Intra-Beam Scattering (IBS) and its application to ERL

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Outline

An attempt to give review talk in 20 minutes ...

- **Single and multiple Coulomb scattering (Touschek and IBS)**
- **General models for Gaussian distribution**
- **Analytic estimate (of halo) for various distributions in linear accelerators**
- **Touschek scaling and multi-pass ERLs**
- **IBS for non-Gaussian distributions**
- **IBS in ERLs:**

- diffusion coefficients for "flattened" distribution

- sliced approximation vs. diffusion coefficients

Coulomb scattering

Charged particle within the beam scatter via Coulomb collisions.

In general, one should distinguish between

1) large-angle single scattering events
2) multiple small-angle scattering events

The scattering by a relatively large angle in a single encounter is often less likely than a net large angle deflection due to the cumulative effect of many small angle scatterings.

In accelerators, both effects were found important:

1) The effect when particles can be lost as a result of single collision, where energy transfer from horizontal to longitudinal direction is amplified due to relativistic γ, is typically called the TOUSCHEK effect.

2) When scattering angles are sufficiently small, addition at random of many such collisions causes beam dimensions to grow (similar to diffusion in gas). The second effect was called by different names, for example, "Multiple Coulomb scattering", "Multiple Touschek effect" and "IBS", which is now a typical convention for circular machines.

A little bit of history of basic IBS theories (sampled)

1. In circular accelerators, multiple Coulomb scattering was first applied to explain emittance growth in electron machines:

   Bruck, Le Duff (1960's) – called "multiple Touschek effect".

2. It was later generalized by Piwinski for protons machines, without making any restrictions on beam temperatures.

   Piwinski (1974) – called "IBS". Assumed smooth lattice of an accelarator.

3. The IBS theory was later extended to include variations of the betatron functions and momentum dispersion function along the lattice:

   Sacherer, Mohl, Hubner & Piwinski (end of 70's) – described in CERN report by Martini (1984).

4. Another approach for calculation of IBS rates based on the scattering matrix formalism from quantum electrodynamics was used:

   Bjorken and Mtingwa (1982-83).

   Since then, a variety of models were developed which provide simple approximate results for some regimes of applicability, as well as some refinements of basic theories were done.

Typical limitations of generally used IBS models

1. The use of non-relativistic scattering cross section.

   (not a significant effect)

1. The use of Gaussian beam distribution to construct expressions for the growth rates.

Some examples which try to address these limitations:

1. Generalization to relativistic cross section (T. Toyomasu, 1992).
2. Analytic treatment of Coulomb collisions for various distributions; also includes space-charge and halo extent (Gluckstern & Fedotov, PRST 1999; N. Pichoff PAC'99).
3. IBS formalism for arbitrary distributions in 3-D, based on diffusion coefficients, was implemented in BETACOOL code in 2007 (comparison with experimental data reported in HB2010 workshop; references therein to other approaches and models for non-Gaussian distributions).

Coulomb scattering in linear accelerators
(Gluckstern & Fedotov, Phys. Rev. ST 1999)

Motivated by studies of HALO formation in high-intensity proton linacs

Both single and multiple scattering was estimated for a variety of distribution functions (in 3-D):

$$\frac{dP}{dt} = \int dr \int d\Omega \int dv_1 f(r, v_1) \int dv_2 f(r, v_2) |v_1 - v_2| \frac{d\sigma}{d\Omega}$$

$$f(\boldsymbol{r}, \boldsymbol{v}) = \begin{cases} N(H_0 - H)^n = N[G(r) - mv^2/2]^n, & H < H_0, \\ 0, & H > H_0, \end{cases}$$

$$H(\boldsymbol{r}, \boldsymbol{v}) = mv^2/2 + kr^2/2 + e\Phi_{sc}(r)$$

11-dimensional integral was evaluated analytically for a variety of distribution functions (from singular to Maxwellian)

Single large-angle scattering:

$$\frac{dP}{cdt} \sim \begin{cases} r_p^2/\epsilon_N^3, & n > 0, \\ (r_p^2/\epsilon_N^3)\ln(\epsilon_N^2/r_p a), & n = 0, \\ (r_p^2/\epsilon_N^3)(\epsilon_N^2/r_p a)^{-n}, & 0 < -n < 1 \end{cases}$$

Multiple small-angle scattering:

$$\frac{dP}{cdt} \sim \frac{r_p^2}{\epsilon_N^3} \ln\left(\frac{1}{\theta_D}\right)$$

- **Scattering was found negligible for proton linear accelerators.**
- **However, it may become important for electron linacs with high phase-space density like in some proposed ERLs.**

Order of magnitude estimate

In lab frame, probability of particles to scatter outside of beam core (fractional loss rate) is [1/m of accelerator]:

$$\frac{1}{c}\frac{dP}{dt} \propto K_n \frac{1}{\gamma} \frac{r_c^2}{\varepsilon_{n,\perp}^2 \varepsilon_{n,z}} \Lambda_n \Lambda_c$$

Here, $\mathbf{K_n}$ is normalization constant which depends on the distribution. $\Lambda_\mathbf{n}$ is an additional logarithm needed for singular and uniform distributions, not needed for a Gaussian distribution, $\Lambda_\mathbf{c}$ is the Coulomb logarithm needed for multiple scattering (PRST'99).

Example:

ERL with short bunches.

small emittance and high charge.

γ	1000
ε, μm (rms, normalized)	1
σ_p	1e-3
σ_z, ps	1

$$\frac{1}{\tau} \propto N \times 8 \cdot 10^{-8} \left[\frac{1}{s}\right]$$

N=1.25*10^{10} (2 nC):

τ^{-1}: 1000 1/s

(could be a noticeable effect for low-energy transport due to $1/\gamma^2$)

However, it was pointed out (Xiao, Borland, PAC'09, ERL09) that local IBS rates could be much stronger due to very small local momentum spread in a longitudinal slice of electron beam in ERL.

will be discussed later in this talk.

Large-angle single scattering and Touschek lifetime

- 1-D (flat beam) Touschek

(Bernardini et al.; Haissinski; Bruck; Le Duff; Volkel, 60's):

$$\frac{1}{N}\frac{dN}{dt} = \frac{\sqrt{\pi}}{\gamma^3}\frac{c r_c^2 N C(\varepsilon)}{(4\pi)^{3/2}\theta_x \sigma_x \sigma_y \sigma_z (\Delta E_m / E)^2}$$

$$\varepsilon \equiv \left(\frac{\Delta E_m / E}{\gamma \theta_x}\right)^2$$

for small ε:

$$C(\varepsilon) = \ln\left(\frac{1}{1.78\varepsilon}\right) - 1.5$$

- 2-D (round beam) Touschek (Miyahara, 1985):

$$\frac{1}{N}\frac{dN}{dt} = \frac{2\pi}{\gamma^4}\frac{c r_c^2 N F(\varepsilon)}{(4\pi)^{3/2}\theta_y \theta_x \sigma_x \sigma_y \sigma_z (\Delta E_m / E)}$$

$$F(\varepsilon) = \frac{1}{\sqrt{\varepsilon}} - 6$$

- 2-D generalized expression (Piwinski, 1998):

Most universal expression.

Already used for transport lines like ERLs:

(Xiao & Borland, PAC'07; Hoffstaetter et al., EPAC'08).

Note: In 2008-09 papers from Cornell this effect is referred to as "IBS" which could lead to some confusion.

Touschek scaling and multi-pass ERLs

For multi-pass ERLs, with most losses expected for decelerated beam, we are interested in estimating accumulated energy distribution of scattered particles.

For this, instead of loss rate outside relative energy acceptance for fixed energy, we express rate in terms of scattering outside specific energy deviation in MeV (not lost).

Scaling becomes apparent using approximate expression for function $F(\varepsilon)$ (Miyahara, 1985):

$$\frac{1}{N}\frac{dN}{dt} = \frac{2\pi}{\gamma^4}\frac{c r_c^2 N F(\varepsilon)}{(4\pi)^{3/2}\theta_y\theta_x\sigma_x\sigma_y\sigma_z(\Delta E_m / E)}$$

$$\varepsilon \equiv \left(\frac{\Delta E_m / E}{\gamma\theta_x}\right)^2$$

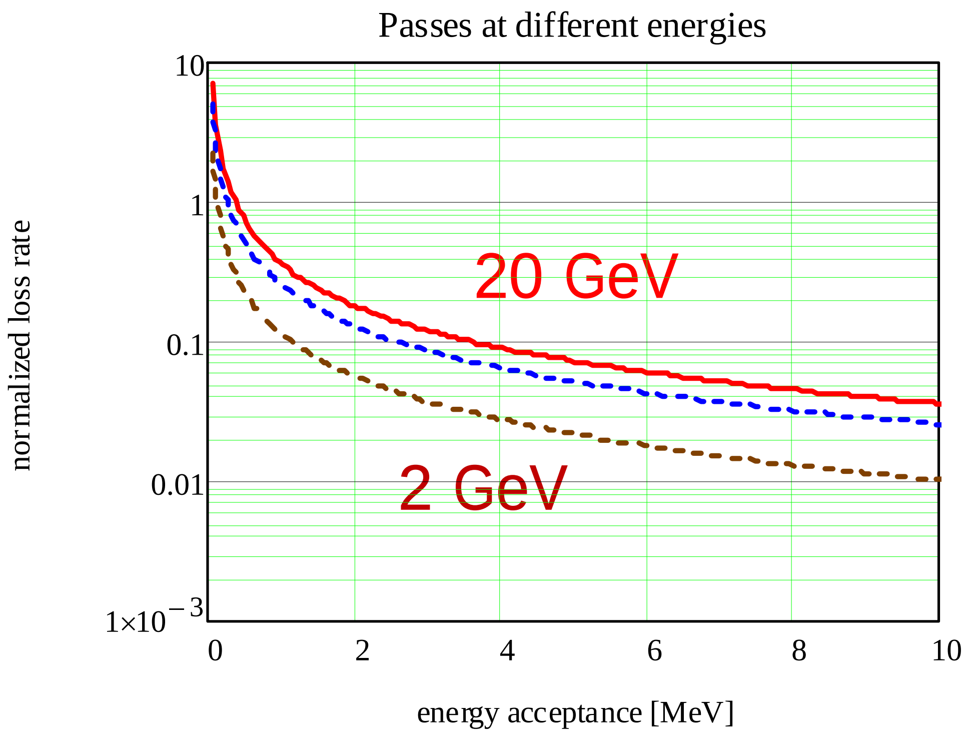
for small ε:

$$F(\varepsilon) = \frac{1}{\sqrt{\varepsilon}} - 6$$

one gets:

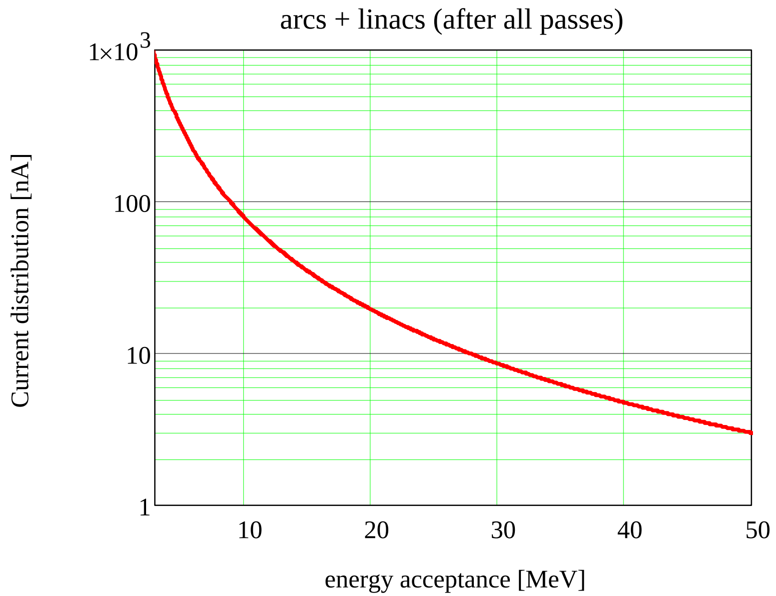
$$\frac{1}{N}\frac{dN}{dt} = \frac{2\pi c r_c^2 N \gamma^{1/2}}{(4\pi)^{3/2}\varepsilon_{ny}\varepsilon_{nx}\sigma_z(\Delta E_m)^2}\sqrt{\frac{\varepsilon_{nx}}{\beta_x}}$$

Touschek scaling with energy (multi-pass ERL for eRHIC)



Contribution from high-energy passes is important.

Example:
Final distribution for eRHIC (20 GeV) after 12 passes.



Sketch of typical IBS calculation

1. Go to Particle Rest Frame.
2. Assume Rutherford scattering cross section to describe scattering between pairs of identical particles.
3. Compute the change of single-particle emittances in a given binary collision.
4. Assume simple Gaussian phase-space distribution.
5. Average over all collisions (all positions, angles and energy deviations).
6. Write expressions for growth rates of beam dimensions in laboratory frame of reference.

Resulting expressions are amplitude independent and in general cannot be used for accurate time evolution of non-Gaussian distributions, especially when one is interested in evolution of distribution in the presence of some other amplitude-dependent force (like electron cooling, for example). This problem was extensively studied in electron cooling community.

3-D IBS model for non-Gaussian distribution

(based on amplitude dependent diffusion coefficients)

After extensive development, the model was implemented in BETACOOL (JINR, Dubna) code in 2007 (A. Sidorin, A. Smirnov et al., BNL-BETACOOL report December 2007).

$$\vec{F} = \frac{\langle \Delta\vec{p} \rangle}{\Delta t} = -\frac{4\pi n e^4 Z_t^2 Z_f^2}{\left(\frac{m_f m_t}{m_f + m_t}\right)} \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{\vec{U}}{U^3} f(v) d^3 v$$

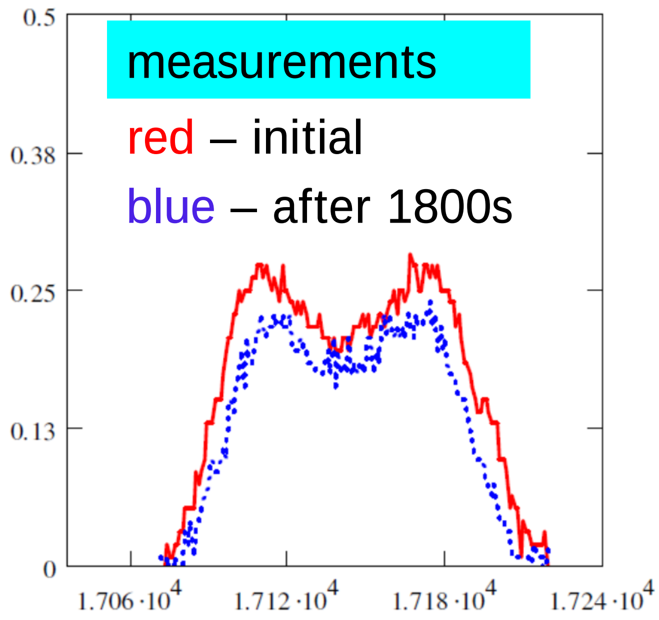
$$\begin{pmatrix} D_{x,x} & D_{x,y} & D_{x,z} \\ D_{x,y} & D_{y,y} & D_{y,z} \\ D_{x,z} & D_{y,z} & D_{z,z} \end{pmatrix}$$

$$D_{\alpha,\beta} = \frac{\langle \Delta p_\alpha \Delta p_\beta \rangle}{\Delta t} = 4\pi n e^4 Z_t^2 Z_f^2 \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{U^2 \delta_{\alpha,\beta} - U_\alpha U_\beta}{U^3} f(v) dv$$

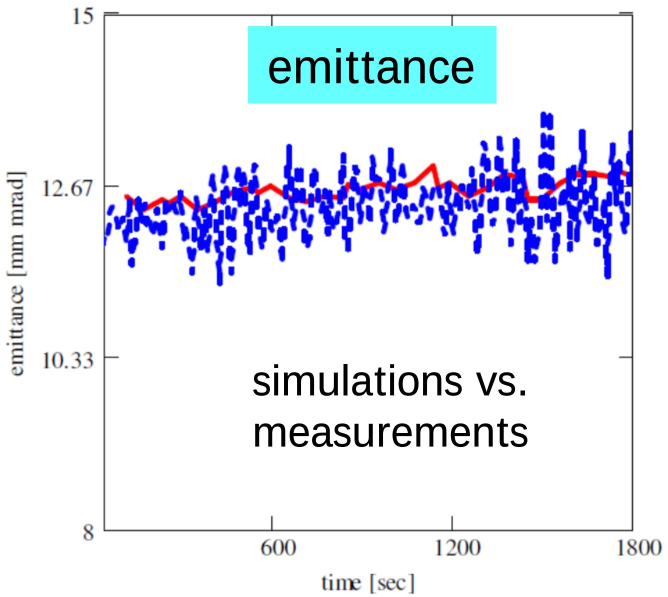
The Fokker-Planck equation is replaced by an equivalent system of Langevin equations. The program solves Langevin equation for each model particle from the particle array. The particle momentum during simulations is changed regularly by action of a friction force and randomly by diffusion (for details see "IBS for non-Gaussian distributions" (HB2010, Switzerland, p. 62) and references therein).

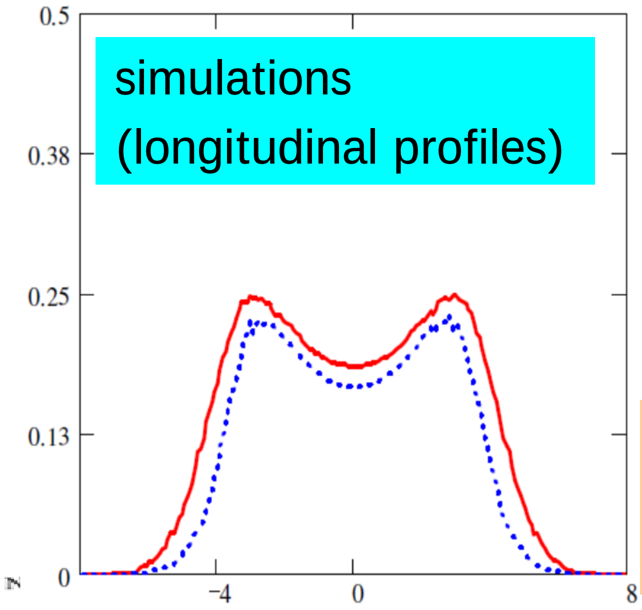
This model of IBS in BETACOOL code is called "local diffusion".

Benchmarking of “local IBS” model with experimental data

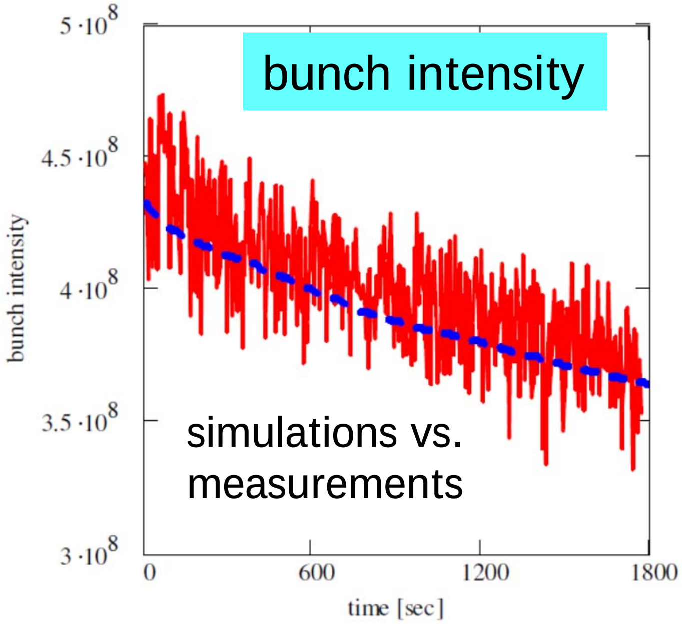


Experimental data from RHIC APEX experiment with hollow longitudinal distribution (Wei et al., HB2004).





“IBS for non-Gaussian distributions” (HB2010, Morschach, Switzerland)



IBS treatment for ERL distribution

- "Local IBS" approach based on amplitude-dependent diffusion coefficients (as in BETACOOL) can be used as well.
- Simplified model based on sliced-beam approach (assuming beam distribution is Gaussian in each individual longitudinal slice) using rates for local slices was proposed (Xiao, Borland'2009). Very large local rates and effect on distribution was reported due to very small local momentum spread.

Can such large local rates result in significant IBS effect?

To look into this question sliced approximation was also implemented in BETACOOL code (A. Smirnov, JINR, Dubna) and compared to "local IBS" approach based on diffusion coefficients.

Should one expect strong effect from large local rates?

Rate definition vs. diffusion coefficients

1. For isotropic Maxwellian distribution, where Δ is rms velocity spread, and Λ is Coulomb logarithm.

$$\tau^{-1} = \frac{1}{\Delta^2}\frac{d\Delta^2}{dt} = \frac{4\pi n(Ze)^4}{m^2}\frac{\Lambda}{\Delta^3}$$

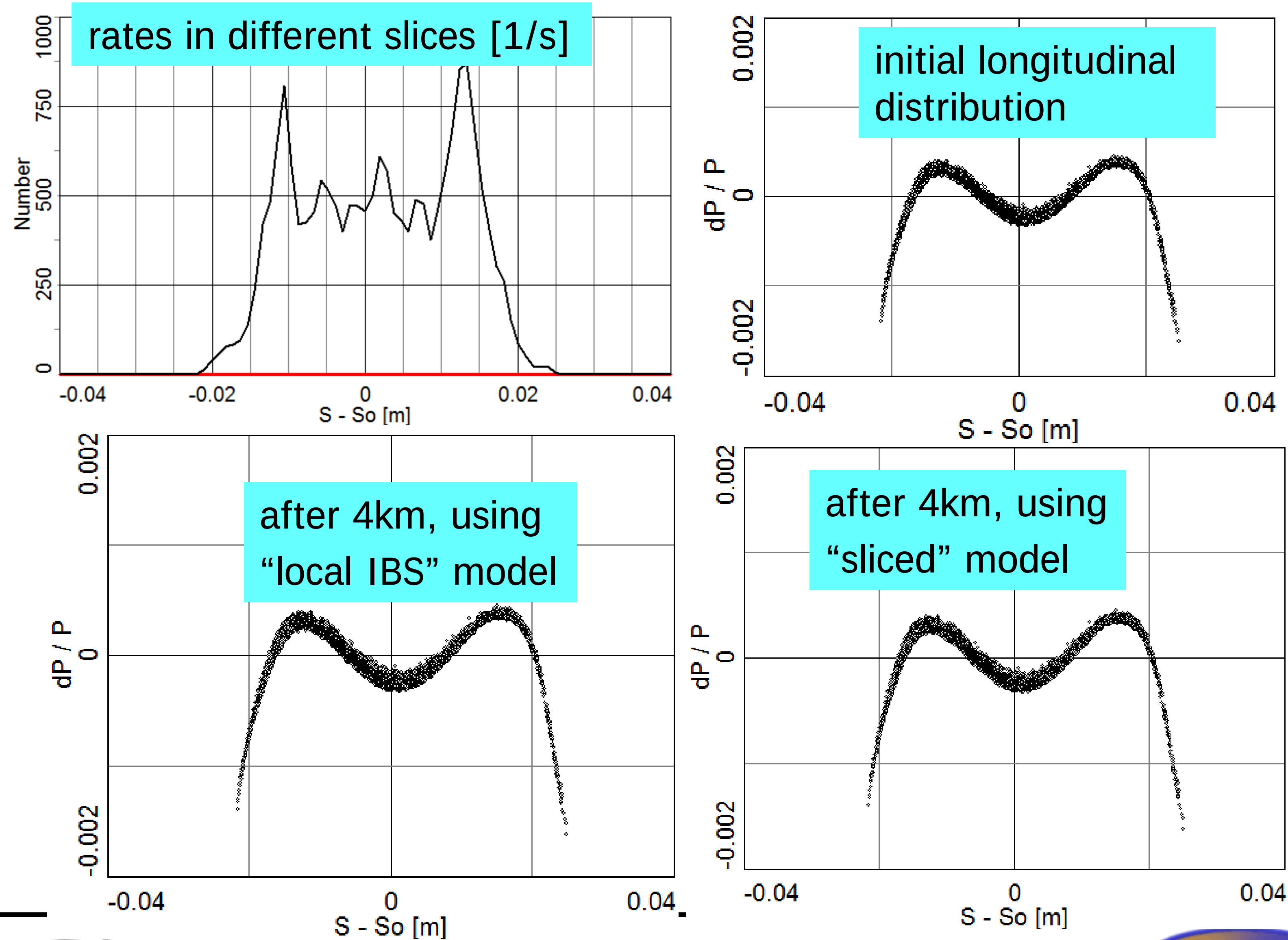
2. For anisotropic velocity distribution when longitudinal velocity spread in beam rest frame is much smaller than transverse ("flattened") (possible situation in a slice of ERL's electron beam distribution due to very small local energy spread).

$$f(\vec{r},\vec{v}) = \frac{n}{\pi\sqrt{\pi\Delta_\perp^2}\Delta_{\|}} e^{-(v_x^2+v_y^2)/\Delta_\perp^2} e^{-v_z^2/(2\Delta_{\|}^2)}$$

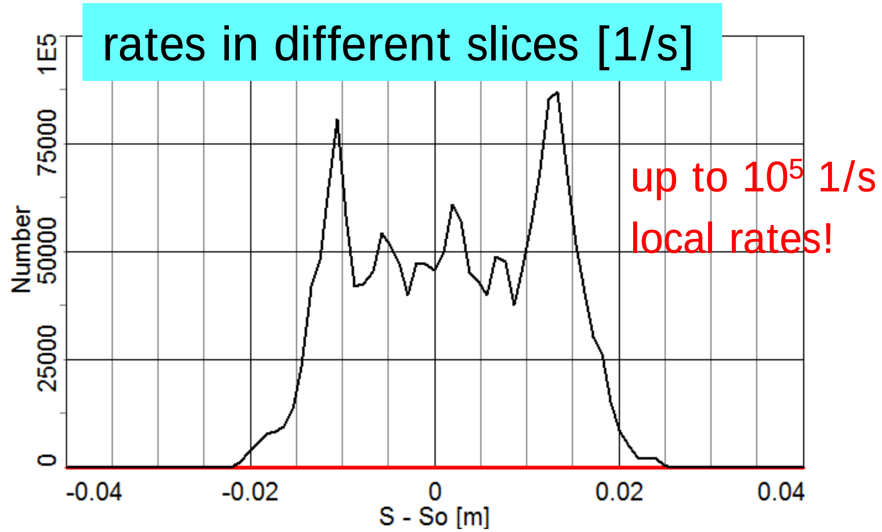
$$D_{\alpha,\beta} = \frac{\langle \Delta p_\alpha \Delta p_\beta \rangle}{\Delta t} = 4\pi m e^4 Z_i^2 Z_f^2 \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{U^2\delta_{\alpha,\beta} - U_\alpha U_\beta}{U^3} f(v) dv$$

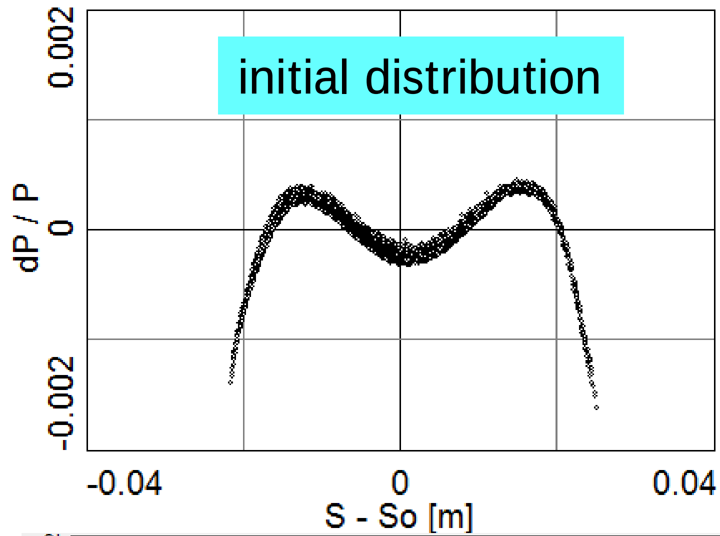
$$\tau_{\|}^{-1} = \frac{1}{\Delta_{\|}^2}\frac{d\vec{v}_z^2}{dt} = 4\pi m(Ze)^4 n\Lambda \frac{1}{\Delta_{\|}^2\Delta_\perp} \qquad D_{zz} = \frac{4\pi n(Ze)^4}{m^2\Delta_\perp}\Lambda_{ibs}\left[\sqrt{\pi}e^{-u^2/2\Delta^2_\perp} I_0\left(\frac{u^2}{2\Delta^2_\perp}\right)\right]$$

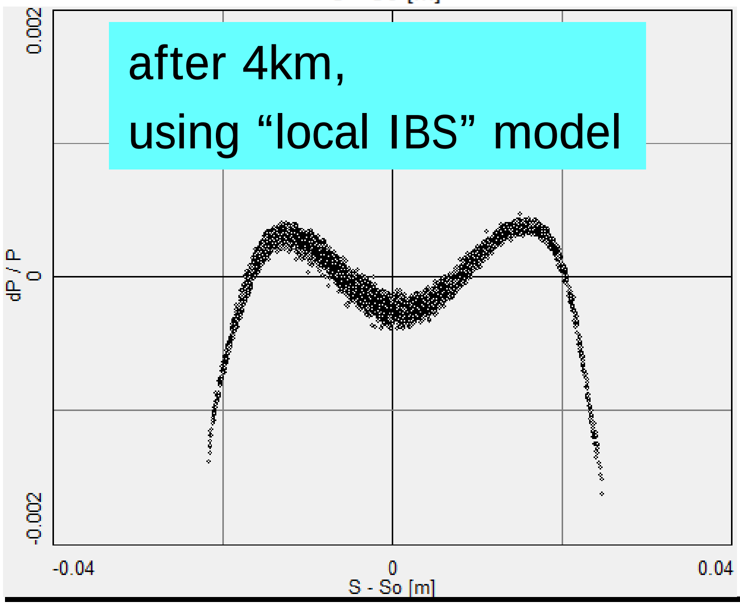
BETACOOL simulations for beam parameters with average over beam longitudinal IBS rate: 15 [1/sec]

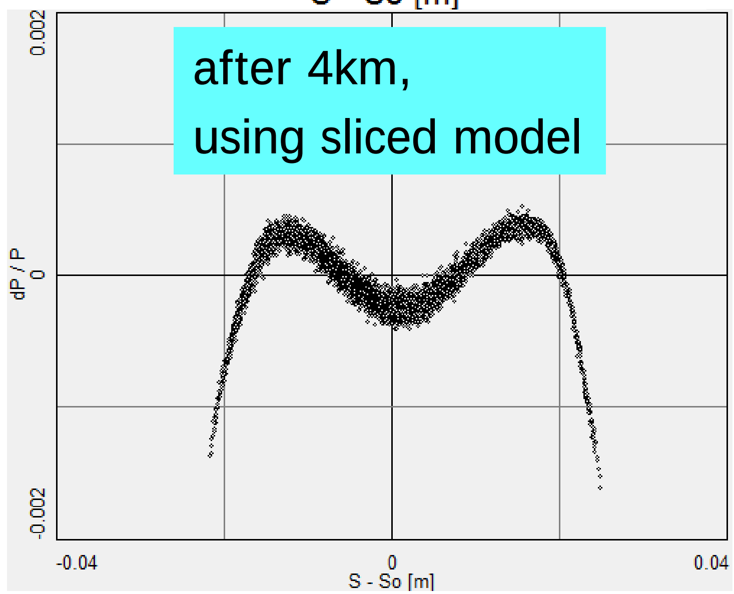


BETACOOL simulations for beam parameters with average longitudinal IBS rate: 1500 1/sec









Summary

- Coulomb scattering can be important in high-intensity ERLs.
- Single large-angle scattering (Touschek effect) can result in significant tails/halo of the distribution which has to be properly collimated (3-D treatment for halo).
- Scaling of Touschek's rate shows significant contribution from highest energies for multi-pass ERLs.
- Multiple small-angle scattering (IBS) should not be a significant effect for typical parameters of proposed ERLs in terms of emittance growth (but could be a source of halo), unless one considers very long transport of high-brightness beams at low energies.
- IBS treatment valid for arbitrary distribution in 3-D (based on diffusion coefficients) was developed in BETACOOL code (JINR, Dubna), which was also benchmarked vs. experimental data. Here it was compared to the sliced-beam approach for ERLs.

Thank you.