MaRIE

(Matter-Radiation Interactions in Extremes)

Emittance Partitioning Through Controlling Eigen-Emittances

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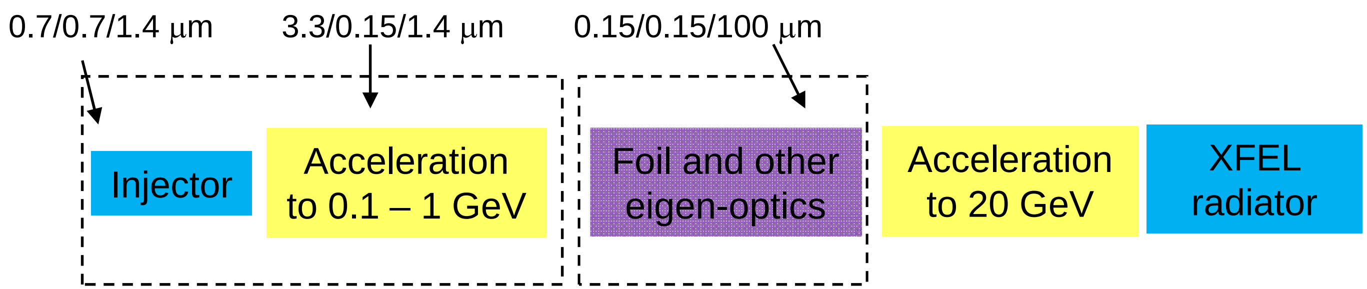
Overview

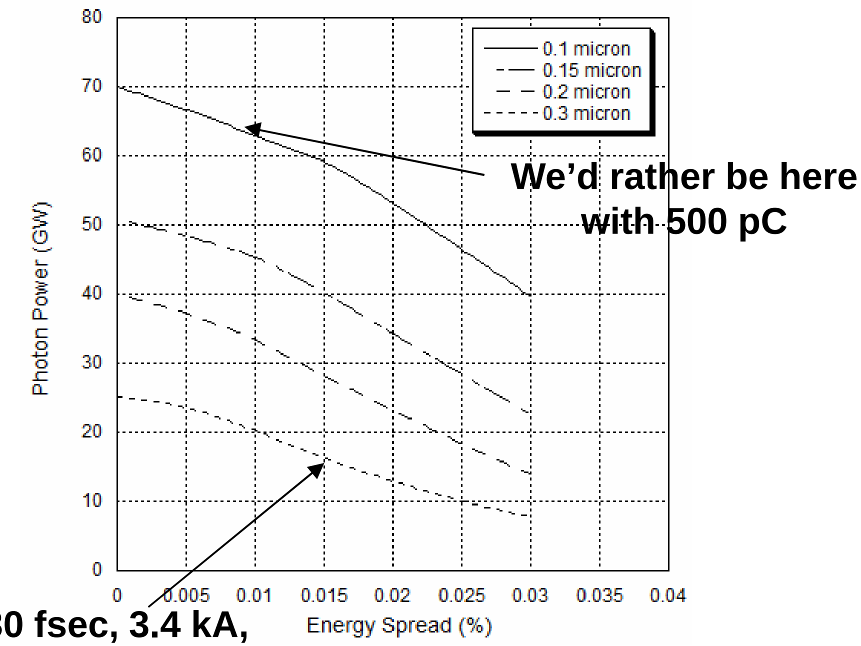
- Using eigen-emittance formalism to increase beam quality
- Flat-beam transforms (FBTs) and other eigen-emittance applications
- Four eigen-emittance schemes with potential to achieve very low emittances

Concept of Eigen-Emittance Partitioning



There is enough "spare" area in the longitudinal phase space to move excess area from the transverse phase spaces

It turns out the key controlling feature is how low the longitudinal energy spread can be kept



100 pC, 30 fsec, 3.4 kA, 0.015% energy spread 0.30 μm emittance

Eigen-Emittance Concept Can Be Used To Control Phase Space Partitioning

- Let σ denote the beam second moment matrix
- The eigenvalues of $J\sigma$ are called eigen-emittances

$$J = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

- Eigen-emittances are invariant under all *linear symplectic* transformations, which include all ensemble electron beam evolution in an accelerator

  however, the eigen-emittances can be *exchanged* among the x-p_x, y-p_y, z-p_z phase planes

- We can control the formation of the eigen-emittances by controlling correlations when the beam is generated (demonstrated in Flat-Beam Transforms (FBTs))
- We recover the eigen-emittances as the beam rms emittances when all correlations are removed

We Use Consistent Units to Describe Eigen-Emittances

$$\varsigma_{can}^{T} = (x, p_x, y, p_y, t, p_t)$$

$$\vec{p} = \vec{p}_{mech} + q\vec{A} \quad p_t = -\gamma mc^2$$

Canonical variables with arbitrary normalization

$$\varsigma_{can}^{T} = (x, (\gamma\beta_x / \gamma_0\beta_0), y, (\gamma\beta_y / \gamma_0\beta_0), c\Delta t, (\Delta\gamma / \gamma_0\beta_0))$$

Canonical variables with the "proper" (traditional) normalization

or: $\boxed{\varsigma^{T} = (x, x', y, y', c\Delta t, \Delta(\gamma\beta)/\gamma_0)}$

We use symplectic transformations along beamline:

$$\sigma_2 = R\sigma_1 R^T$$

$$\vec{\varsigma}_2 = R\vec{\varsigma}_1$$

$$J_6 = R^T J_6 R$$

$$J_6 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

What does Symplectic Mean in an RMS sense?

- Lorentz force law follows from a Hamiltonian:

$$H = c\sqrt{\left(\vec{p}_c - q\vec{A}(\vec{r},t)\right)^2 + m^2c^2} + q\phi(\vec{r},t)$$

- All electrodynamic motion satisfies Liouville's theorem

- If the Hamiltonian is quadratic in beam coordinates (transformation is lienar), then

$$J_6 = R^T J_6 R$$

- If the Hamiltonian is higher order in beam coordinates, the *rms* symplectic condition is no longer a requirement

$$J_6 \neq R^T J_6 R$$

Examples of Symplectic and Non-Symplectic Correlations

Axial field on the cathode (magnetized photoinjector) is an example of a nonsymplectic correlation (once the beam leaves the field region)

$$I + C_{total} = \left(I + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -a & 0 & 0 & 0 \end{pmatrix} \right) \left(I + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right) = \left(I + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 \\ -a & 0 & 0 & 0 \end{pmatrix} \right)$$

$$\mathbf{L} = \left|\langle xy' - yx' \rangle\right| / 2 = |a| \left(\sigma_x^2 + \sigma_y^2 \right) / 2$$

$$a = \frac{e}{2\gamma\beta mc} B_{cath} \left(\frac{R_{cath}}{R_{beam}} \right)^2$$

$$\sigma_{axial\ \ field} = \begin{pmatrix} \sigma_x^2 & 0 & 0 & -a\sigma_x^2 \\ 0 & \sigma_{x'}^2 + a^2\sigma_y^2 & a\sigma_y^2 & 0 \\ 0 & a\sigma_y^2 & \sigma_y^2 & 0 \\ -a\sigma_x^2 & 0 & 0 & \sigma_{y'}^2 + a^2\sigma_x^2 \end{pmatrix}$$

A skew-quad is an example of a symplectic transformation:

$$R_{skew} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & a & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\sigma_2 = R_{skew} \sigma_0 R_{skew}^T = \begin{pmatrix} \sigma_x^2 & 0 & 0 & a\sigma_x^2 \\ 0 & \sigma_{x'}^2 + a^2\sigma_y^2 & a\sigma_y^2 & 0 \\ 0 & a\sigma_y^2 & \sigma_y^2 & 0 \\ a\sigma_x^2 & 0 & 0 & \sigma_{y'}^2 + a^2\sigma_x^2 \end{pmatrix}$$

Easy to Write 4-D Eigen-Emittance Solution

Can find the eigen-emittances using the conservation of the 4-D determinant and of the "Raj" trace:

$$-\frac{1}{2}Tr\left(J\sigma J\sigma\right)$$

We can always make beam waists, eigen-emittances are then:

$$\sigma_{beam} = \begin{pmatrix} \bar{\sigma}_1^2 & 0 & D & B \\ 0 & \bar{\sigma}_2^2 & E & F \\ D & E & \bar{\sigma}_3^2 & 0 \\ B & F & 0 & \bar{\sigma}_4^2 \end{pmatrix}$$

(6D solution doesn't have simple form)

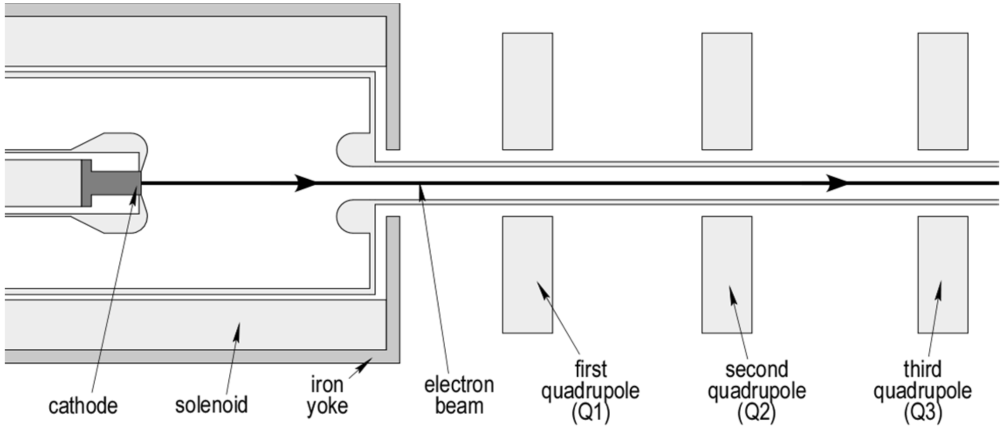
$$\boxed{\varepsilon_{eig,\pm}^2 = U \pm V}$$

where:

$$U = \frac{1}{2}\left(\bar{\sigma}_1^2\bar{\sigma}_2^2 + \bar{\sigma}_3^2\bar{\sigma}_4^2 - 2BE + 2FD\right) \quad V^2 = \frac{1}{4}\left(\bar{\sigma}_1^2\bar{\sigma}_2^2 + \bar{\sigma}_3^2\bar{\sigma}_4^2 - 2BE + 2FD\right)^2 - \left(\bar{\sigma}_1^2\bar{\sigma}_2^2\bar{\sigma}_3^2\bar{\sigma}_4^2 - F^2\bar{\sigma}_1^2\bar{\sigma}_3^2 - E^2\bar{\sigma}_1^2\bar{\sigma}_4^2 - D^2\bar{\sigma}_2^2\bar{\sigma}_4^2 - B^2\bar{\sigma}_2^2\bar{\sigma}_3^2 + D^2F^2 + E^2B^2 - 2EBDF\right).$$

Hard to see from this form, but any correlation makes the difference between 2 eigen-emittances larger

The Flat Beam Transform (FBT) is an Example



$$\sigma_{FBT} = \begin{pmatrix} \sigma_x^2 & 0 & 0 & -a\sigma_x^2 \\ 0 & \sigma_{x'}^2 + a^2\sigma_x^2 & a\sigma_x^2 & 0 \\ 0 & a\sigma_x^2 & \sigma_x^2 & 0 \\ -a\sigma_x^2 & 0 & 0 & \sigma_{x'}^2 + a^2\sigma_x^2 \end{pmatrix}$$

Observed emittances: $\varepsilon_{beam} = \sqrt{\varepsilon_0^2 + \mathbf{L}^2}$

$$\mathbf{L} = \frac{e|B_{cath}|R_{cath}^2}{8\gamma\beta cm} = |a|\sigma_x^2 = \frac{1}{2}\left|\langle xy' - yx'\rangle\right|$$

Eigen-emittances: $\varepsilon_{eig,-} = \frac{\varepsilon_0^2}{2\mathbf{L}}$

$\mathbf{L}^2 >> \varepsilon_0^2$

$\varepsilon_{eig,+} = 2\mathbf{L}$

FBT is protected from nonlinearities by symmetry and conservation of canonical angular momentum

These are always zero

$$\sigma_{XY} = \begin{pmatrix} \langle xy\rangle & \langle xy'\rangle \\ \langle x'y\rangle & \langle x'y'\rangle \end{pmatrix}$$

So you can always define intrinisic emittances so the FBT equations hold

$$\varepsilon_0 = \sqrt{\varepsilon_{beam}^2 - \mathbf{L}^2}$$

We Have Thought About Four Ways to Get Low Emittances

1. Thin pancake with axial field

2. Asymmetric beam with laser tilt

3. Magnetized photoinjector and nonsymplectic foil/undulator (using ISR or Bremstrahlung)

4. General three-dimensional couplings

We are currently evaluating these options

We typically consider an “ideal” photoinjector with nominal emittances (x,y,z) of 0.7/0.7/1.4 μm, with target eigen-emittances of 0.15/0.15/30 μm, but 4:1 ratio in final transverse emittances almost as good

(z-emittance can actually be as high as 200 μm)

The problem comes down to how low the energy spread (and longitudinal emittance) can be maintained

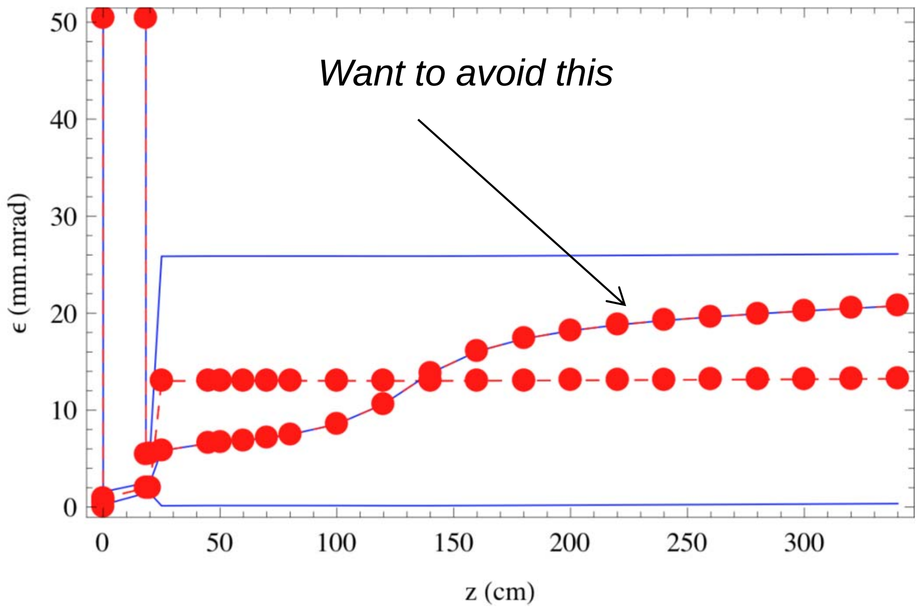
Super-Thin Pancake Off Photocathode

1. Start with a super-short pancake of charge, emittances of 2.1/2.1/0.15 μm, all in a magnetized photoinjector

2. Use a FBT to adjust these numbers to 0.15/30/0.15 μm

3. Use an EEX to swap y and z and end up with 0.15/0.15/30 μm

Problem with this approach is that the phase space volume is not conserved in conventional photoinjectors (*especially growth in the z phase space*).

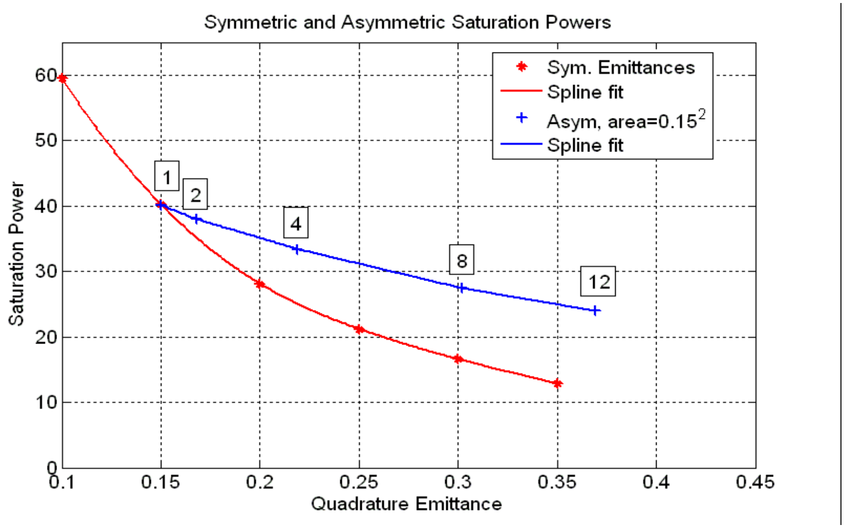


Asymmetric Beam with Titled Drive Laser

1. Start with 5.3:1 ellipticity at cathode (1.61/0.3/1.4 μm)

2. Use a 83° laser tilt (2.3/0.43-mm radius cathode, 3.3-psec long pulse)

3. Eigen-emittances are: 0.075/0.3/30 μm, about 15% decrease in x-ray flux:

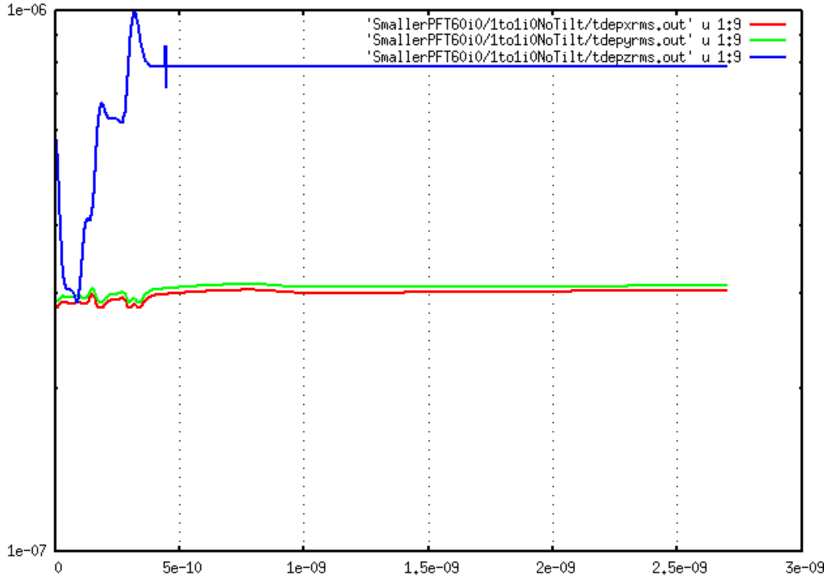


Problem with this approach is that there is no conservation property that helps us and space charge nonlinearities may be an issue, we're studying this (later)

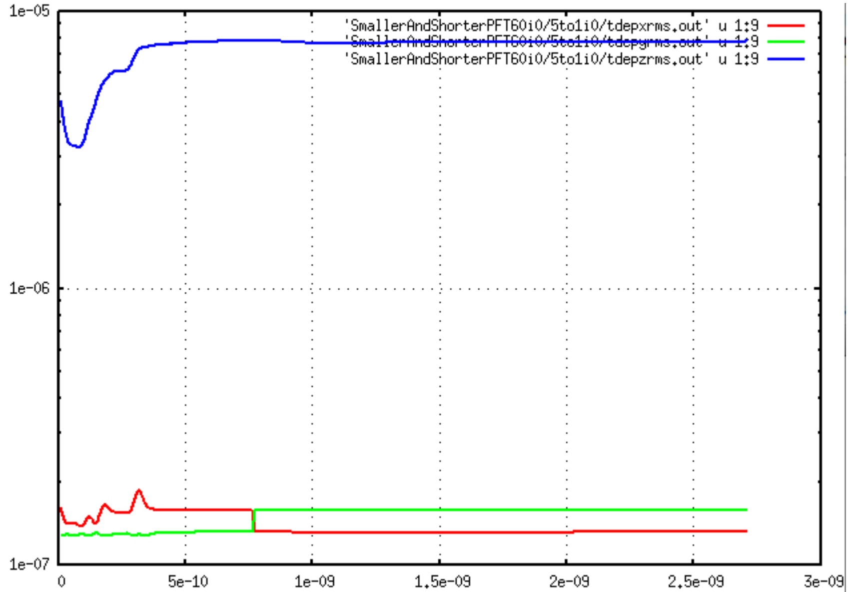
Asymmetric Beam with Titled Drive Laser

IMPACT simulations verify the basic idea, but small decreases in the transverse emittance can lead to large increases in the longitudinal emittance due to rf effects

Emittance plotted versus longitudinal position below, equivalent to 100 pC simulations using IMPACT, with 10x charge typical of 0.1 μm emittances



1:1, no laser tilt



5:1, with laser tilt

Magnetized Photoinjector and Nonsymplectic Element

1. Start with round beam at cathode (0.7/0.7/1.4 μm)

2.FBT in the usual way gives 3.3/0.15/1.4 μm

3.Can use ISR from an undulator or wedge-shaped foil to generate correlation between *x* and energy

ISR: $\Delta E[\mathrm{MeV}] = 6.31\cdot 10^{-4}\, E^{2}[\mathrm{GeV}]\, B^{2}[\mathrm{T}]\, L[\mathrm{m}]$ leads to too long undulators if under a few GeV, wedge-shaped foil may be better

4. Use a wedge-shaped foil at 1 GeV to provide say 100 keV more attenuation at one horizontal end of the beam than the other, final eigen-emittances might be 0.2/0.2/90 μm (there is an emittance hit)

Problem with this approach is that there is both a transverse emittance growth and an energy spread (both from scattering), but it looks promising

IEEE Transactions on Nuclear Science, Vol. NS-30, No. 4, August 1983

REDUCTION OF BEAM EMITTANCE BY A TAPERED-FOIL TECHNIQUE*

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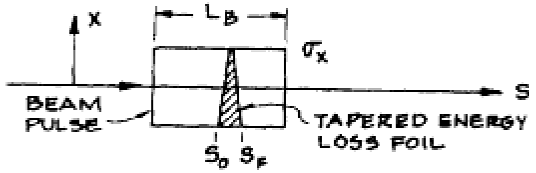


Figure 1. Geometry of the Tapered Energy-Loss Foil

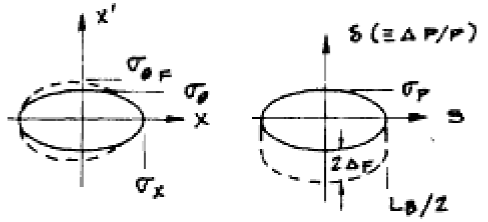


Figure 2. Transverse and Longitudinal Phase Spa Occupied by Beam Before (solid) and After (dashed) the Tapered Energy-Loss Foil.

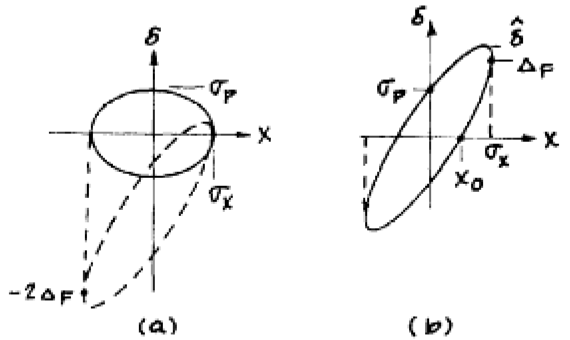


Figure 3. (a) x,δ Phase Space Occupied by Beam Before (solid) and After (Dashed) the Tapered Foil (b) After the Foil with Momentum Renormalized to the Center Momentum.

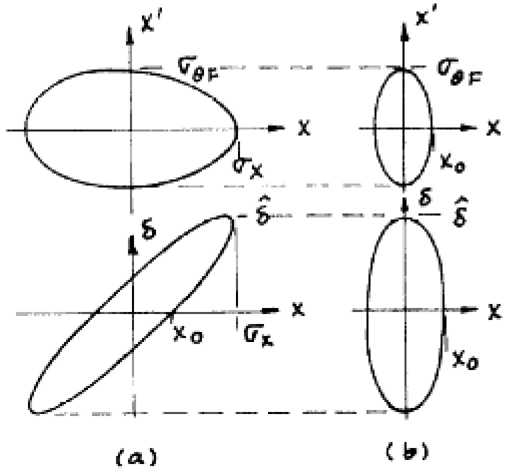


Figure 5. x,x' and x,δ Phase Space Occupied (a) At S_F, the Exit Side of the Tapered Foil and (b) At S1, the Image Point with Zero Dispersion.

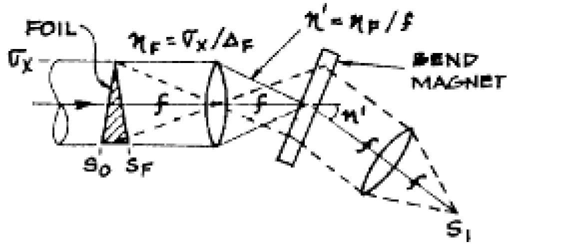


Figure 4. One-Dimensional Beam Transport System that Removes Dispersion and Has One-to-One Imaging of S_F at S_1. Focal length of each lens is f.

Peterson's optics has the same horizontal emittance reduction but does not recover the third eigen-emittance (we probably need both). Peterson's optics also point out the value of an alternative x'-z' transform.

First reported by Claud Bovet LBL-ERAN89, June 1970.

Foil Idea May Work, Stimulating Other Concepts

We nominally start with a magnetized photoinjector to get $\varepsilon_{x,n} / \varepsilon_{y,n} / \varepsilon_{z,n}$ = 3.3/0.15/1.4 μm

Non-symplectic element separates issues and simplifies design.

Induced angular scattering and increased energy spread limit effectiveness, still might get factors of ten improvement

Intrinsic energy spread and emittance

$$\varepsilon_{x,\,final} = \frac{\left(\left(\frac{\Delta\gamma}{\gamma}\right)_{ind}^2 + \left(\frac{\Delta\gamma}{\gamma}\right)_{int}^2\right)^{1/2}}{\left(\frac{\Delta\gamma}{\gamma}\right)_{slew}}\left(\varepsilon_{ind}^2 + \varepsilon_{x,int}^2\right)^{1/2} \qquad \varepsilon_{z,\,final} = \gamma\left(\frac{\Delta\gamma}{\gamma}\right)_{slew}\sigma_z$$

You can do an exact eigen-emittance recovery, if you wish, but it's hard, prone to second-order effects, and you don't need to – simple asymmetric chicane works fine

$$M_{\text{s-chicane}} = \begin{pmatrix} 1 & L_1 & 0 & \eta_1 \\ 0 & 1 & 0 & 0 \\ 0 & \eta_1 & 1 & \varepsilon_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & L_2 & 0 & -\eta_2 \\ 0 & 1 & 0 & 0 \\ 0 & -\eta_2 & 1 & \varepsilon_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & L_t & 0 & \Delta\eta \\ 0 & 1 & 0 & 0 \\ 0 & \Delta\eta & 1 & \varepsilon_t \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$L_t = L_1 + L_2$$

$$\Delta\eta = \eta_1 - \eta_2$$

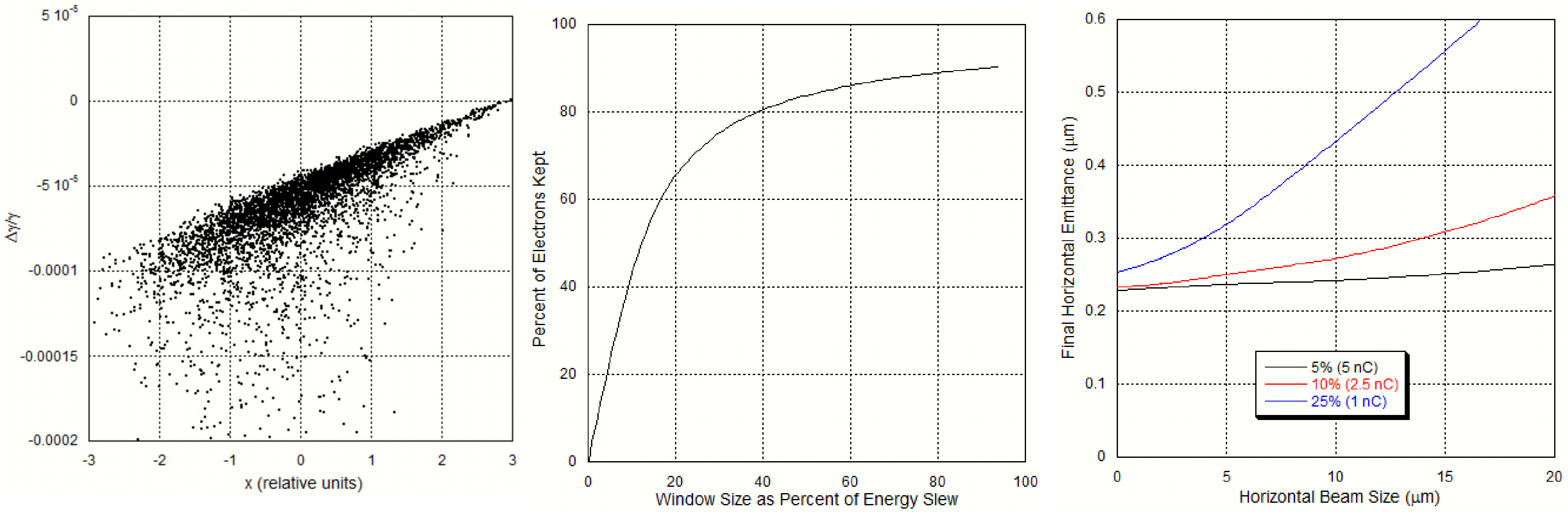
$$\varepsilon_t = \varepsilon_1 + \varepsilon_2$$

$$\Delta\eta = 0.049\ \text{m}$$

$$\varepsilon_x^2\varepsilon_z^2 = \varepsilon_{x0}^2\varepsilon_{z0}^2 + \eta^2\left(\varepsilon_{x0}^2 + \varepsilon_{z0}^2\right)\left\langle x_0'^2\right\rangle\left\langle z_0'^2\right\rangle + \eta^4\left\langle x_0'^2\right\rangle^2\left\langle z_0'^2\right\rangle^2$$

The growth in the product of the emittances of only about 1%.

Wedge Foil Results with 250 pC of Final Charge



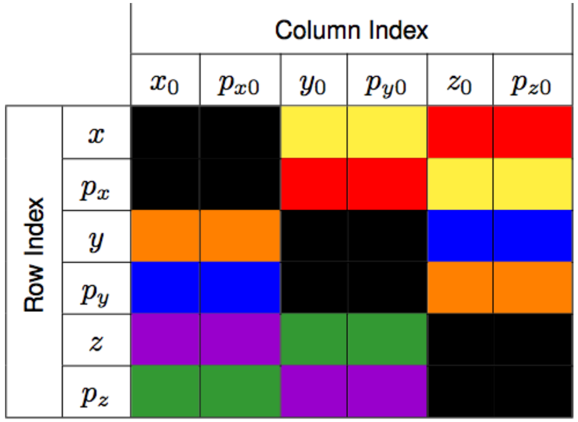
Requires some amount of scraping. Fairly insensitive to fraction kept (10% or less), energy (100 MeV to 1 GeV), and factors of a few for magnitude of energy slew (target is 0.25/0.25/90 μm).

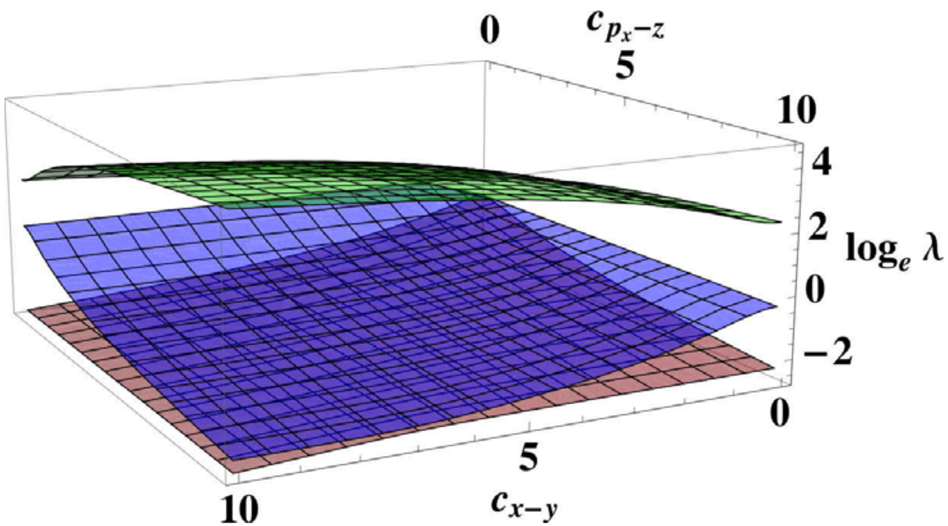
Dominated by beam's intrinsic slice energy spread.

You Can Also Consider General 6-D Couplings

1. Start with round beam at cathode (0.7/0.7/1.4 μm)

2. Pick combination where row index is function of column index; issue here is to identify some combinations that are least sensitive to photoinjector nonlinearities, ongoing research (later)





Summary

- Future XFEL designs will require higher brightness electron beams
- Exploiting eigen-emittances may lead to a new way of achieving very low transverse emittances by moving excess transverse phase space into the longitudinal dimension
- There are several ways to implement eigen-emittance concept
- Two-stage generation of beam correlations (using a non-symplectic beamline element) may be a practical application of eigen-emittances