Experimental Studies of Resonance Crossing with a Paul Trap

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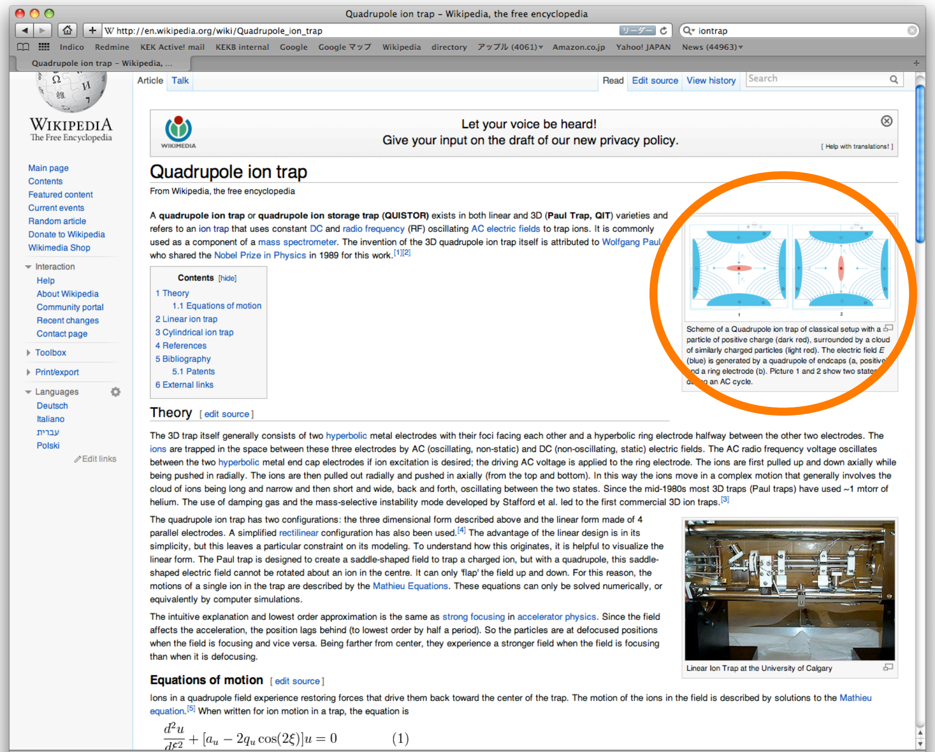
(Hiroshima Univ.)

Use a Paul Trap?

- Paul trap found in Wikipedia.



http://en.wikipedia.org/wiki/Quadrupole_ion_trap

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Quadrupole ion trap

From Wikipedia, the free encyclopedia

A **quadrupole ion trap** or **quadrupole ion storage trap (QUISTOR)** exists in both linear and 3D (**Paul Trap, QIT**) varieties and refers to an ion trap that uses constant DC and radio frequency (RF) oscillating AC electric fields to trap ions. It is commonly used as a component of a mass spectrometer. The invention of the 3D quadrupole ion trap itself is attributed to Wolfgang Paul who shared the Nobel Prize in Physics in 1989 for this work.[1][2]

Contents [hide]

Theory [edit source]

The 3D trap itself generally consists of two hyperbolic metal electrodes with their foci facing each other and a hyperbolic ring electrode halfway between the other two electrodes. The ions are trapped in the space between these three electrodes by AC (oscillating, non-static) and DC (non-oscillating, static) electric fields. The AC radio frequency voltage oscillates between the two hyperbolic metal end cap electrodes if ion excitation is desired; the driving AC voltage is applied to the ring electrode. The ions are first pulled up and down axially while being pushed in radially. The ions are then pulled out radially and pushed in axially (from the top and bottom). In this way the ions move in a complex motion that generally involves the cloud of ions being long and narrow and then short and wide, back and forth, oscillating between the two states. Since the mid-1980s most 3D traps (Paul traps) have used ~1 mtorr of helium. The use of damping gas and the mass-selective instability mode developed by Stafford et al. led to the first commercial 3D ion traps.[3]

The quadrupole ion trap has two configurations: the three dimensional form described above and the linear form made of 4 parallel electrodes. A simplified rectilinear configuration has also been used.[4] The advantage of the linear design is in its simplicity, but this leaves a particular constraint on its modeling. To understand how this originates, it is helpful to visualize the linear form. The Paul trap is designed to create a saddle-shaped field to trap a charged ion, but with a quadrupole, this saddle-shaped electric field cannot be rotated about an ion in the centre. It can only 'flap' the field up and down. For this reason, the motions of a single ion in the trap are described by the Mathieu Equations. These equations can only be solved numerically, or equivalently by computer simulations.

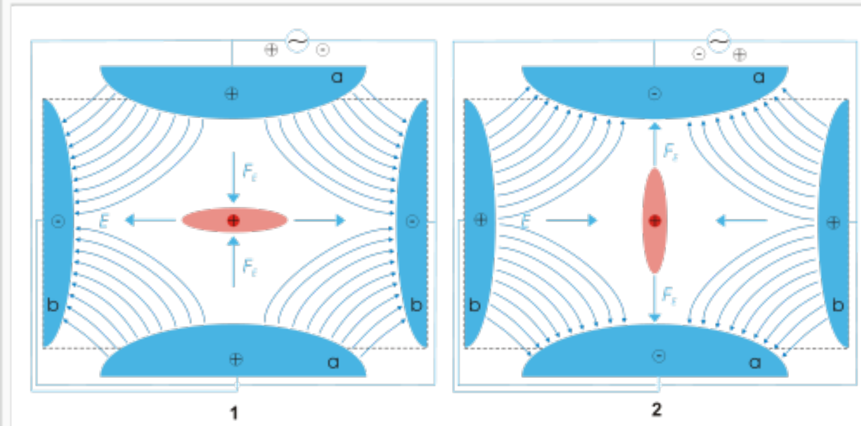
The intuitive explanation and lowest order approximation is the same as strong focusing in accelerator physics. Since the field affects the acceleration, the position lags behind (to lowest order by half a period). So the particles are at defocused positions when the field is focusing and vice versa. Being farther from center, they experience a stronger field when the field is focusing than when it is defocusing.

Linear Ion Trap at the University of Calgary

Equations of motion [edit source]

Ions in a quadrupole field experience restoring forces that drive them back toward the center of the trap. The motion of the ions in the field is described by solutions to the Mathieu equation.[5] When written for ion motion in a trap, the equation is

$$\frac{d^2u}{d\xi^2} + [a_u - 2q_u\cos(2\xi)]u = 0 \qquad (1)$$



Scheme of a Quadrupole ion trap of classical setup with a particle of positive charge (dark red), surrounded by a cloud of similarly charged particles (light red). The electric field E (blue) is generated by a quadrupole of endcaps (a, positive) and a ring electrode (b). Picture 1 and 2 show two states during an AC cycle.

http://en.wikipedia.org/wiki/Main_Page

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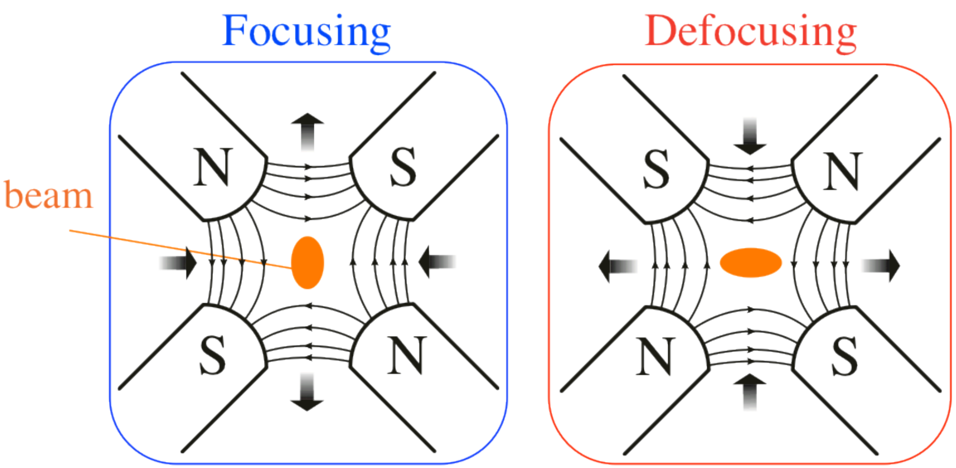
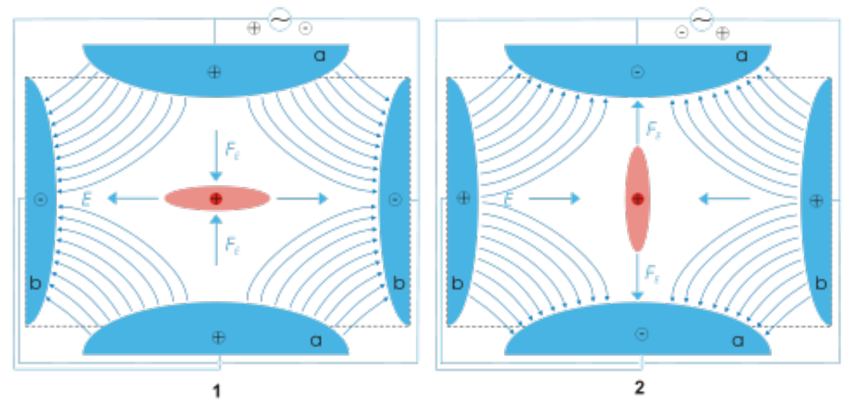
Accelerator





Scheme of a Quadrupole ion trap of classical setup with a particle of positive charge (dark red), surrounded by a cloud of similarly charged particles (light red). The electric field *E* (blue) is generated by a quadrupole of endcaps (a, positive) and a ring electrode (b). Picture 1 and 2 show two states during an AC cycle.

This is exactly an AG focusing system!

http://en.wikipedia.org/wiki/Main_Page

Outline

1. Introduction
   - Background and motivation
   - S-POD, an experimental tool for study of beam physics

2. Multi-particle Simulation
   - Resonance-band distribution in S-POD
   - Resonance crossing

3. Experiments
   - Resonance-band distribution in S-POD
   - Resonance crossing
   - Comparison with the multi-particle simulation

Background and Motivations

- Fixed-field accelerators have the potential to be a high-power accelerator owing to its high repetition rate.
- The beam optics potentially varies during the beam acceleration.
  - e.g. Non-Scaling FFAG (NS-FFAG) ring, EMMA
    Cell tune varies from ~0.3-4 to ~0.17
    The beam transverses one and more resonance bands.
- The resonance crossing may limit the machine performance.
- Past theoretical studies
  - Emitance growth is negligible or tolerable when the crossing speed is sufficiently high or/and the resonance is not so strong.

Today's Talk

Experimental (and numerical) studies on betatron resonance crossing ***not*** using any accelerators, but using a ***plasma trap.***

Study Beam Dynamics with a Plasma Trap

- Charged particle beam is…
  a kind of non-neutral plasma confined in a machine.
- After some algebra, we reach following two Hamiltonians

Charged particle beam
(beam rest frame)

$$H_{beam} = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} K(s)\left(x^2 - y^2\right) + \frac{q}{M\gamma^3 (\beta c)^2}\phi$$

Non-neutral plasma
(laboratory frame)

$$H_{plasma} = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} K(\tau)\left(x^2 - y^2\right) + \frac{q}{Mc^2}\phi$$

$$\tau \equiv c \times t$$

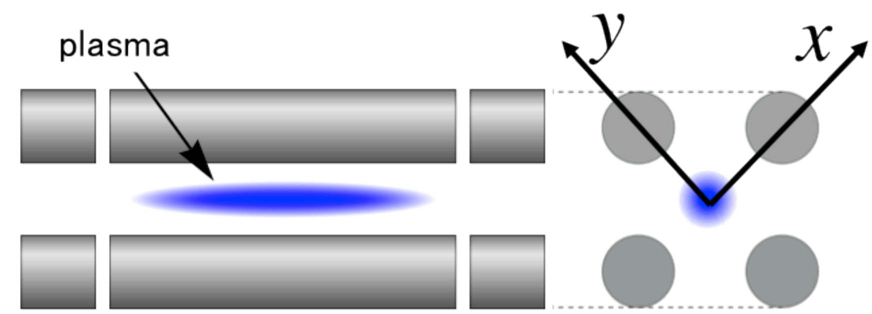
with the Vlasov-Poisson equation

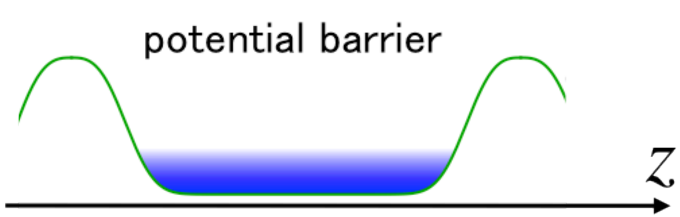
$$\frac{\partial f}{\partial t} + [f, H] = 0 \qquad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi = -\frac{q}{\varepsilon_0}\iint f\, dp_x dp_y$$

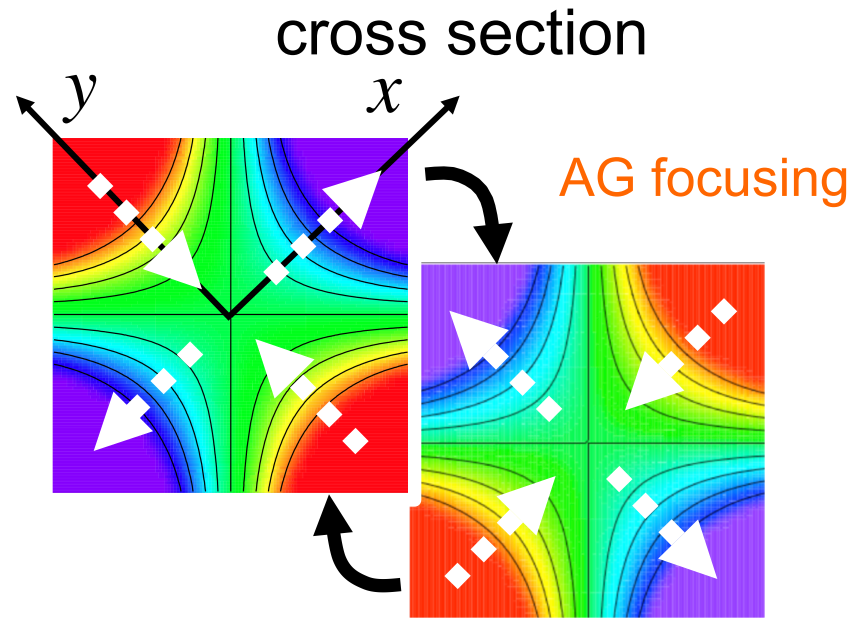
- We can use this physical equivalence to study beam dynamics.

H. Okamoto and H. Tanaka, NIM A 437 (1999) p.178.

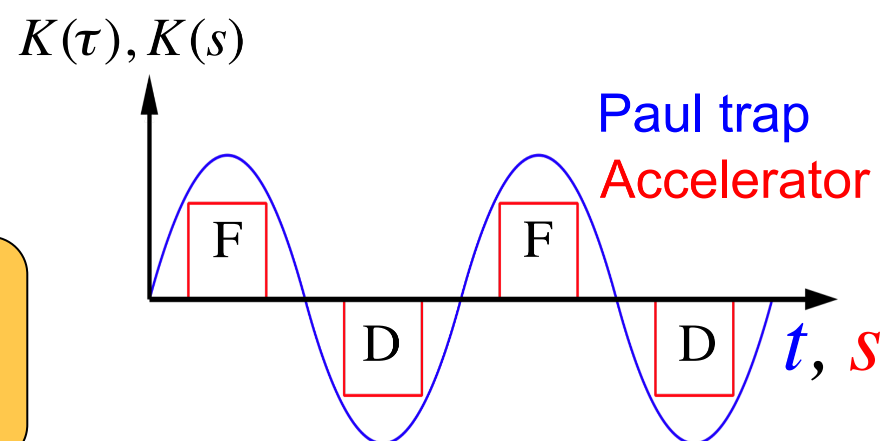
Linear Paul trap







- Transverse confinement
  - *RF quadrupole electric field*
- Longitudinal confinement
  - *Static potential barrier*

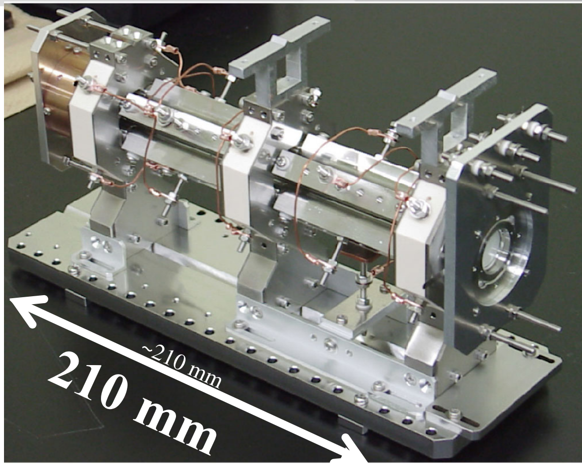


Transverse 2D approximation $(\partial/\partial z \approx 0)$

$$H_{plasma} \approx \frac{p_x^2 + p_y^2}{2} + \frac{1}{2}K(\tau)\left(x^2 - y^2\right) + \frac{q}{Mc^2}\phi$$

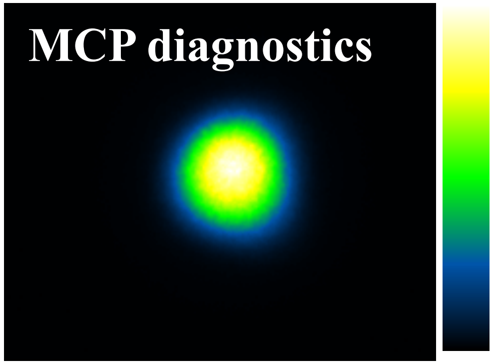
Advantages

- <u>Very compact and low cost</u>
  - Several tens of thousands dollars for the whole system

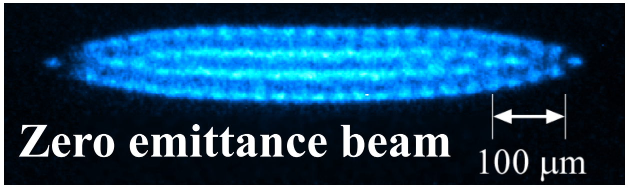


- <u>High flexibility of fundamental parameters</u>
  - Beam density, operating point, lattice function, etc.

- <u>High resolution & high precision measurements</u>
  - Faraday cup, micro-channel plate
    Laser induced fluorescence (LIF)

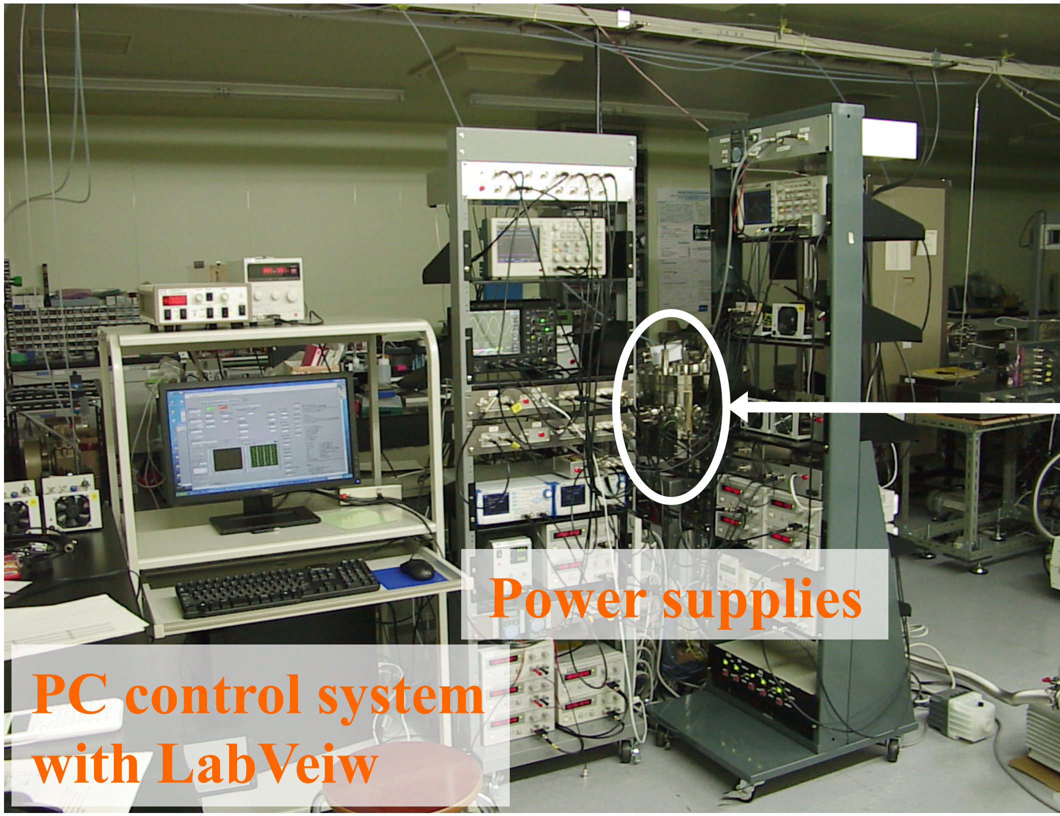


- <u>Radio-activation free</u>
  - Experiment with any strong beam instability.

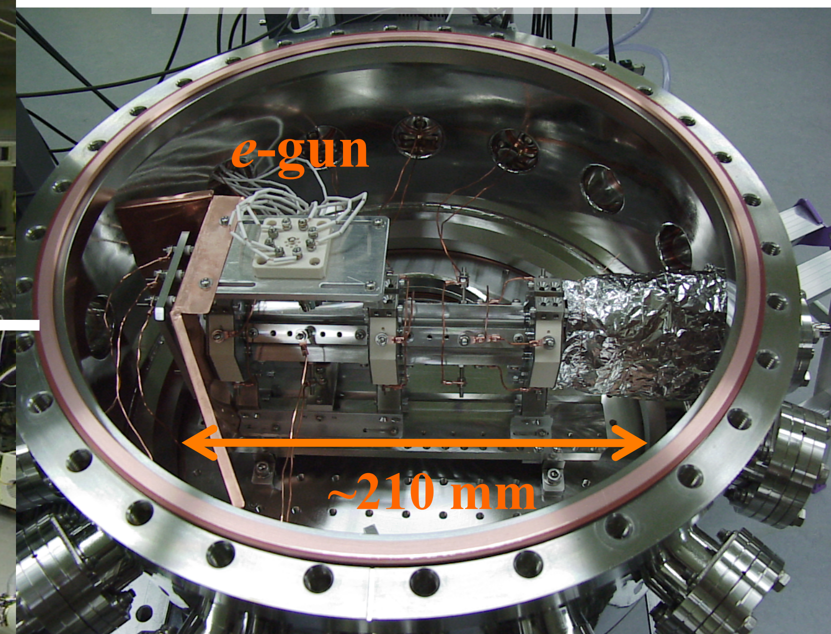


Linear Paul Trap System at Hiroshima Univ.

- S-POD - Simulator for Particle Orbit Dynamics –



Vacuum chamber

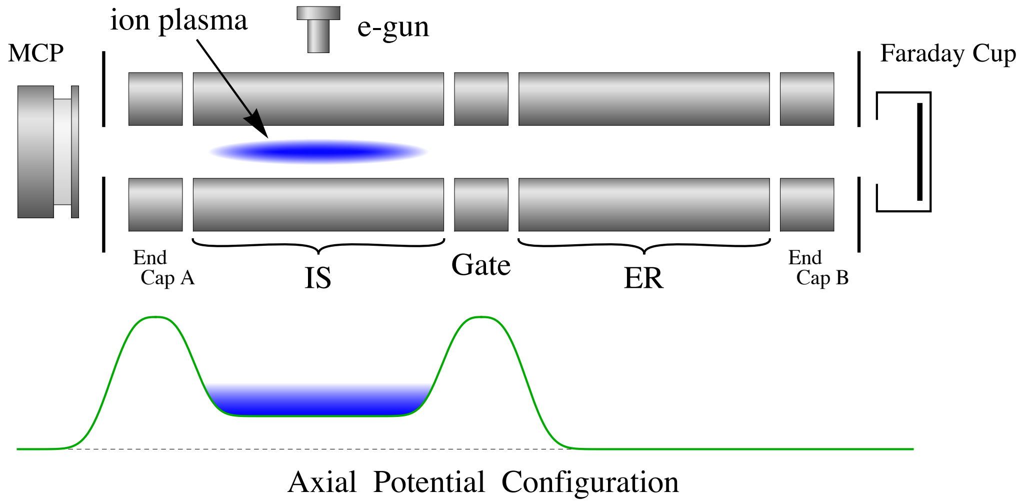


- There are three S-POD systems
  - **S-POD I** : crystalline beam, nano-ion beam, etc.
  - **S-POD II**, **S-POD III** : resonant instability, etc.

Experimental Setup

- Multi-sectioned linear Paul trap

Ion species : $^{40}Ar^{+}$
Operating frequency : 1 MHz
RF amplitude : 0 ~ 92 V



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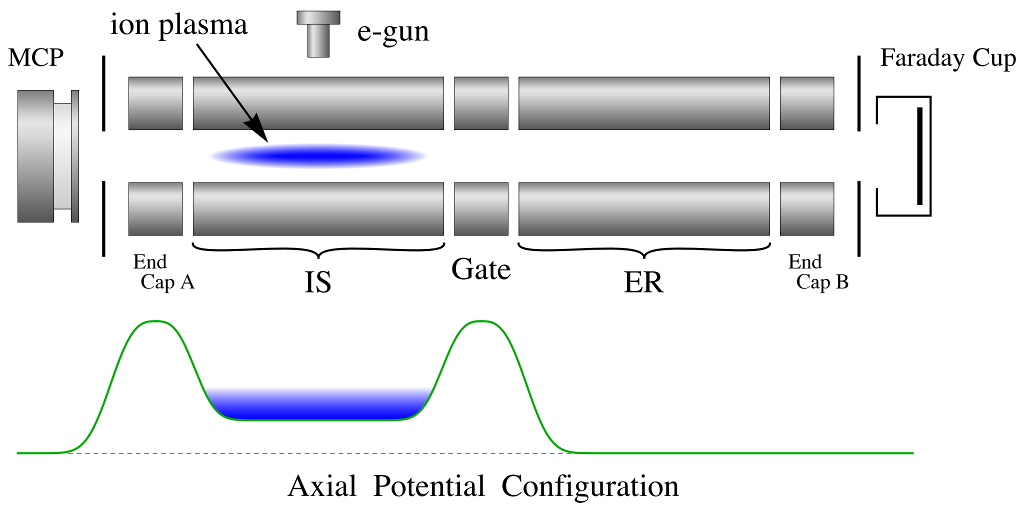

1. $^{40}Ar^{+}$ ions are generated by electron impact ionization with an electron.

Experimental Setup

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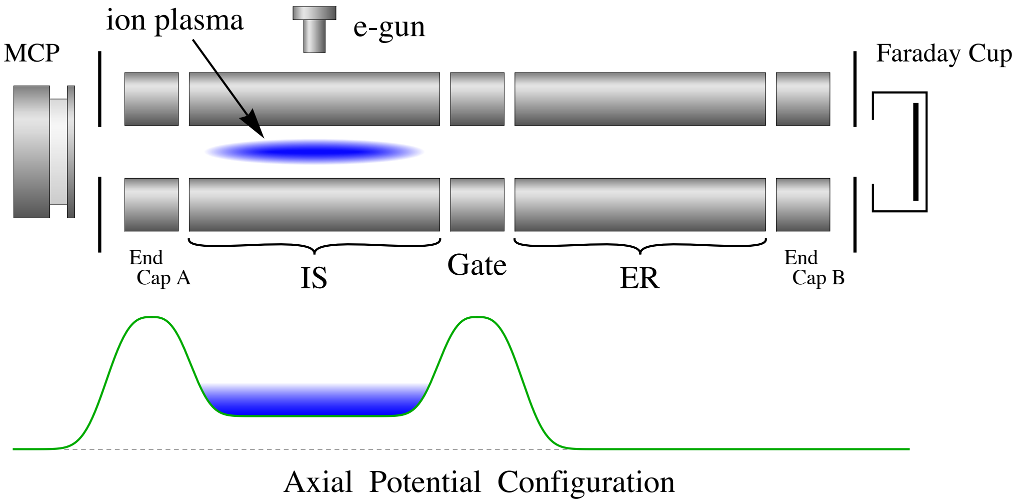

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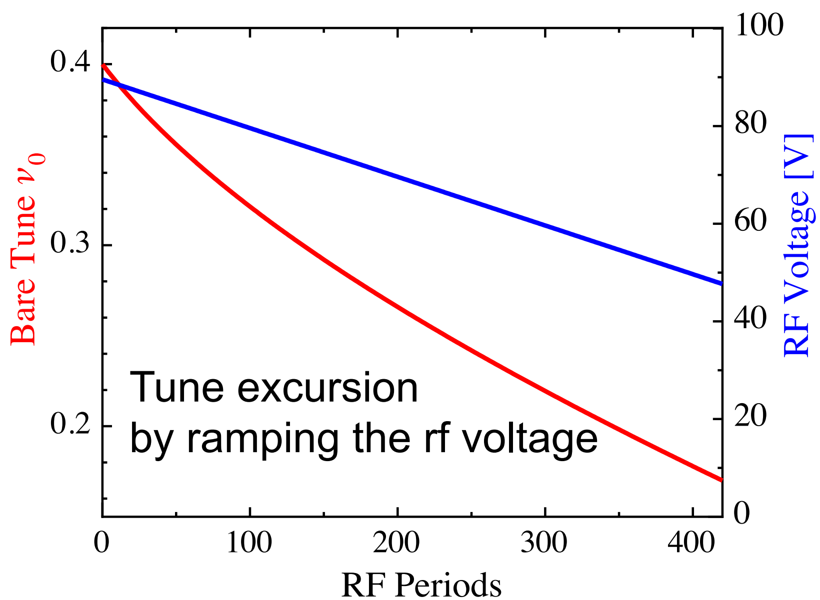

1. $^{40}Ar^{+}$ ions are generated by electron impact ionization with an electron.
2. Storage plasma W/ or W/O tune excursion.

Experimental Setup

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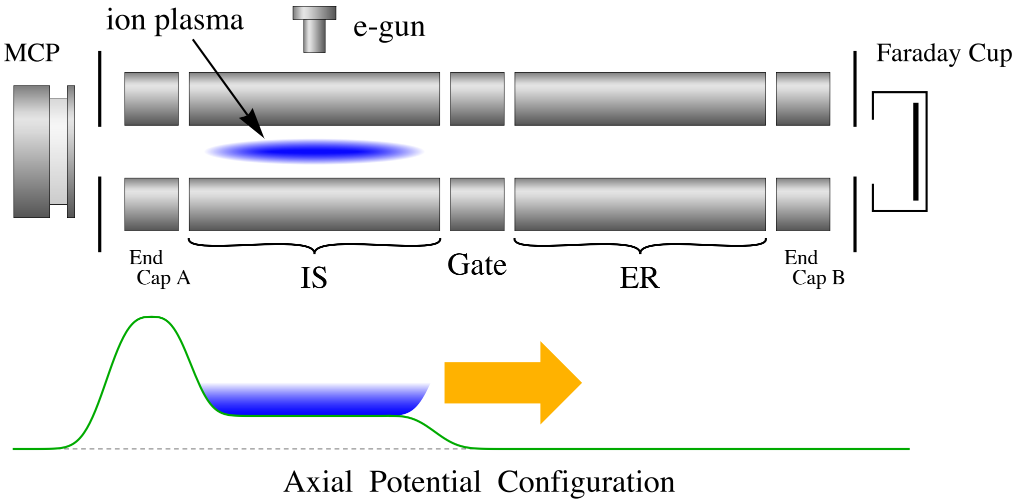

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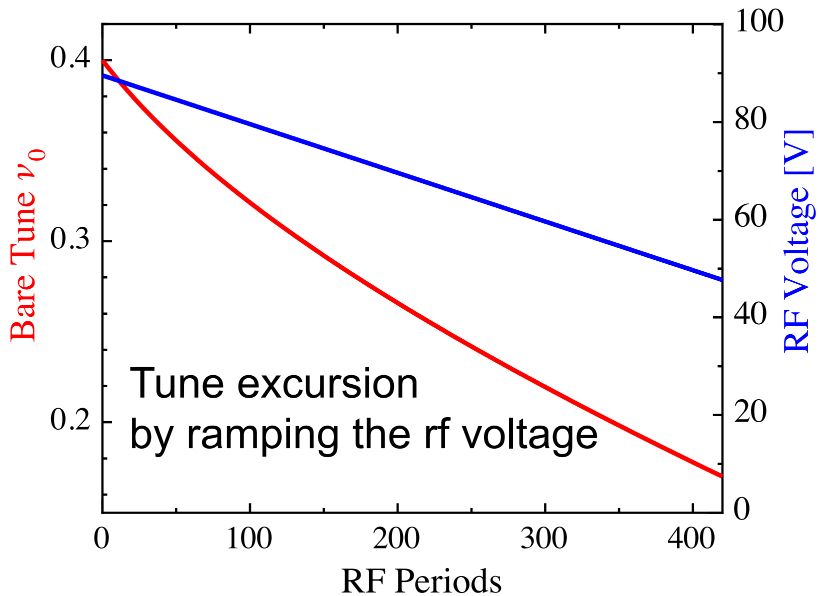
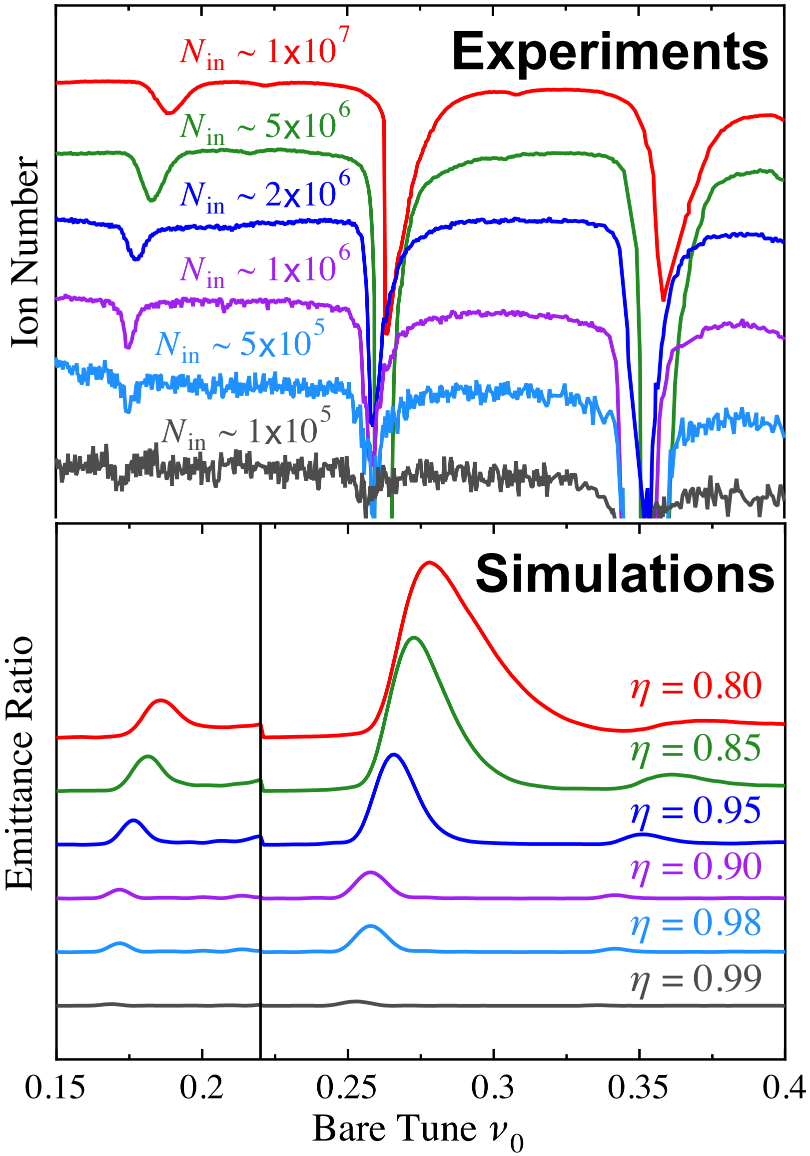


1. $^{40}Ar^{+}$ ions are generated by electron impact ionization with an electron.
2. Storage plasma W/ or W/O tune excursion.
3. Shut down the bias potential on Gate to send the plasma toward a FC detector.

Stop-band Distribution in S-POD



- **Experiment**
  - Tune survey on # of surviving particle after 10msec storage.
- **Simulation**
  - Emittance growth after 0.1msec

*Coherent resonance condition**

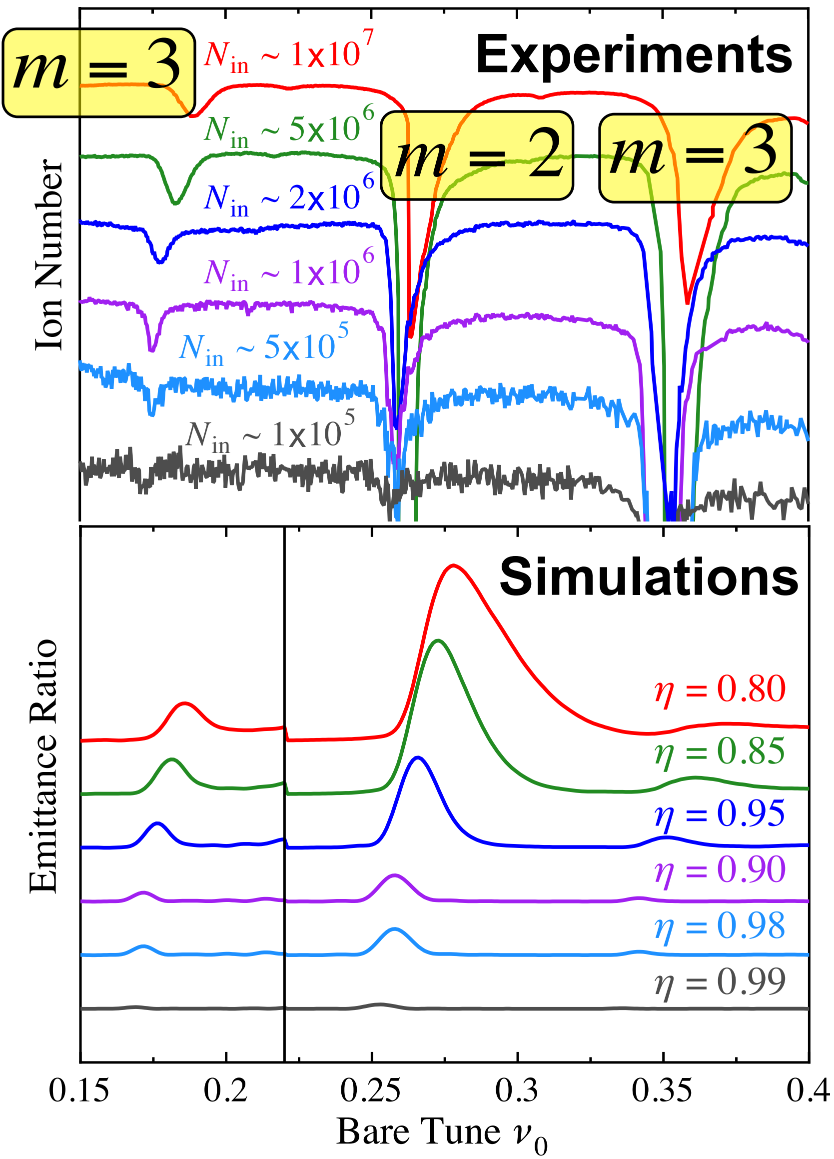
$$\nu_0 - C_m \Delta\nu \approx \frac{n}{2m}$$

m : mode number
n : integer
C_m : constant < 1

- The Instabilities at ν_0~1/3 and 1/6 are likely enhanced by mechanical misalignment of electrodes in S-POD.
- Linear coherent resonance at ν_0~1/4 is rapidly increased as the beam becomes denser.

* H. Okamoto and Y. Yokoya, NIM A 482 (2002) 51.

Stop-band Distribution in S-POD



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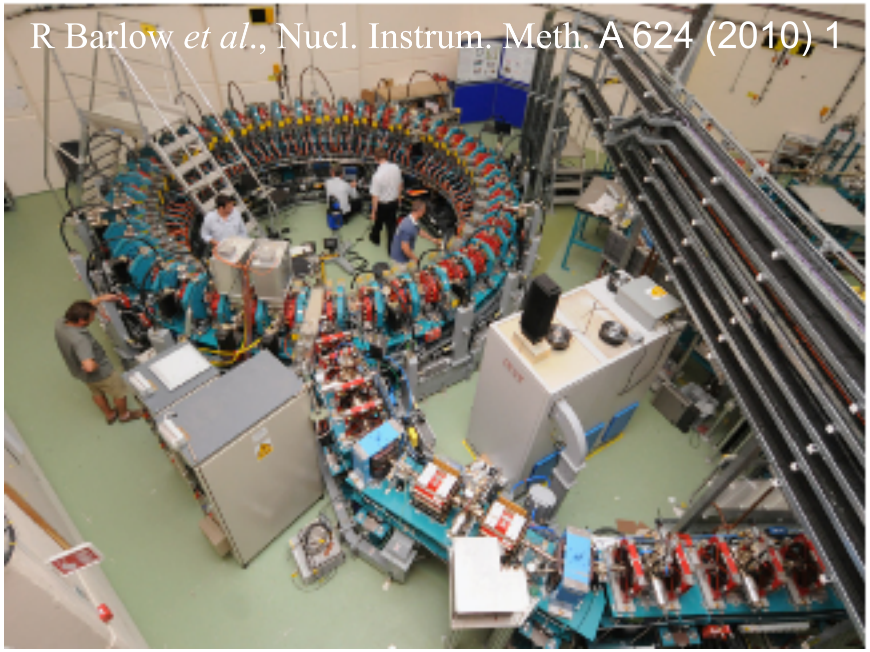
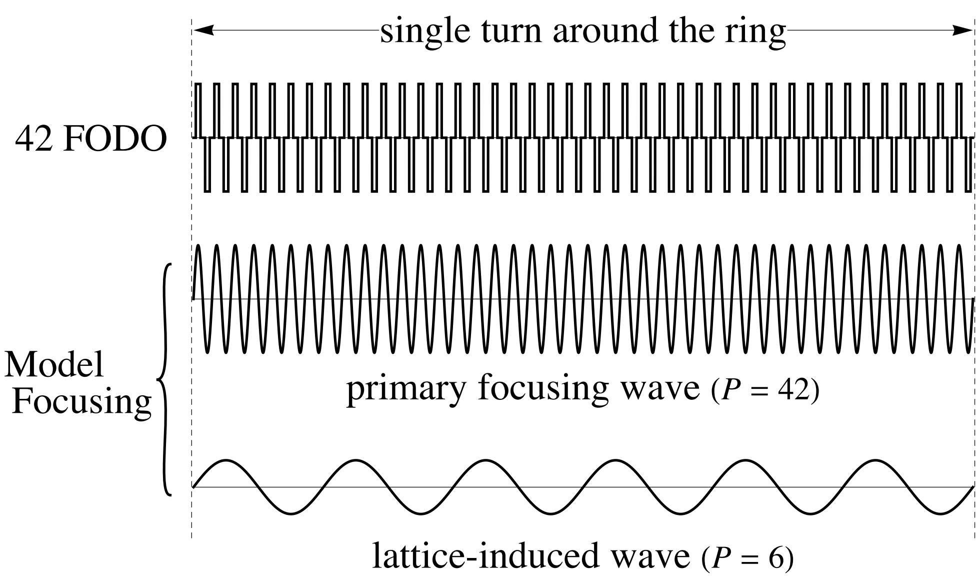
NS-FFAG EMMA and S-POD

- EMMA
  - Composed of 42 quadrupole-doublet cells along the ring.
- S-POD
  - 42 rf periods correspond to 1 turn around the EMMA lattice.
  - Possible to study the effects of the lattice symmetry breaking by superimposing one or more lower-frequency rf waves.

EMMA



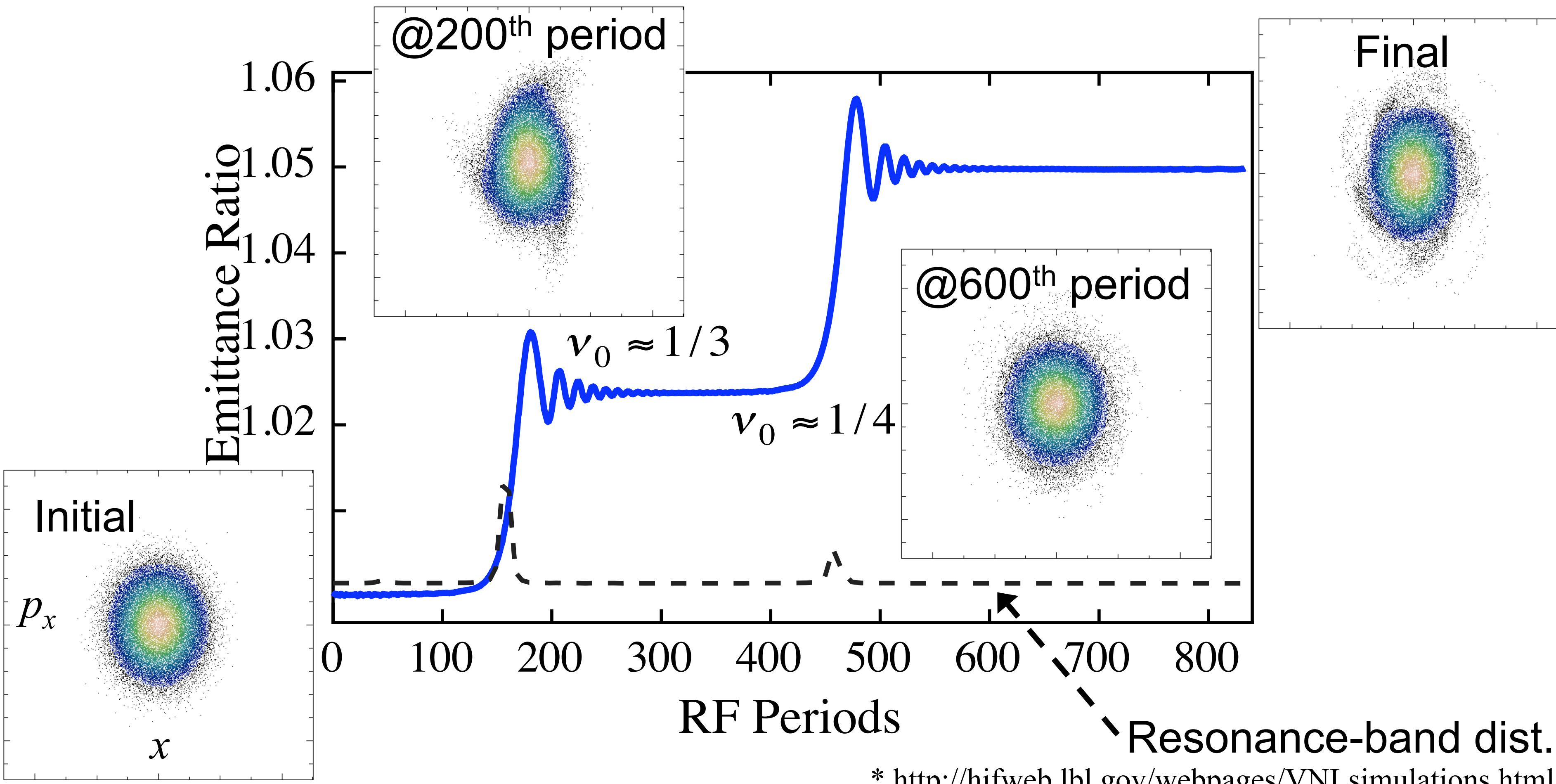
S-POD



Resonance Crossing – Low Density – PIC Simulation with the WARP code*

- Tune sweeping range (0.40 -> 0.17).
- 840 rf periods sweeping (= 20 turns along the EMMA ring)

Tune depression
$\eta = 0.99$

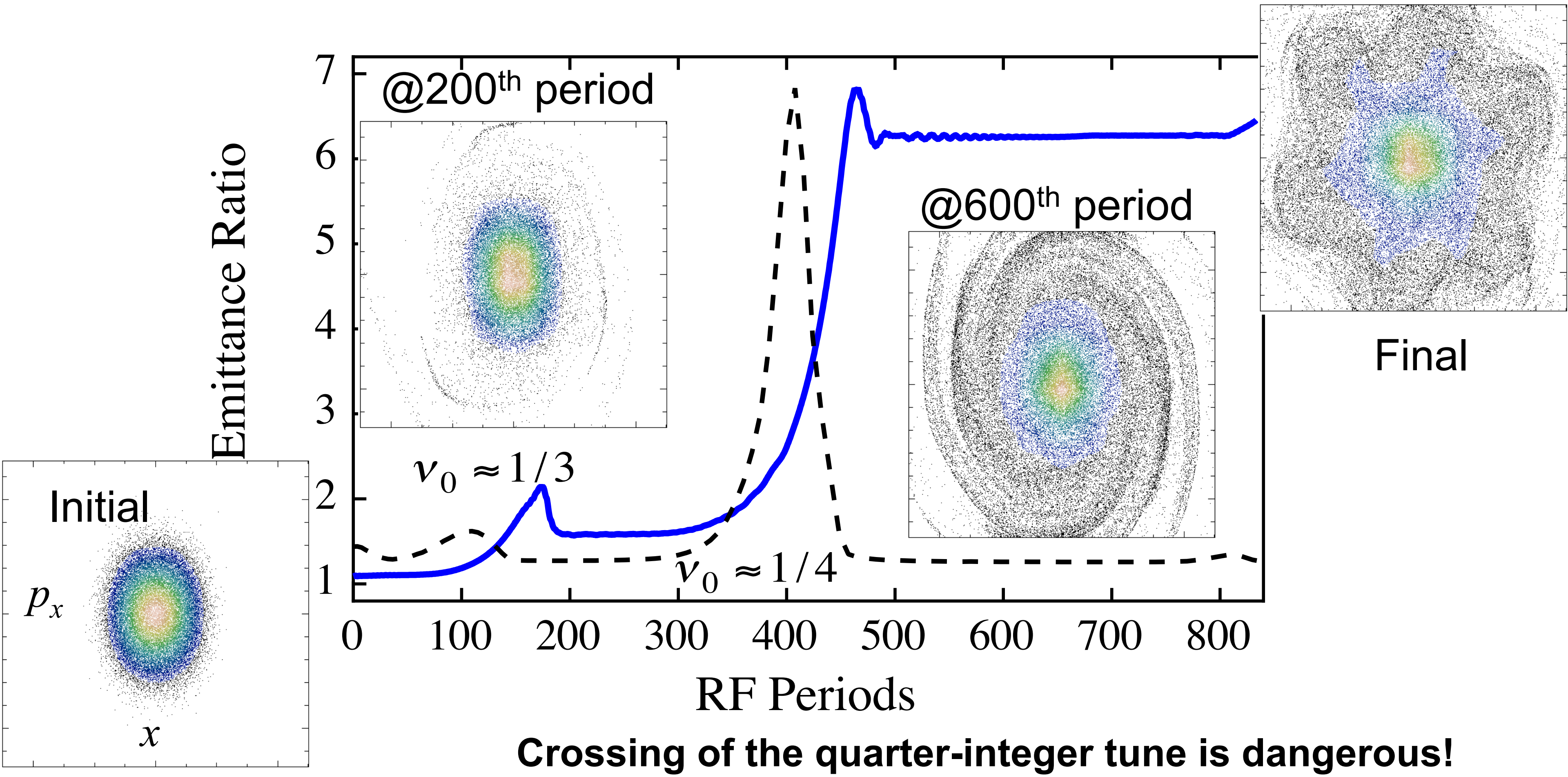


* http://hifweb.lbl.gov/webpages/VNLsimulations.html

Resonance Crossing – High Density – PIC Simulation

- Tune sweeping range (0.40 -> 0.17)
- 840 rf periods sweeping (= 20 turns along the EMMA ring)

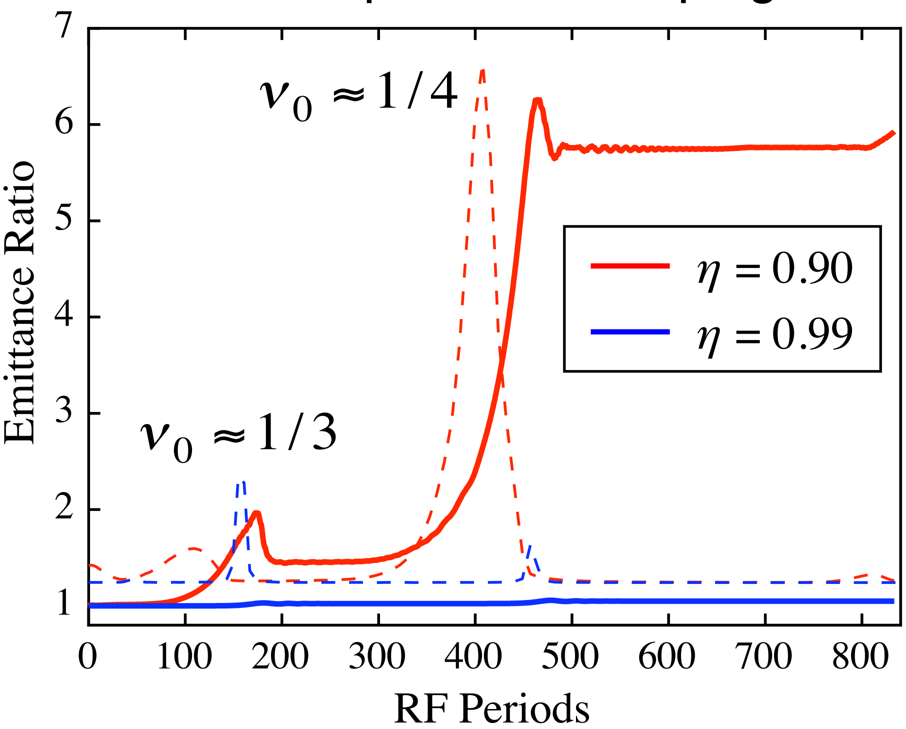
Tune depression
$\eta = 0.90$



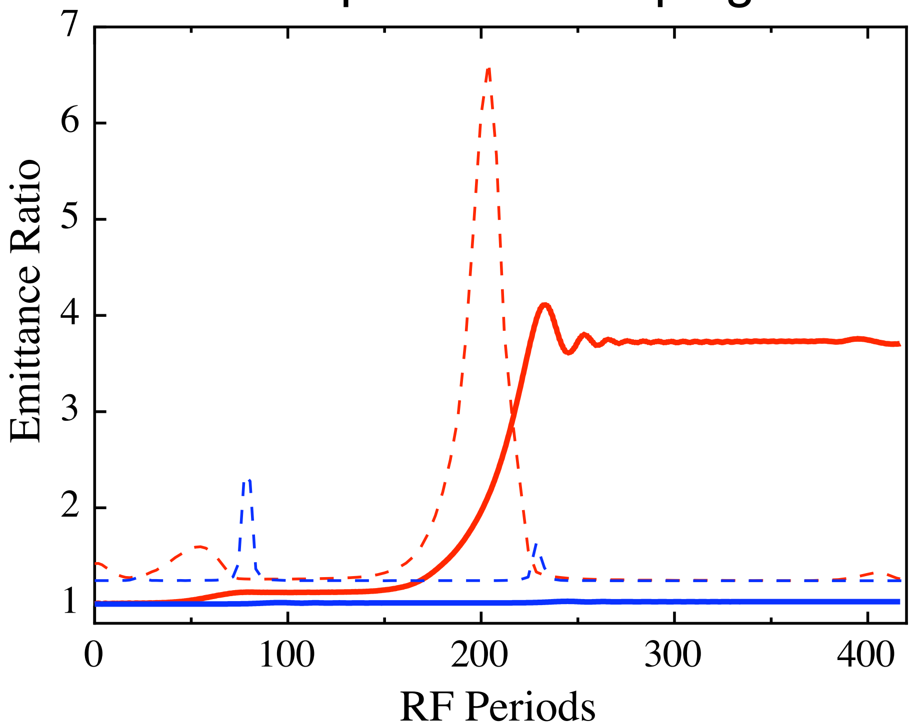
Crossing of the quarter-integer tune is dangerous!

Resonance Crossing - Crossing Speed Dependency – PIC Simulation

840 rf periods sweeping



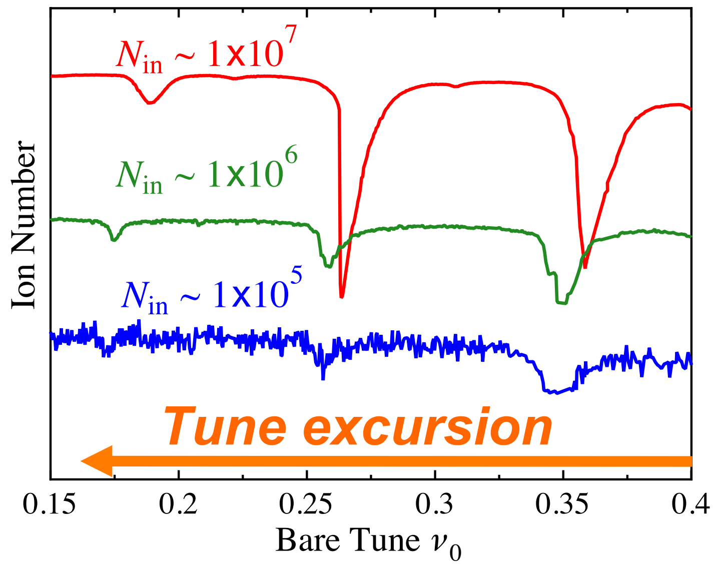
420 rf periods sweeping



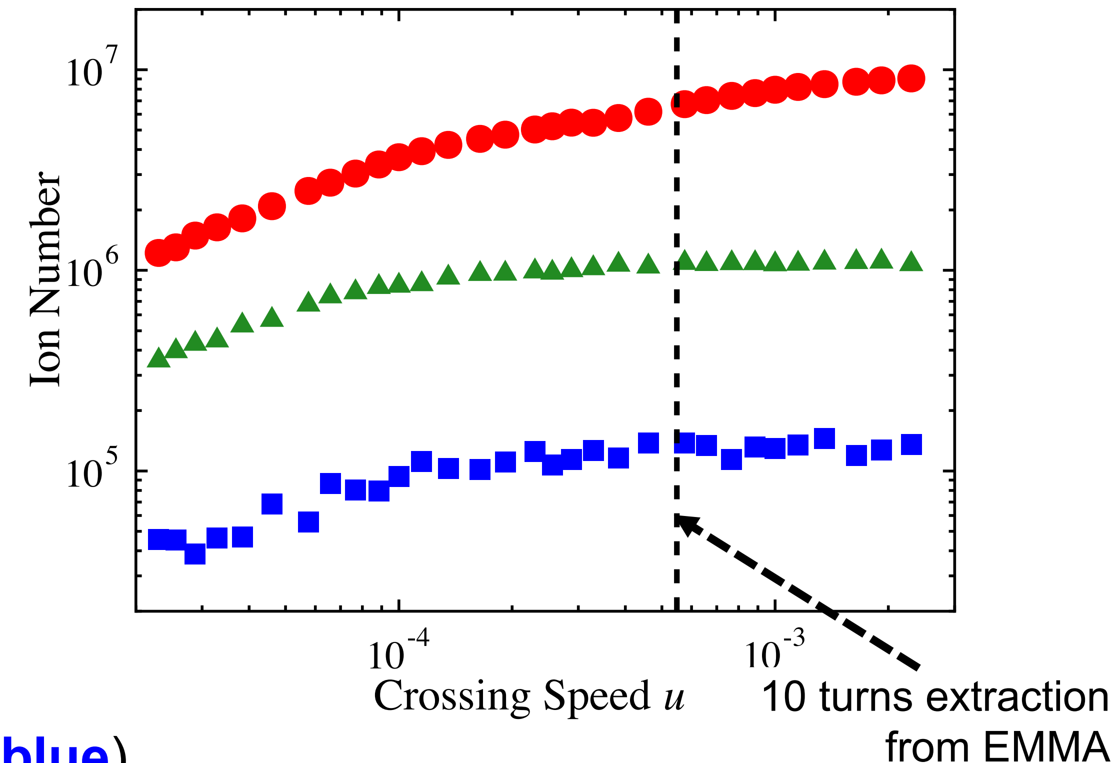
- Faster crossing mitigates degradation of beam quality.
- Serious emittance growth is caused by the linear coherent resonance even with rather fast crossing.

Resonance Crossing Experiment

- Ions are ejected to the Faraday cup right after the tune sweeping.



Ion number vs. crossing speed

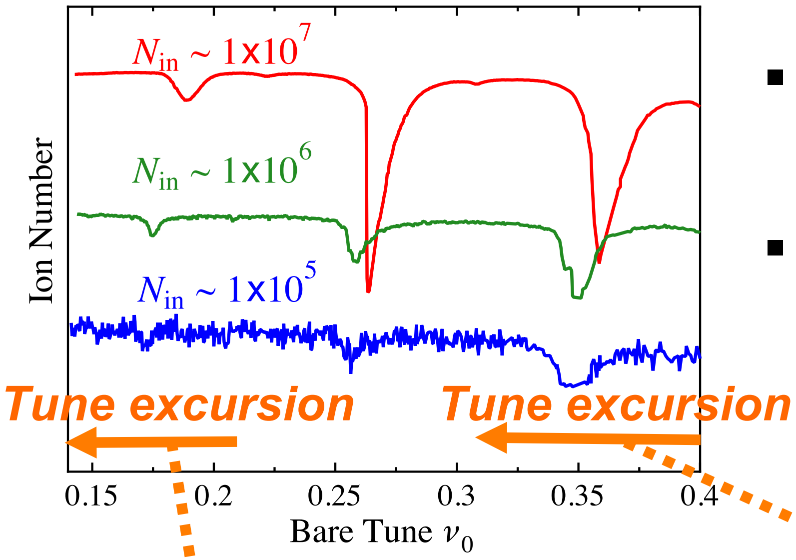


Crossing speed ***u***

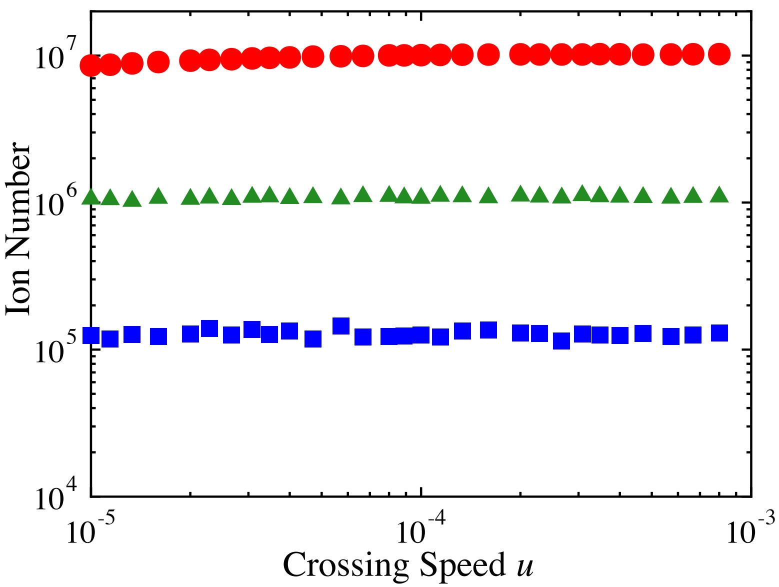
$$u \equiv \frac{\text{Crossing width}}{\text{Number of cell}} = \frac{\delta}{n_{\mathrm{rf}}}$$

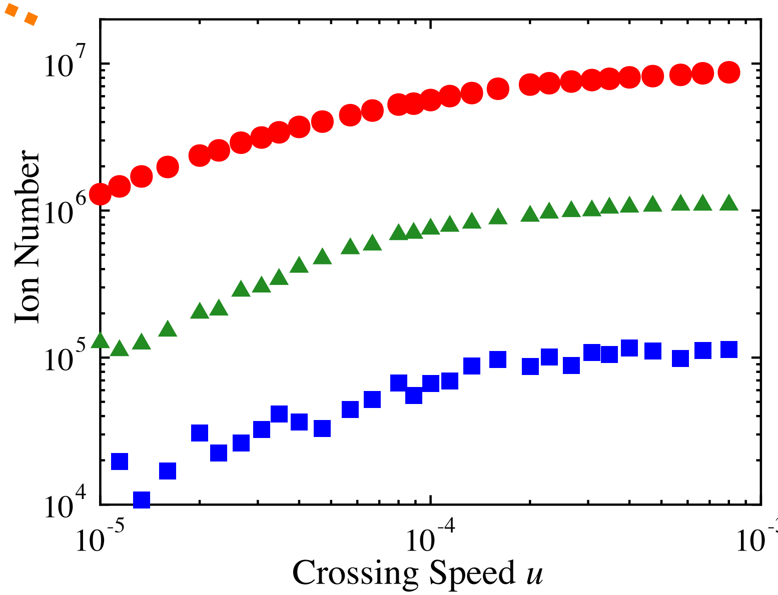
- Low density cases (**green** & **blue**)
  - Ion losses are suppressed when the crossing speed is sufficiently high.
- High density case (**red**)
  - Ion Loss is not negligible even with rather high-speed crossing.

Single Resonance Crossing

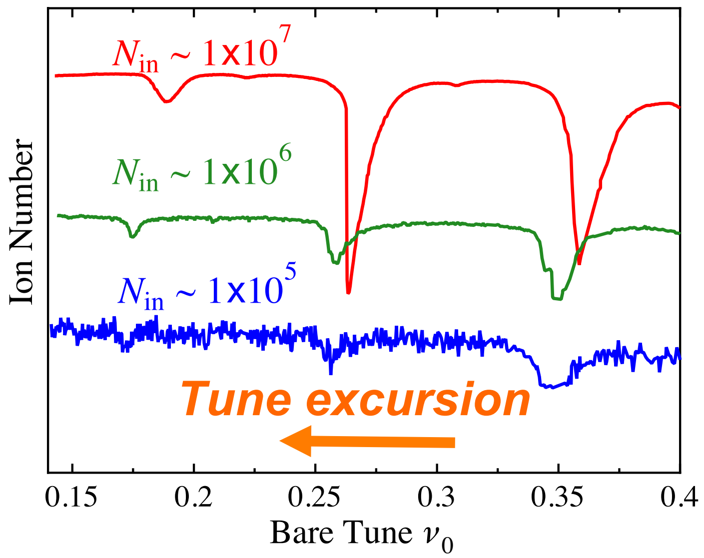


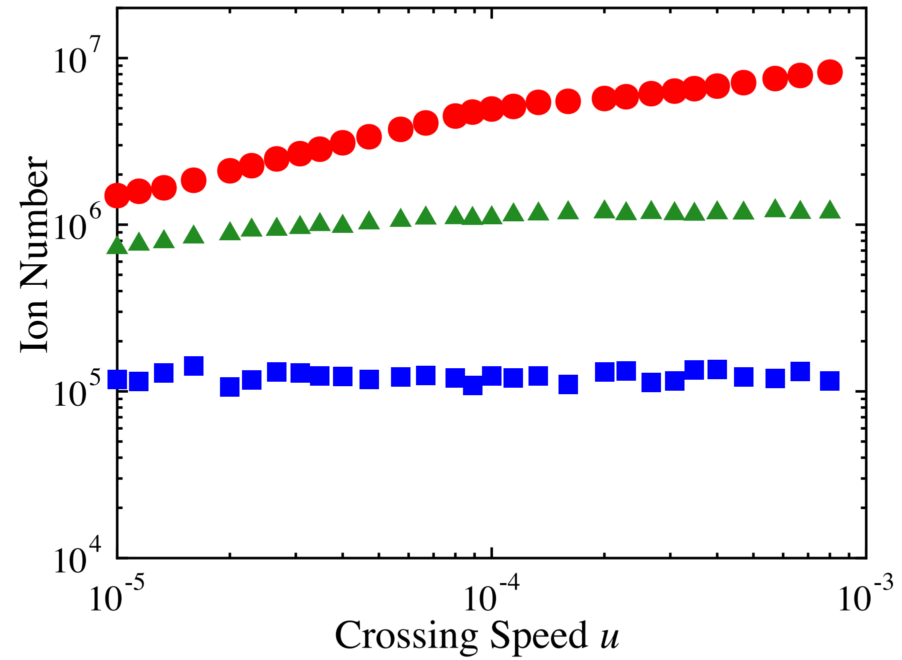
- Crossing of resonance at ν_0~1/6
  - Negligible
- Crossing of resonance at ν_0~1/3
  - Considerable ion losses
  - Enhanced by mechanical misalignment of the electrodes.





Single Resonance Crossing





- Linear coherent resonance at ν_0~**1/4**

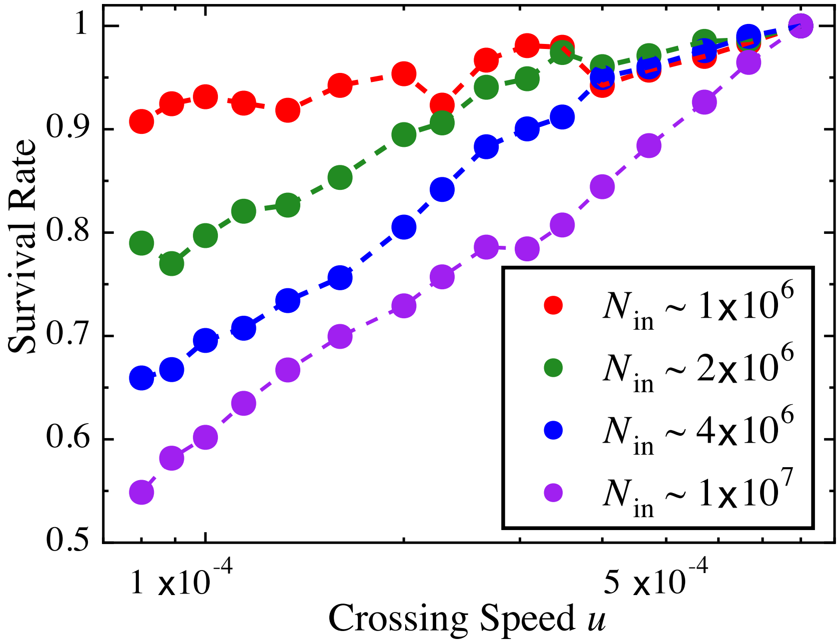
  - Ion loses are remarkably enhanced as particle number is increased.

  - This instability is instinct and independent of lattice errors.

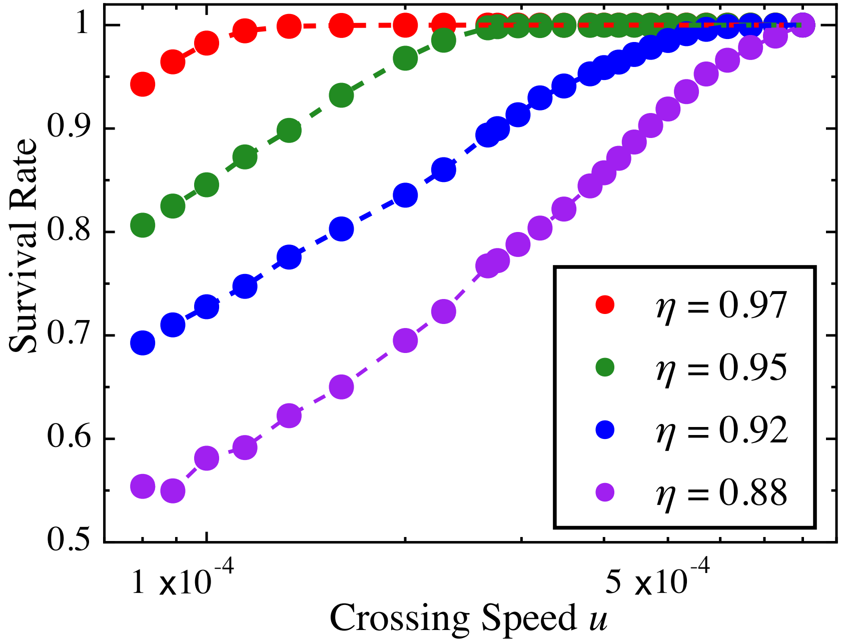
  - Can be a troublesome issue for future high-density NS-FFAG beams.

Comparison with PIC Simulation on the Crossing of ν_0~1/4 Resonance

Experiments

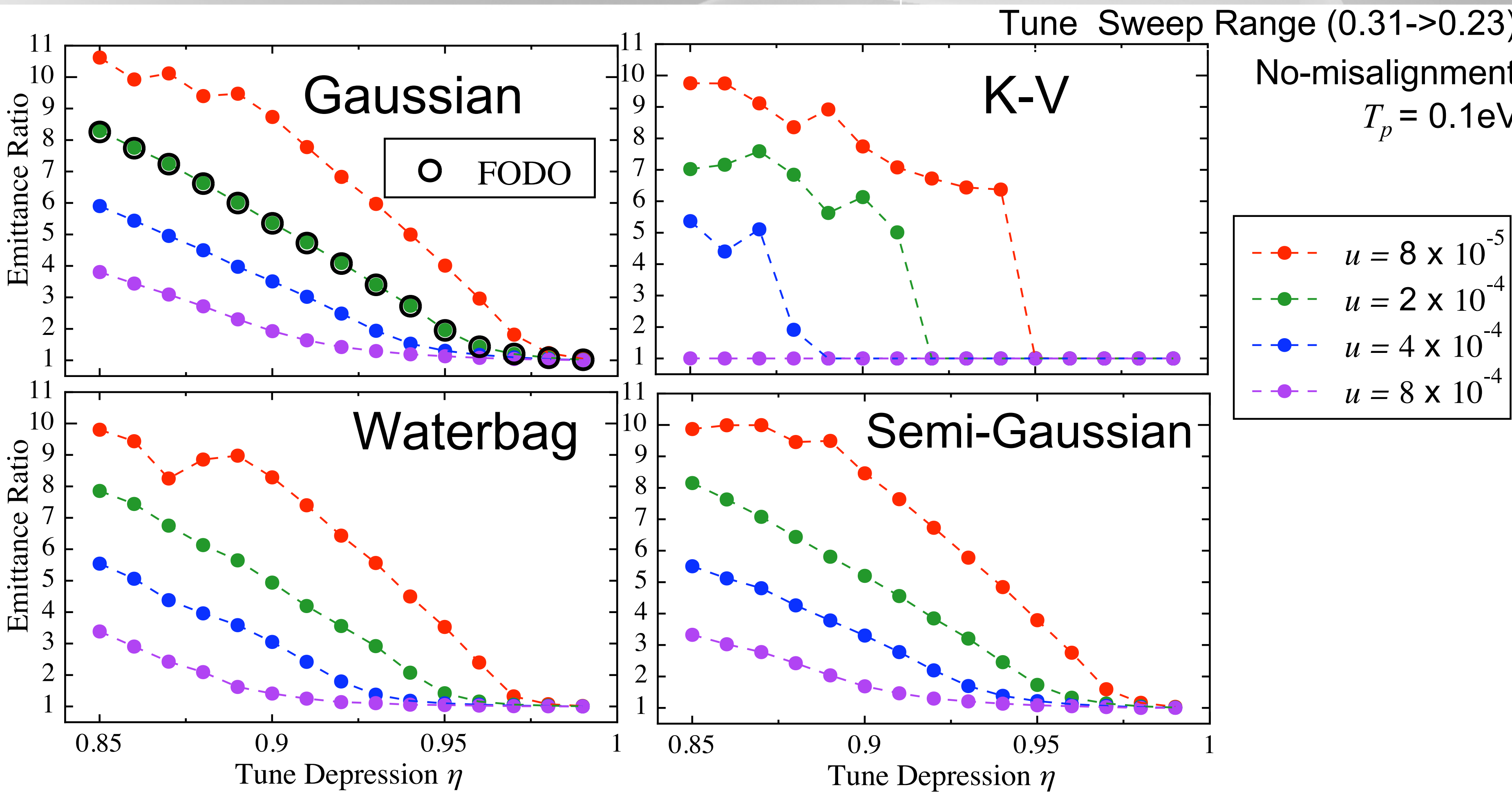


Simulations



- Well reproduce the experimental curve.
- Slower crossing or higher density beam results larger particle losses.
- Even with rather higher crossing speed, ion losses are not negligible in high-density beams.
- The performance of high-power NS-FFAGs maybe limited by this resonance.

Emittance Growth by the Crossing of ν_0~1/4 Resonance - PIC Simulation -

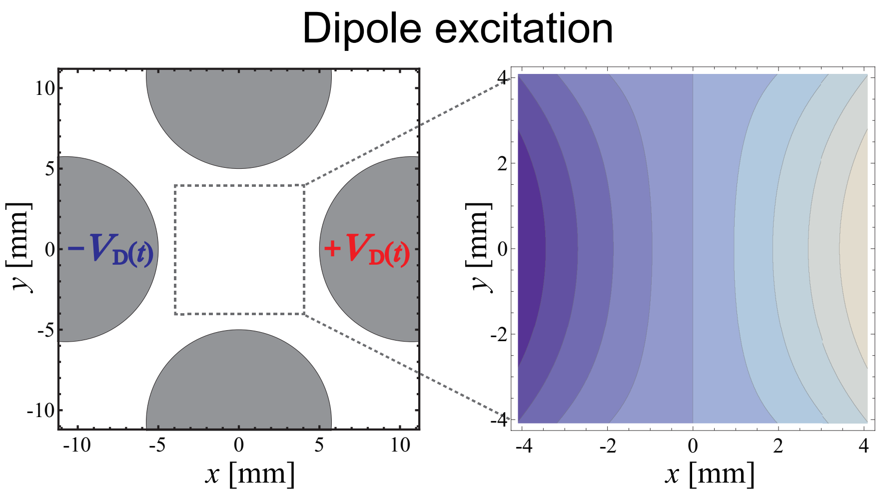


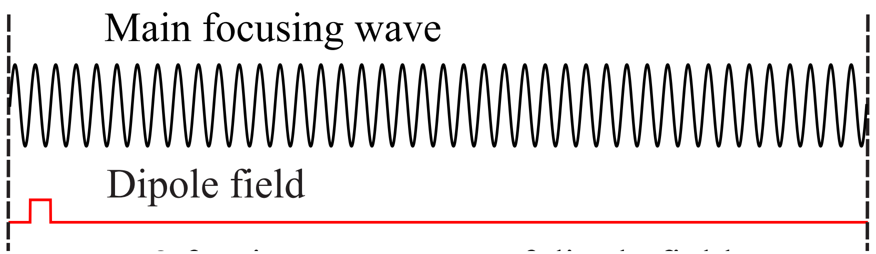
- All distributions show qualitatively same behavior
- K-V beam can cross the resonance without large emittance growth as long as the tune depression and/or crossing speed is below the threshold value.

S-POD Application to Integer Resonance Crossing

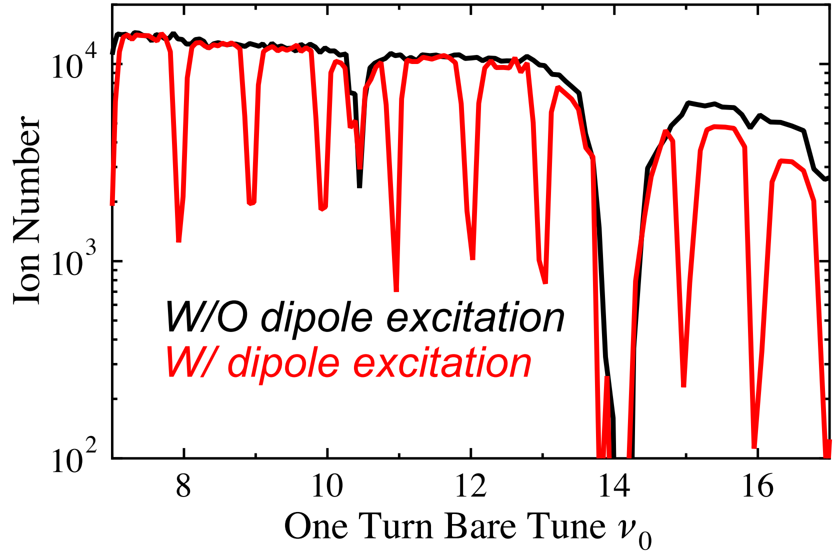
S.L.Sheehy *et al.*, IPAC'13 2677.

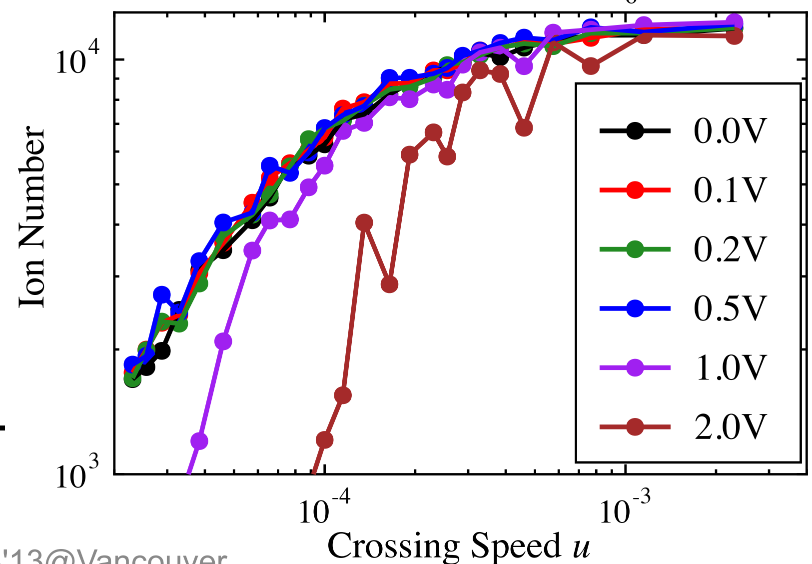
- Crossing of multiple integer resonances in EMMA NS-FFAG.





Dipole perturbation wave to emulate unexpected field from the injection septum.





Summary

- The S-POD system is employed to systematic study of betatron resonance crossing.
- Numerical campaign using the WARP PIC code is also conducted.
- As for low density beams, emittance dilution is negligible or tolerable when the crossing speed is sufficiently high or the resonance is not so strong.
- As for high density beams, linear coherent instability is dangerous even with rather high crossing speed.
- PIC simulation on emittance growth caused by the crossing of ν_0~1/4 resonance.
  - Gaussian, K-V Semi-Gaussian and Waterbag beams have same tendency.
  - K-V beam is the most stable as long as the tune depression and crossing speed is below the threshold value.

Acknowledgements

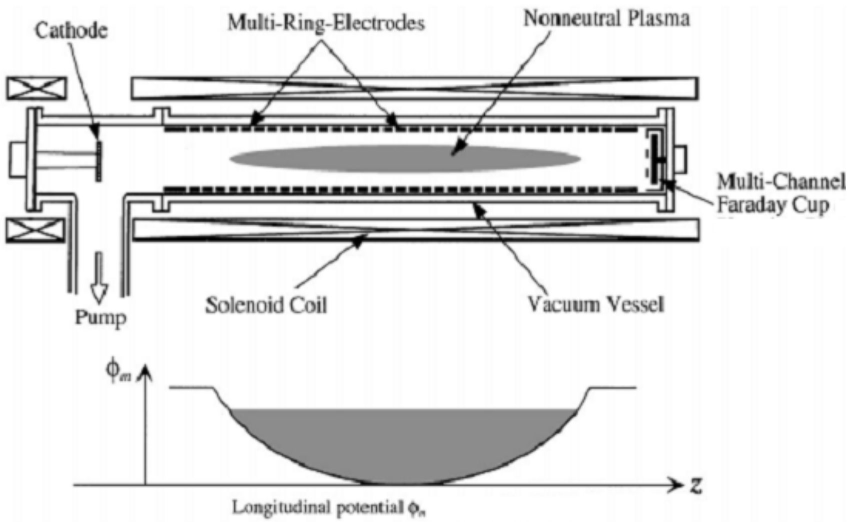
- Members of the beam physics Lab. @ Hiroshima Univ.
  - Staffs
    Hiromi Okamoto, Hiroyuki Higaki, Kiyokazu Ito
  - Students
    K. Fukushima, K. Moriya
    and many past students who contributed to the project.

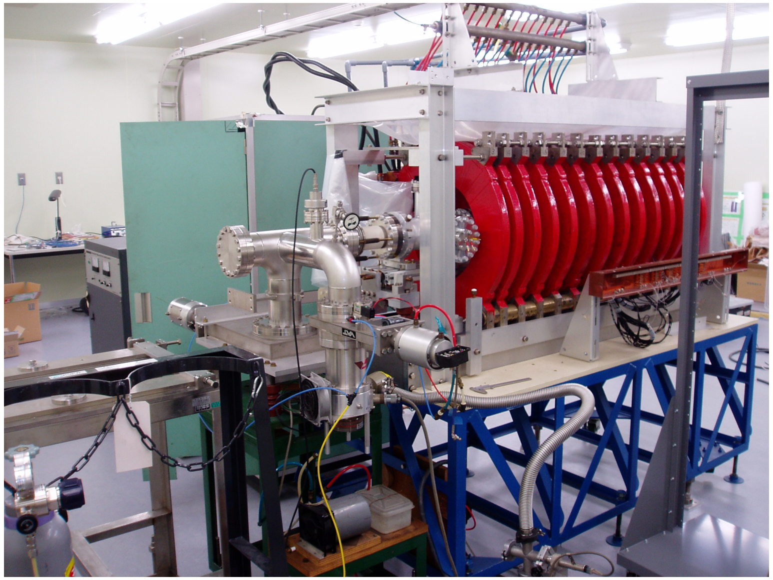
- Collaborators
  - Andrew M. Sessler(LBNL), Steven M. Lund(LLNL), David P. Grote
    Jean-Luc Vay

Thank you for your attention!

Appendix

Penning-Malmberg trap





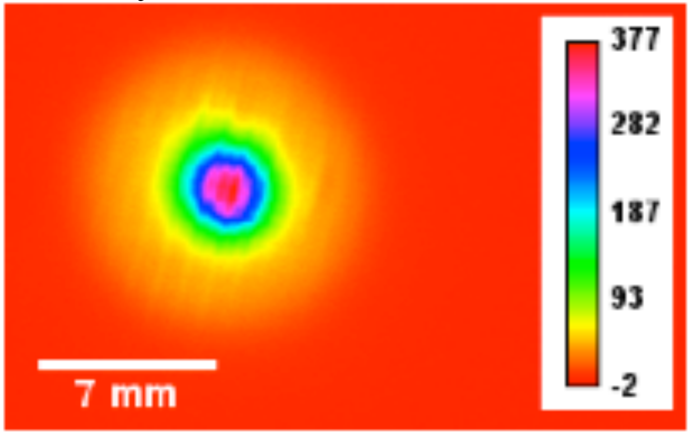
- Transverse confinement
  - *Axial magnetic field*
- Longitudinal confinement
  - *Static potential barrier*

2D Hamiltonian in a rotating frame

$$H_{plasma} \approx \frac{p_x^2 + p_y^2}{2} + \frac{1}{2}K(x^2 + y^2) + \frac{q}{Mc^2}\phi$$

Smooth focusing lattice

Study of beam halo formation

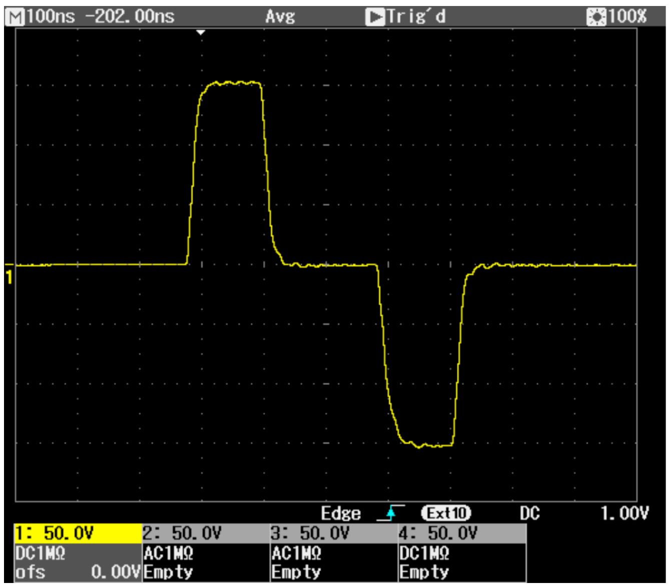


M. Endo *et al*., Proc. of the 9th Annual Meeting of Particle Accelerator Society of Japan, pp. 427 - 429 (2011)

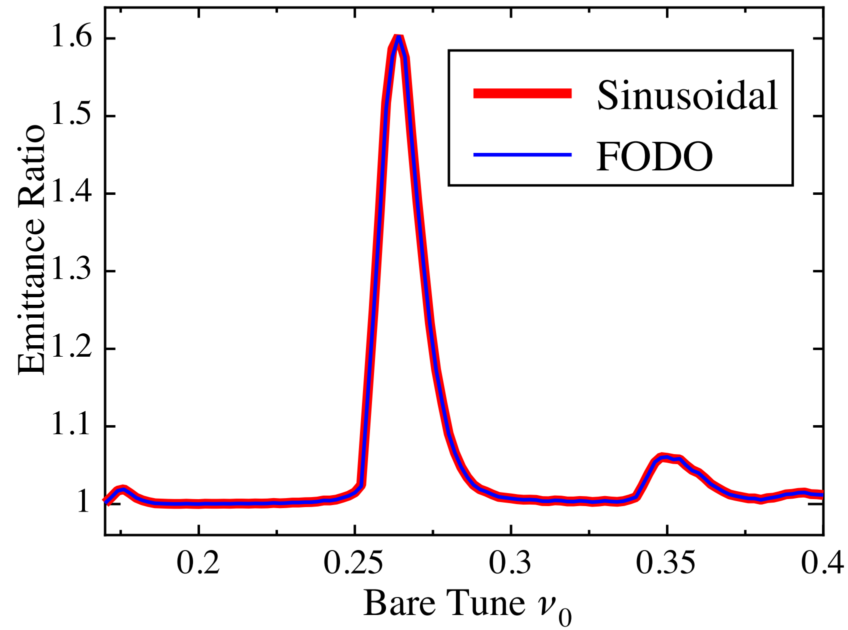
Choice of Lattice Function

- The S-POD system can generates a wide variety of lattice functions.

Doublet lattice emulation in S-POD*



Stop-band distribution (PIC simulation)



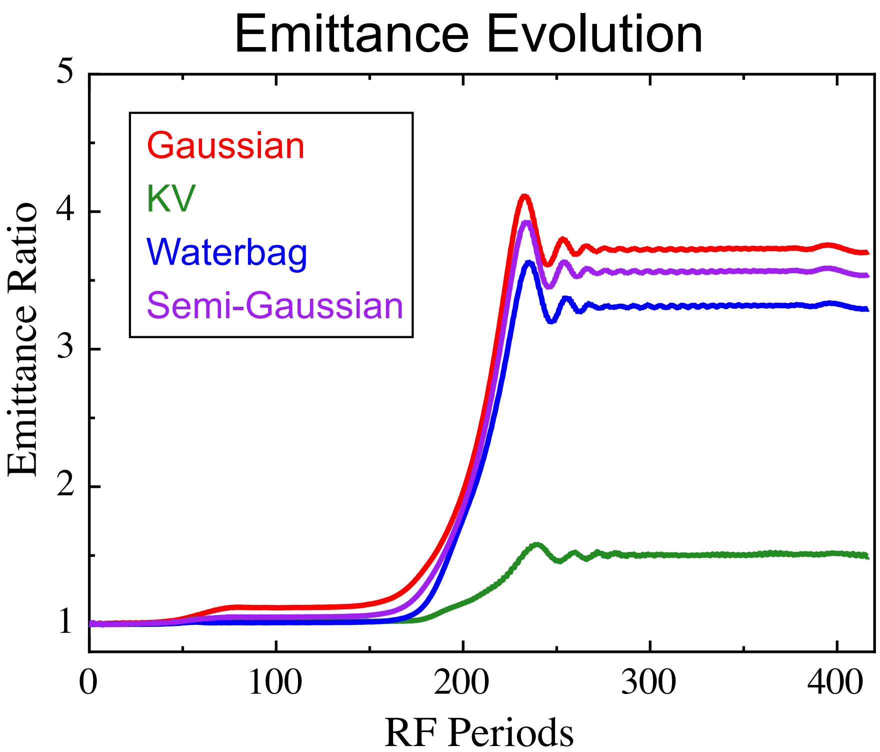
- FODO lattice and sinusoidal focusing system have almost an identical resonance structure.
- We employ the sinusoidal focusing just for technical simplicity.

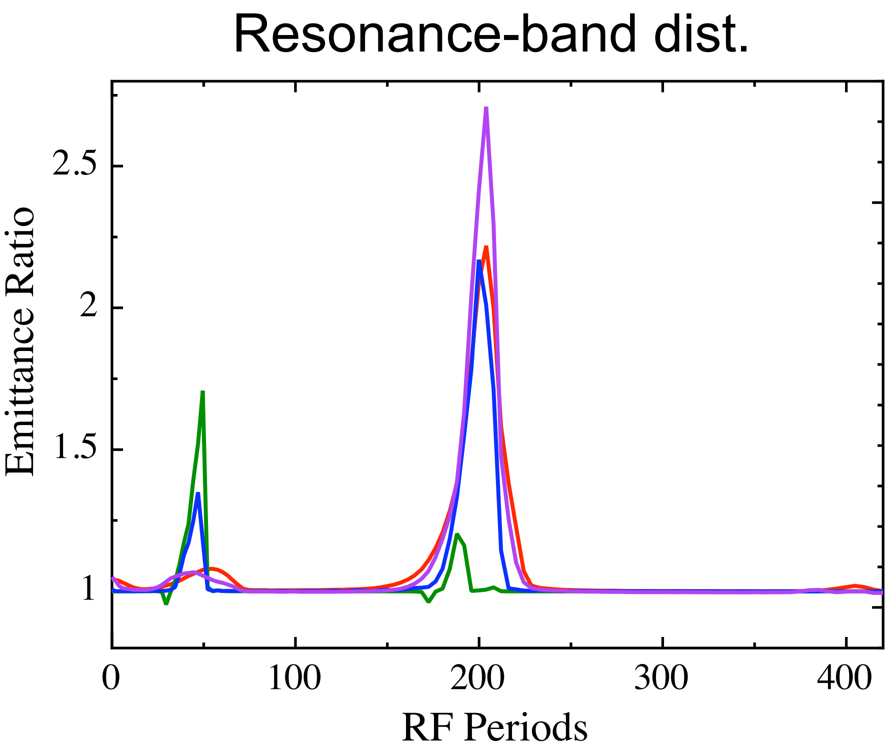
* K. Fukushima et al., Nucl. Instrum. Meth. A, to be published.

Resonance Crossing – Distribution Dependency – PIC Simulation

Tune depression: $\eta = 0.90$

420 rf periods sweeping



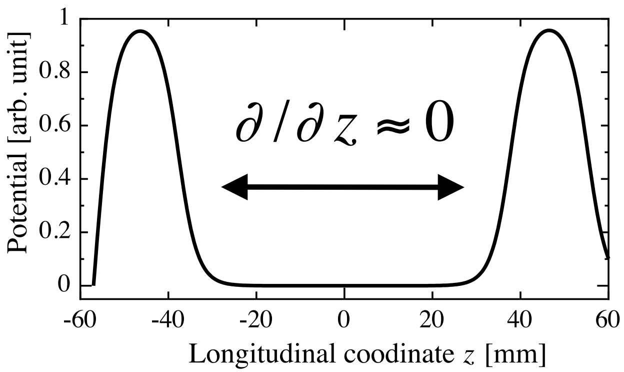


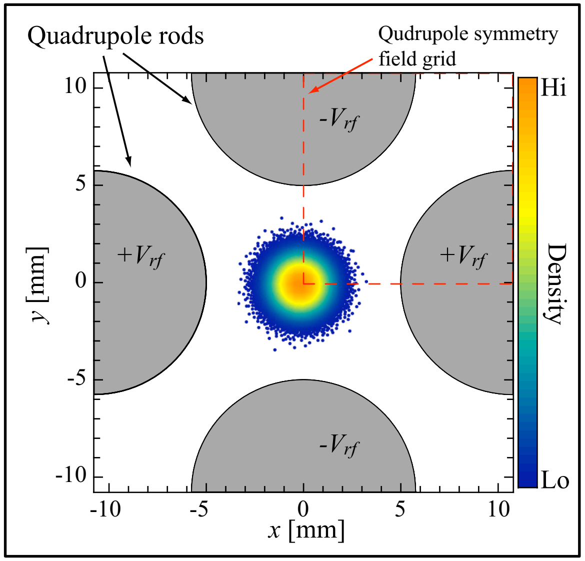
- Qualitatively same emittance evolution.
- KV beam results the smallest emittance growth.

Simulation Setup

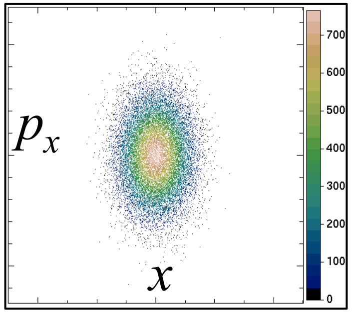
- Transverse 2D simulation using PIC code WARP*
  - The 2D approximation is reasonable because…

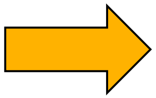
    Longitudinal potential wall is square-like.





- Launch a plasma matched to the focusing force and see how does it evolve.





?

Simulation Parameters

Initial distribution:

Gaussian, KV, Waterbag, Semi-Gaussian

Temperature: 0.1 ~ 0.3 eV

Tune depression η: 0.8 ~ 1.0

# of simulation particles : 10^5

# of integration step : 200 per 1 rf period

* http://hifweb.lbl.gov/webpages/VNLsimulations.html