrt{\left(\frac{z}{L}+\frac{1}{2}\right)^2+A^2}\right]$$

Electric field induced by Many electrons

where $L = \sqrt{12}\sigma_z$ **:Full length of bunch**

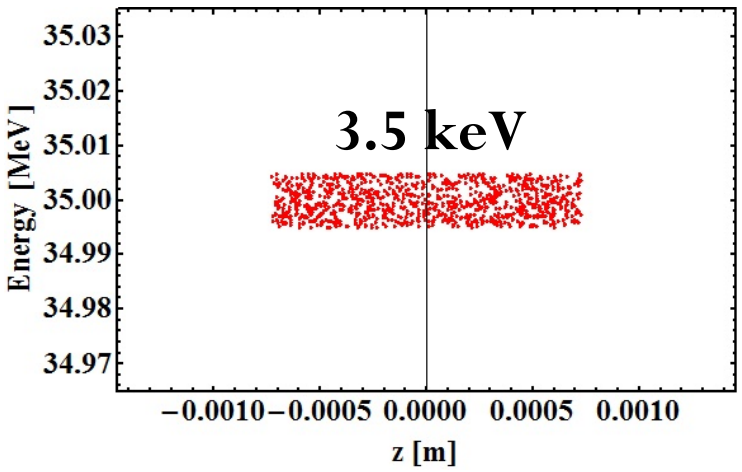
A = α/γL, Q is charge

α is radius, ε_0 is permittivity.

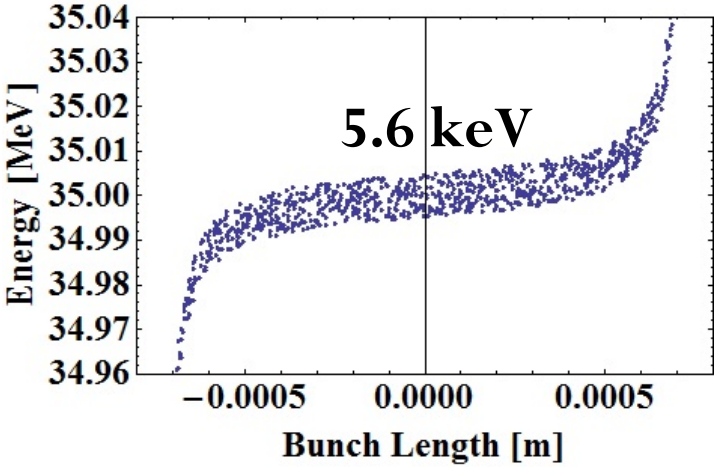
The head and tail of bunch was accelerated and decelerated, respectively

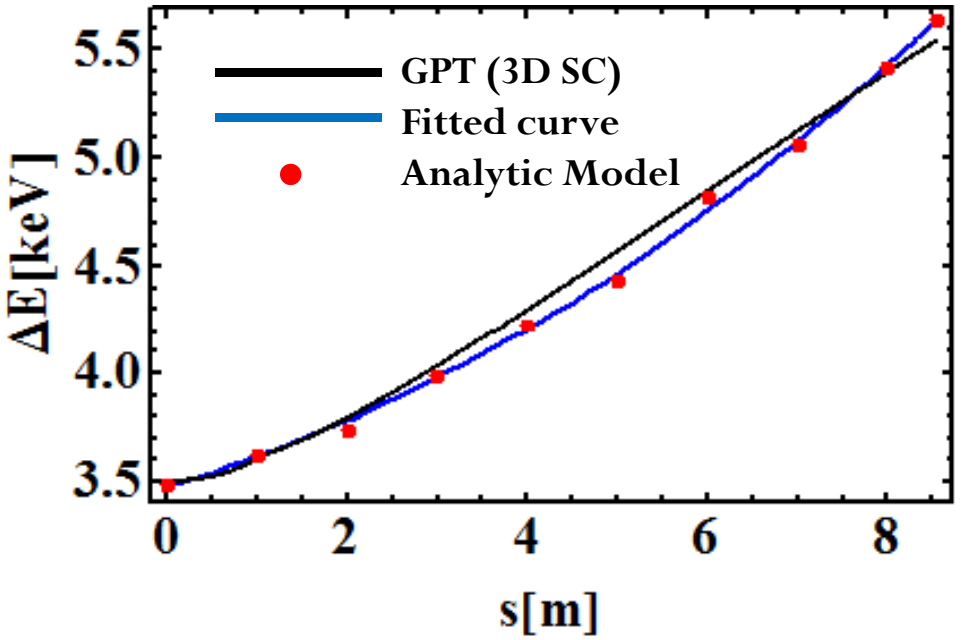
Energy spread growth

Longitudinal Space Charge Effect



35 MeV, 8.54 m Drift space



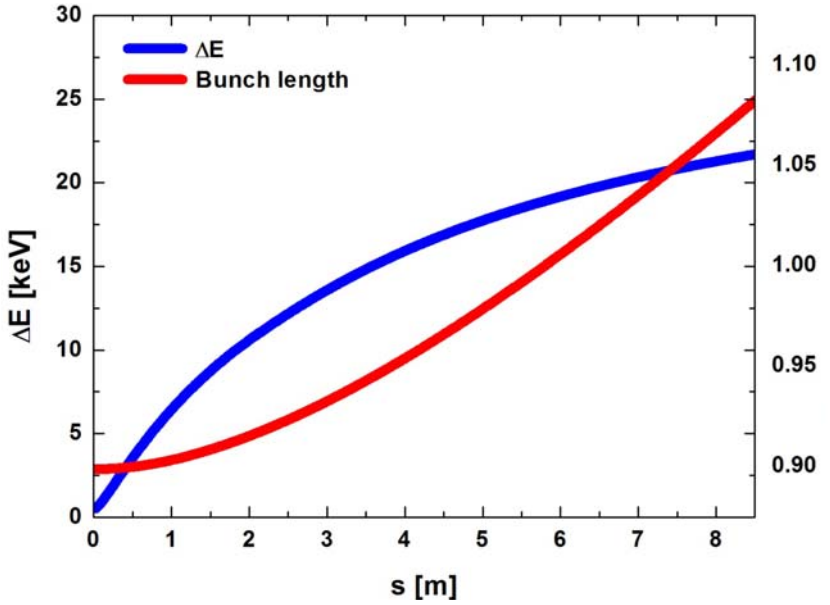


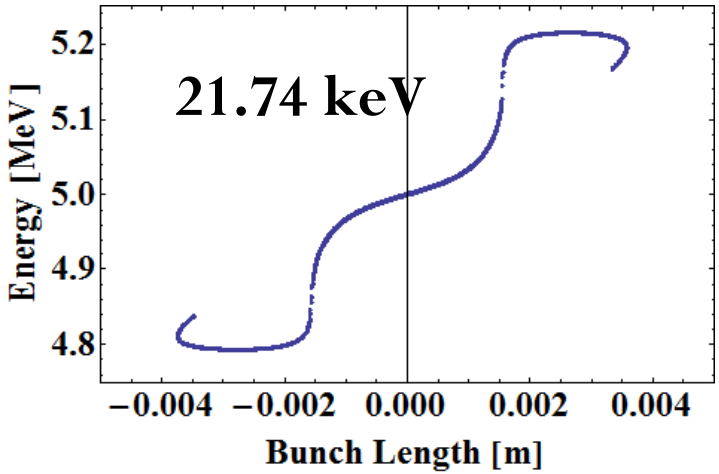
$$\Delta E = k_0 + k_1 s + k_2 s^2 + k_3 s^3$$

Fitted parameters

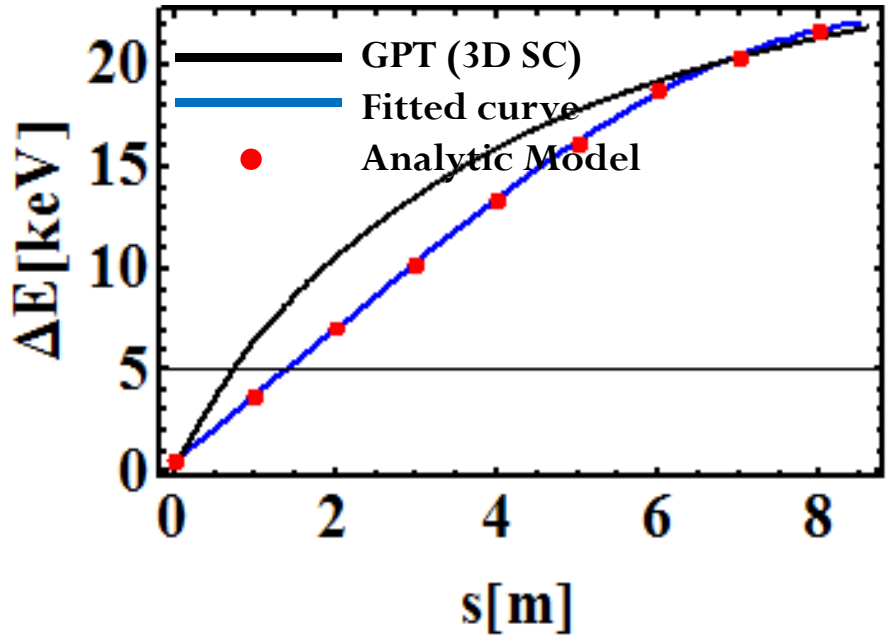
$k_0 = 3.49 \pm 0.03$	*Initial energy spread*
$k_1 = 0.0956 \pm 0.0335$	*1st order SC effect*
$k_2 = 0.0234 \pm 0.00943$	*2nd order SC effect*
$k_3 = -0.000635 \pm 0.000721$	*3rd order SC effect*

Longitudinal Space Charge Effect





5 MeV, 8 m Drift space

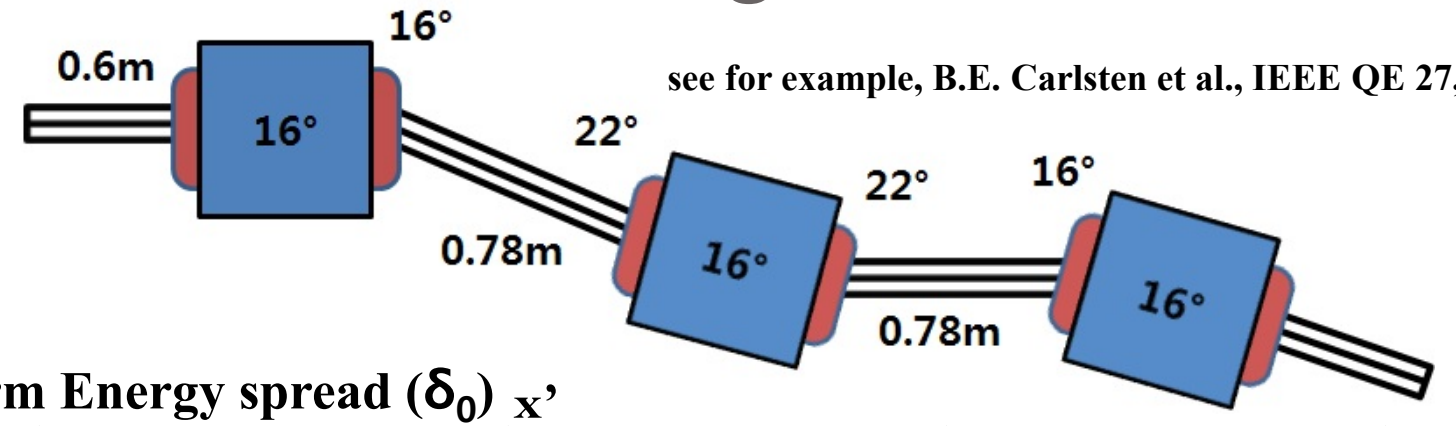


$$\Delta E = k_0 + k_1 s + k_2 s^2 + k_3 s^3$$

Fitted parameters

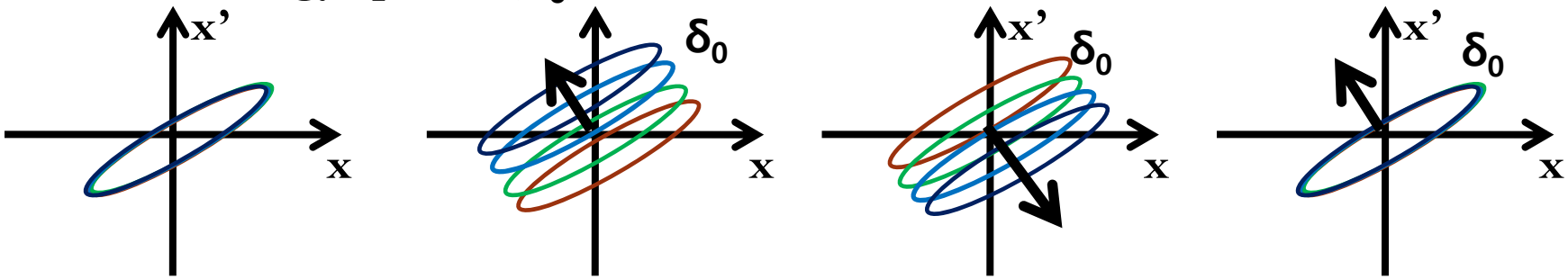
$k_0 = 0.453 \pm 0.0945$	*Initial energy spread*
$k_1 = 3.20 \pm 0.109$	*1st order SC effect*
$k_2 = 0.0924 \pm 0.033$	*2nd order SC effect*
$k_3 = -0.020 \pm 0.0027$	*3rd order SC effect*

SC effect in merger section



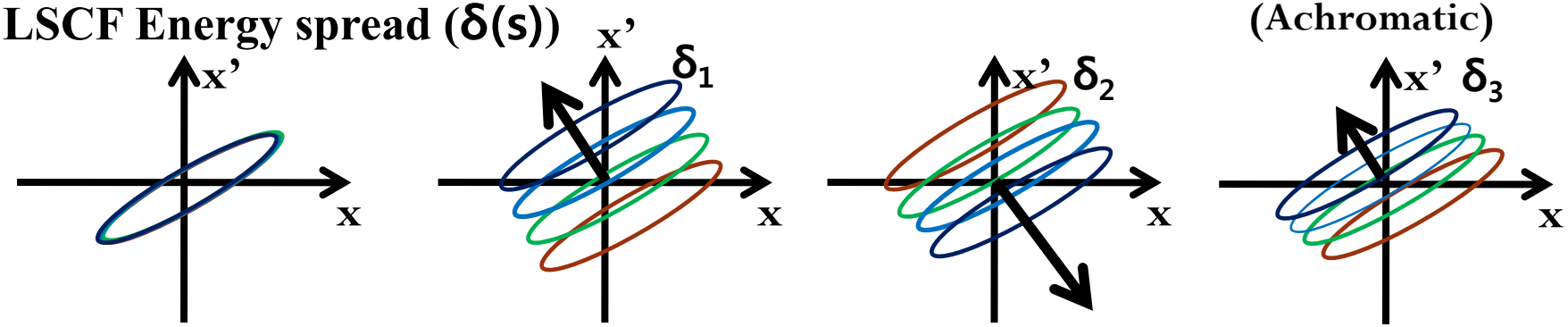
see for example, B.E. Carlsten et al., IEEE QE 27, 2580 (1991).

Uniform Energy spread (δ_0)



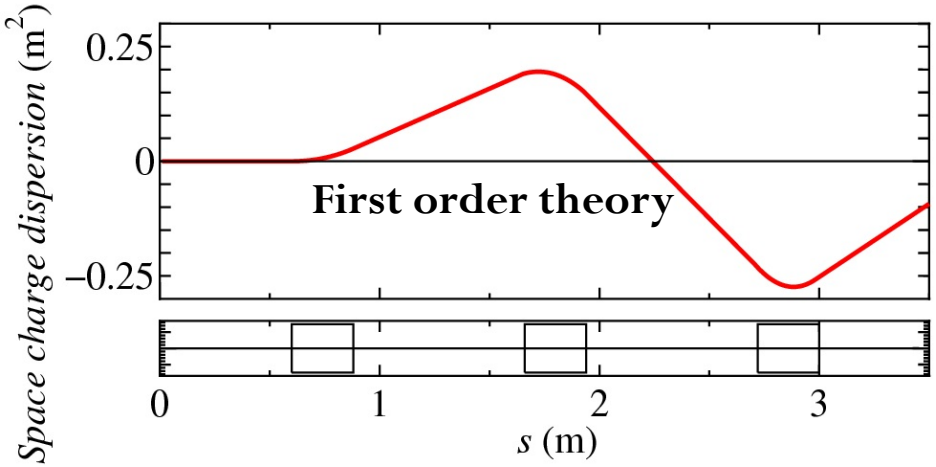
Initial Position | After 1st Bending | After 2nd Bending | Exit of merger (Achromatic)

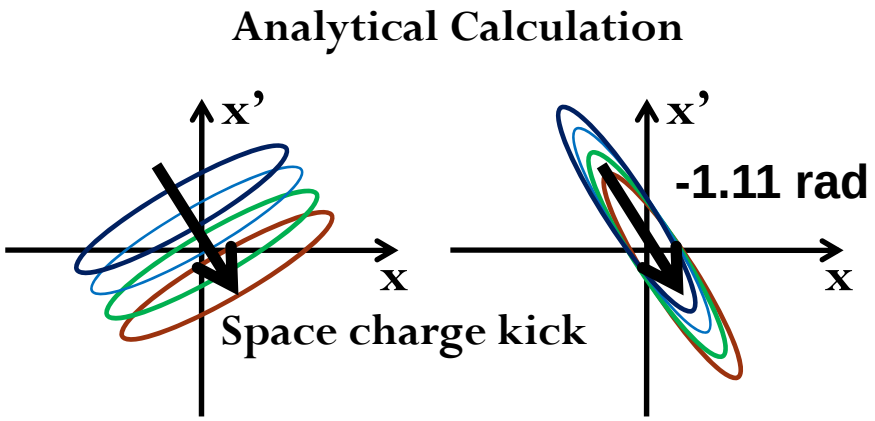
LSCF Energy spread ($\delta(s)$)



Initial Position | After 1st Bending | After 2nd Bending | Exit of merger

SC effect in merger section

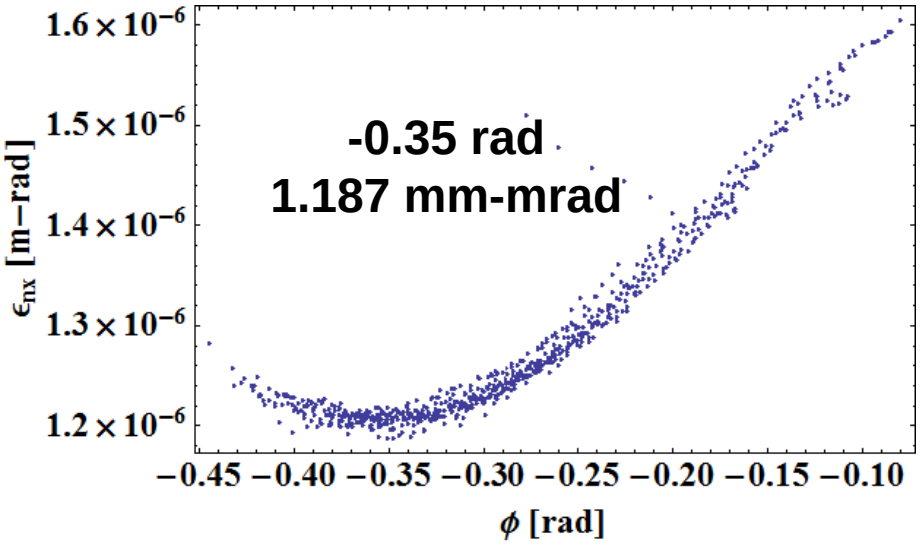




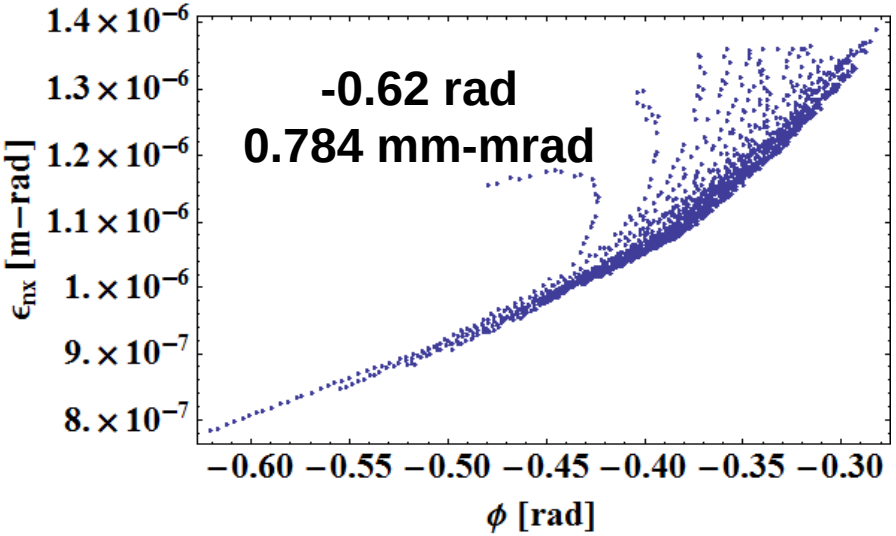
$$x'' + \frac{x}{\rho^2} = \frac{1}{\rho}(\delta_0 + \delta_{SC} + k_1(s-s_0) + k_2(s-s_0)^2 + k_3(s-s_0)^3)$$

$$\begin{pmatrix} x_f \\ x'_f \\ \delta_0 \\ \delta_{SC} \\ k_{1f} \\ k_{2f} \\ k_{3f} \end{pmatrix} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) & \rho(1-\cos\theta) & \rho^2(\theta-\sin\theta) & \rho^3(\theta^2+2\cos\theta-2) & \rho^4(\theta^4+6\sin\theta-6\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta & \sin\theta & \rho(1-\cos\theta) & 2\rho^2(\theta-\sin\theta) & 3\rho^3(\theta^2+2\cos\theta-2) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho\theta & (\rho\theta)^2 & (\rho\theta)^3 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ x'_i \\ \delta_0 \\ \delta_{SC} \\ k_{1i} \\ k_{2i} \\ k_{3i} \end{pmatrix}$$

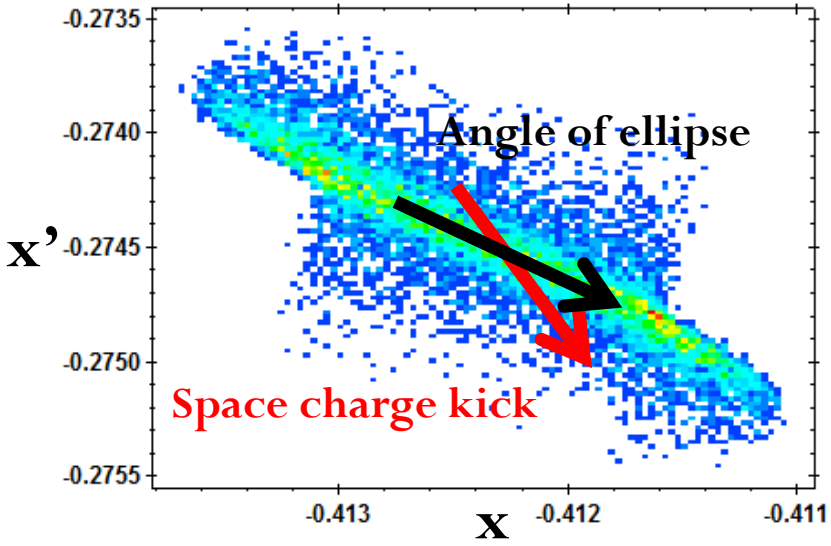
SC effect in merger section



Horizontal CS parameter scan



Vertical CS parameter scan



Energy: 5 MeV ε_{nx} = 0.1 mm-mrad

Charge: 77 pC ε_{ny} = 0.1 mm-mrad

$\Delta E/E_0$: 0.01 %

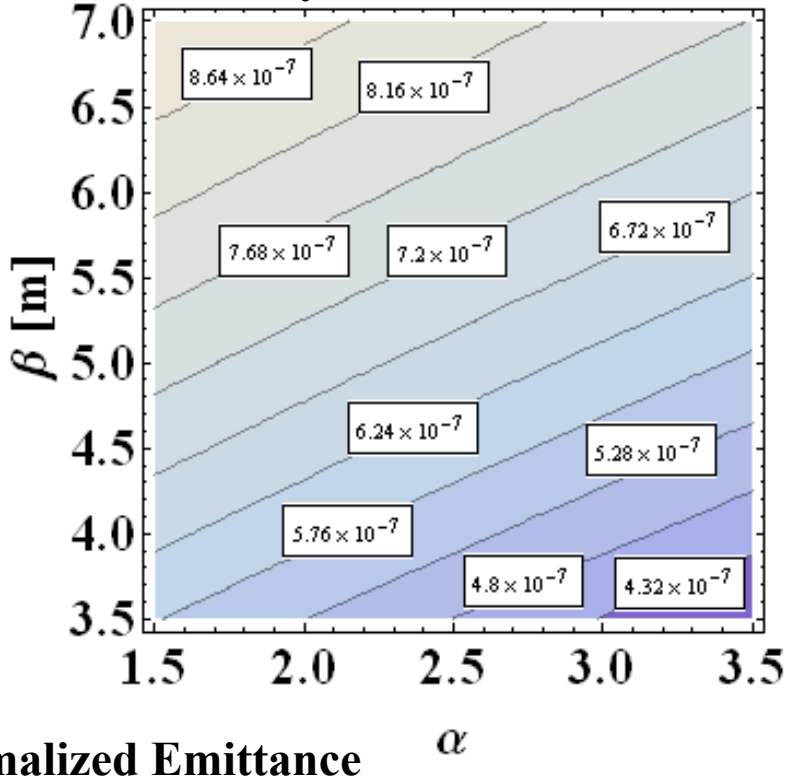
B unch length(rms): 0.9 mm (3 ps)

Macro-particles : 10 k

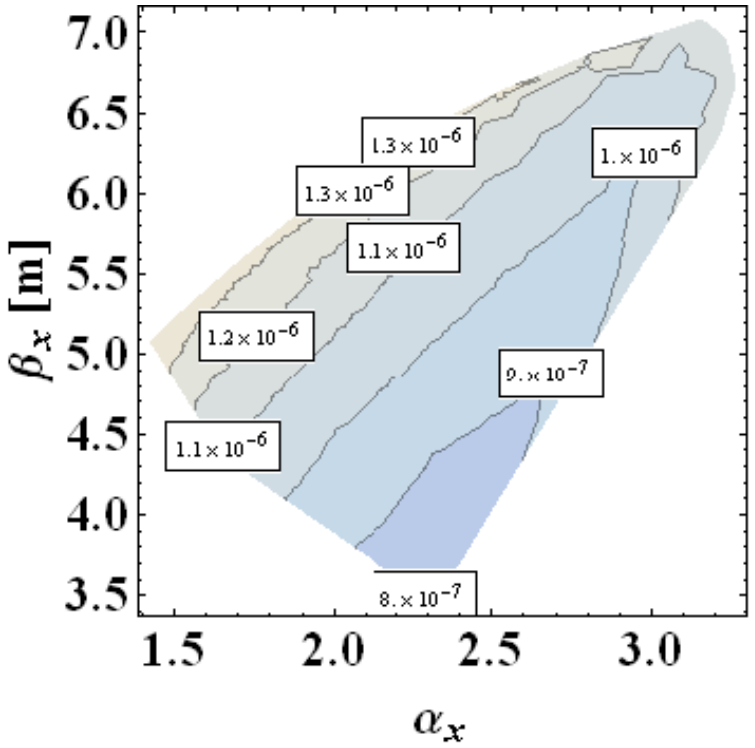
Horizontal emittance ~0.78 mm-mrad

SC effect in merger section

Analytical calculation



Numerical calculation



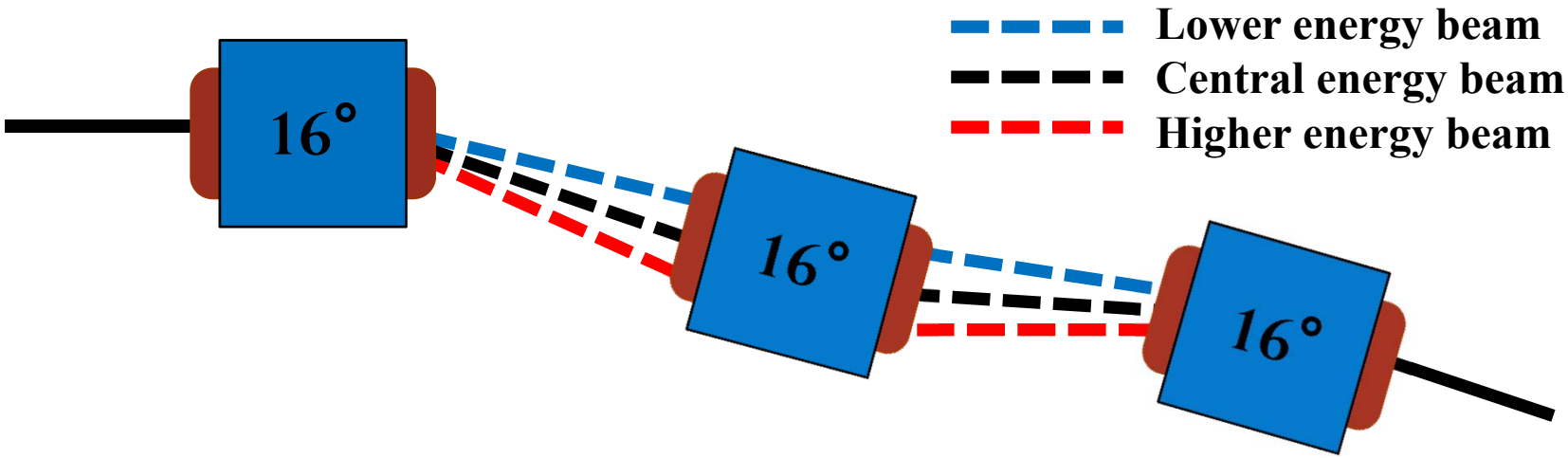
Normalized Emittance

$$\varepsilon_{nx} = ((\varepsilon_0\beta + D^2)(\varepsilon_0\gamma + D'^2) - (\varepsilon_0\alpha_x - DD')^2)\gamma_E$$

where $(D, D') = \Delta k_{rms}(\zeta_x, \zeta_x')$, $\Delta k_{rms} = 0.207\ \%/\mathrm{m}$

see for example, V.N. Litvinenko, R. Hajima and D. Kayran, NIM-A557, 165-175 (2006)

Effect of the energy spread



- A path length difference for particles with a relative energy deviation δ is given by:

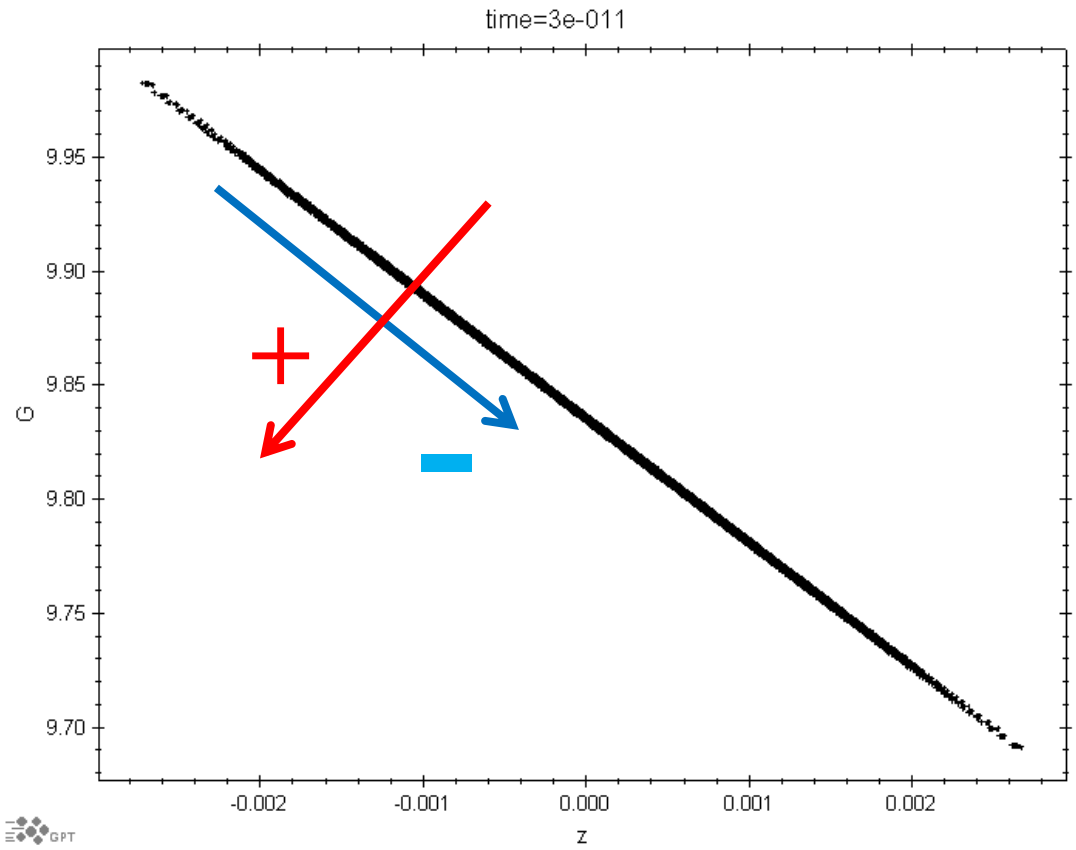
$$\Delta z = \eta\delta = R_{56}\delta + T_{566}\delta^2 + U_{5666}\delta^3 \ldots\ldots$$

Therefore, the growth of emittance in the merger section can be reduced by the changing of the energy spread of injected beam.

The merger has the following momentum compaction factors

$$R_{56} = -20.54mm \qquad T_{556} = 65.81mm$$

Effect of the energy spread



$$\left|d_z\right| = \frac{\sigma_E}{\sigma_z} = \frac{25keV}{0.9mm}$$

$$= -27.78\ [MeV/m]$$

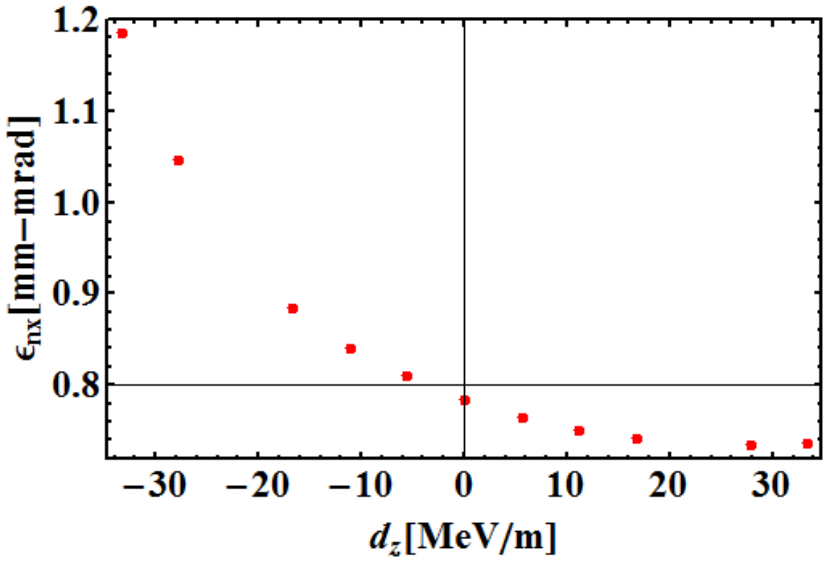
Energy : 5 MeV
Charge : 77 pC
ε_{nx} = 0.1 mm-mrad
ε_{ny} = 0.1 mm-mrad
Bunch length(rms) : 0.9 mm (3 ps)
Macro-particles : 10 k
Energy spread : 0.1 %

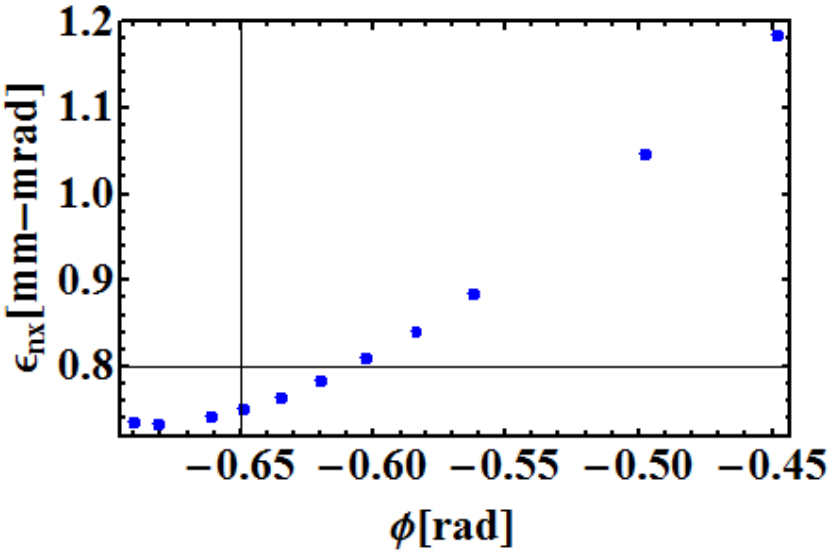
A path length difference due to the energy spread in merger

$$\Delta z = R_{56}\delta + T_{566}\delta^2 \approx 0.1mm \qquad R_{56} = -20.54mm$$

$$T_{556} = 65.81mm$$

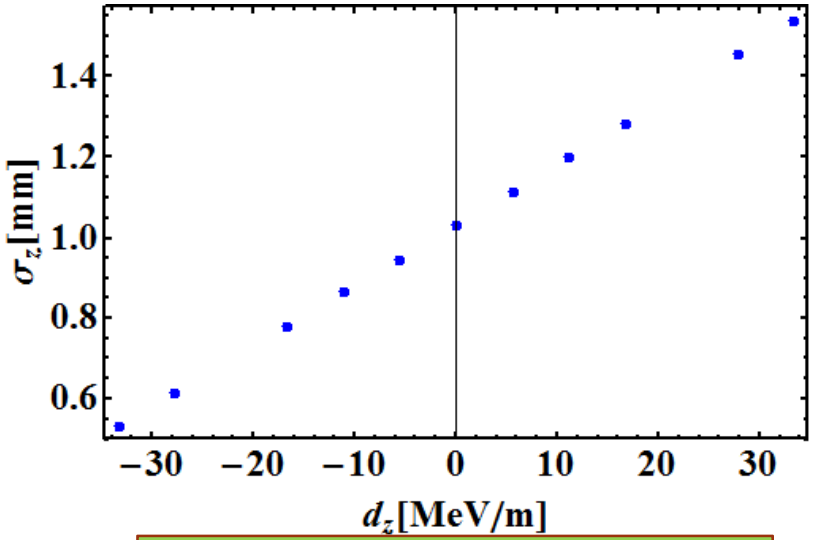
Effect of the energy spread





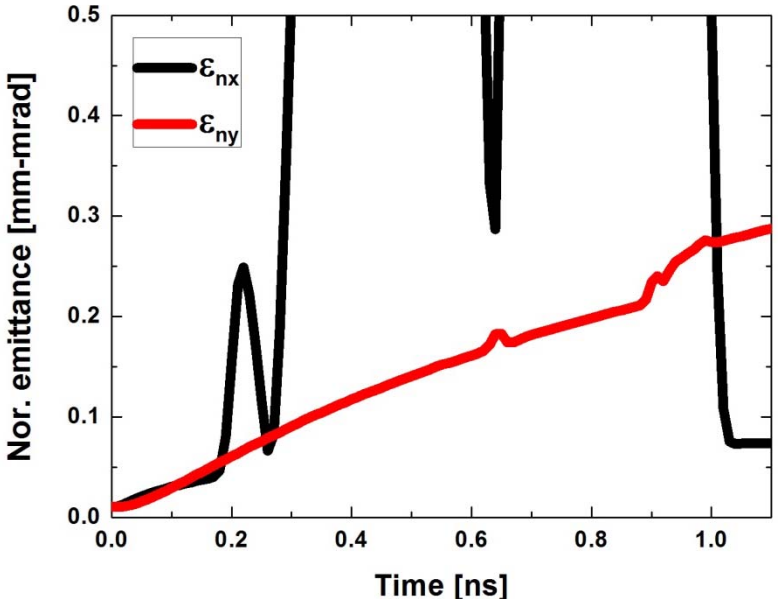
Energy : 5 MeV ε_{nx} **= 0.1 mm-mrad**
Charge : 77 pC ε_{ny} **= 0.1 mm-mrad**
Bunch length(rms) : 0.9 mm (3 ps)
Macro-particles : 10 k

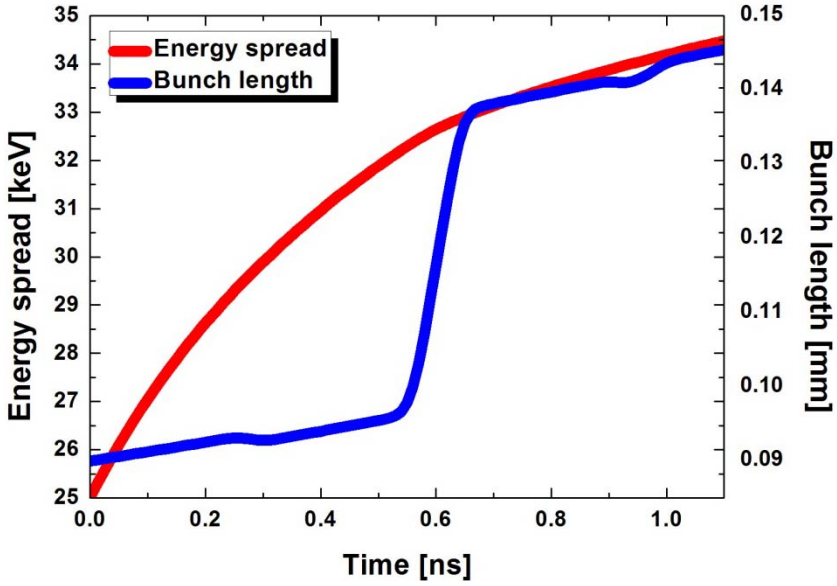
$$d_z = \frac{\sigma_E}{\sigma_z} = -40 \sim 40\ [MeV/m]$$

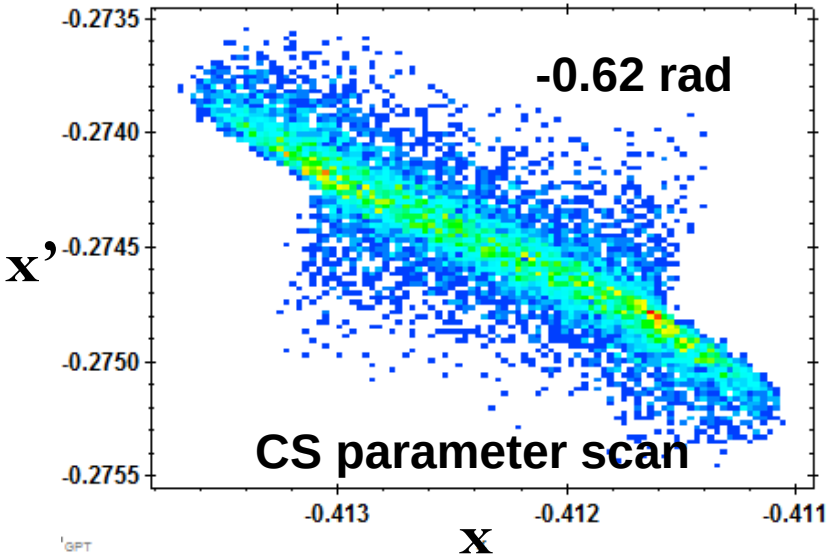


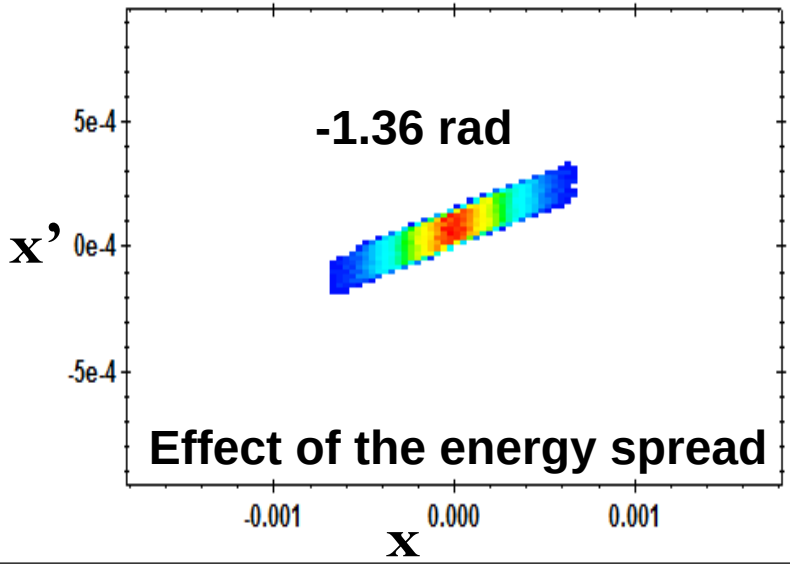
Horizontal emittance ~0.735 mm-mrad

Effect of the energy spread









Summary

The study of the space charge effect in the merger section of compact-ERL was performed to preserve the ultra-low transverse emittance in the ring.

In the very low energy case, around 5 MeV, the effect of changing of the length of bunch due to the SC effect was observed.

The minimum emittance, 0.78 mm-mrad, was achieved by changing of the Courant-Snyder parameter in merger section,.

The effect of the energy spread of injected beam was investigated. It is one of the useful parameter for minimization of the growth of transverse emittance in merger section.

Thank you for your attention !