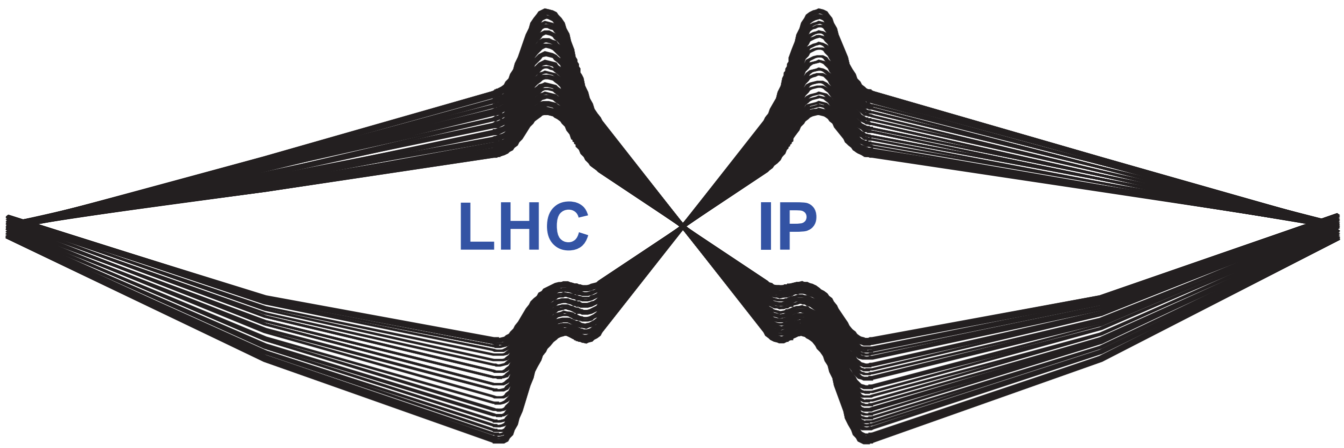
THE RAY-TRACING CODE ZGOUBI



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CYCLOTRONS'13, Vancouver, BC, Canada, 16-20 Sept. 2013

Contents

CYCLOTRONS'13, Vancouver, BC, Canada, 16-20 Sept. 2013

1/ **I'll be commenting the version of Zgoubi that I maintained myself, over the years.**

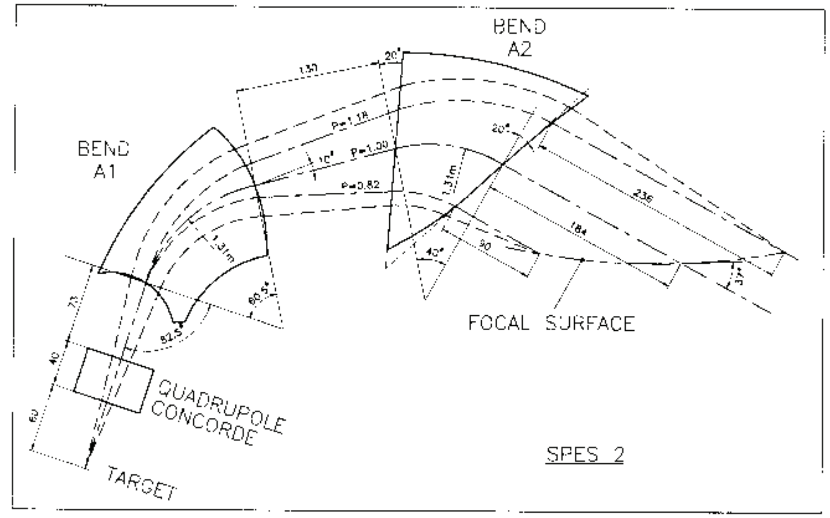
I won't discuss developments done by other groups/people.

2/ **It is available on a development site, together with its "Users' Guide" and its graphic/analysis interface "zpop", and many operational examples**

http://sourceforge.net/projects/zgoubi/

A lot of articles and other technical reports can be found on the DOE OSTI site

http://www.osti.gov/bridge/



... **was written in 1972,** **at Saclay,**
for SPES2, **by J. C. Faivre and D. Garreta**

- **The equation of motion in magnets writes**

$$d(m\vec{v}) \;=\; q\,\vec{v}\times\vec{b}\,dt$$

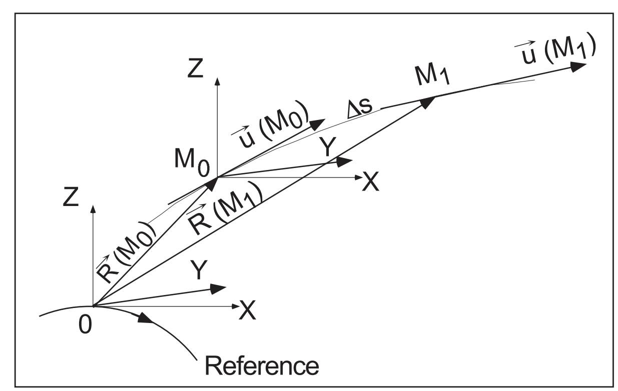
Introduce reduced notations, $\vec{u}=\frac{\vec{v}}{v},\quad \vec{B}=\frac{\vec{b}}{B\rho}$, **then :**

$$\boxed{\vec{u}\,' = \vec{u}\times\vec{B}}$$

- **Solved using truncated Taylor expansions of** $\vec{R}$ **and** $\vec{u}=\vec{v}/v$ **:**

$$\vec{R}(M_1)\approx\vec{R}(M_0)+\vec{u}(M_0)\,\Delta s+\vec{u}\,'(M_0)\,\frac{\Delta s^2}{2!}+\ldots+\vec{u}\,'''''(M_0)\,\frac{\Delta s^6}{6!}$$

$$\vec{u}(M_1)\approx\vec{u}(M_0)+\vec{u}(M_0)\,\Delta s+\vec{u}\,''(M_0)\,\frac{\Delta s^2}{2!}+\ldots+\vec{u}\,'''''(M_0)\,\frac{\Delta s^5}{5!}$$



• Over 40+ years... oodles of magnetic elements have been installed

What you want to simulate :	*Keyword :*
Semi-analytical models :	
Decapole	DECAPOLE, MULTIPOL
Dipole	BEND, DIPOLE[S, -M], MULTIPOL, QUADISEX
Dodecapole	DODECAPO, MULTIPOL
FFAG magnets	FFAG, FFAG-SPI
Multipole	MULTIPOL, QUADISEX, SEXQUAD
Octupole	OCTUPOLE, MULTIPOL, QUADISEX, SEXQUAD
Quadrupole	QUADRUPO, MULTIPOL, SEXQUAD
Sextupole	SEXTUPOL, MULTIPOL, QUADISEX, SEXQUAD
Solenoid	SOLENOID
Helical dipole	HELIX
Field maps :	
1-D, cylindrical symmetry	BREVOL
2-D, mid-plane symmetry	CARTEMES, POISSON, TOSCA
2-D, no symmetry	MAP2D
2-D, polar mesh	POLARMES
3-D	TOSCA
4-D : time !	

AN EXAMPLE OF A "KEYWORD" : MULTIPOL

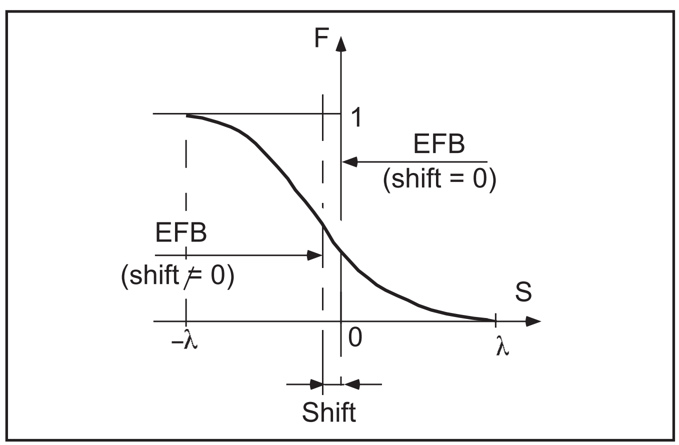
- **Field and derivatives as needed in the Taylor series**

$$\frac{\partial^{i+j+k}\vec{B}_n(X,Y,Z)}{\partial X^i \partial Y^i \partial Z^k} \qquad i+j+k=0 \textbf{ to } 4 \tag{1}$$

are obtained by differentiation of the scalar potential

$$V_n(X,Y,Z) = (n!)^2 \left(\sum_{q=0}^{\infty} (-1)^q \frac{G^{(2q)}(X)(Y^2+Z^2)^q}{4^q q!(n+q)!} \right) \left(\sum_{m=0}^{n} \frac{sin\left(m\frac{\pi}{2}\right) Y^{n-m} Z^m}{m!(n-m)!} \right) \tag{2}$$

- $G(s)$ **is a longitudinal form factor which simulates the "field fall-off"**



EXAMPLE (2005+) – Virtual EMMA FFAG / ON-LINE MODEL

(several companion posters and papers at CYC'13)

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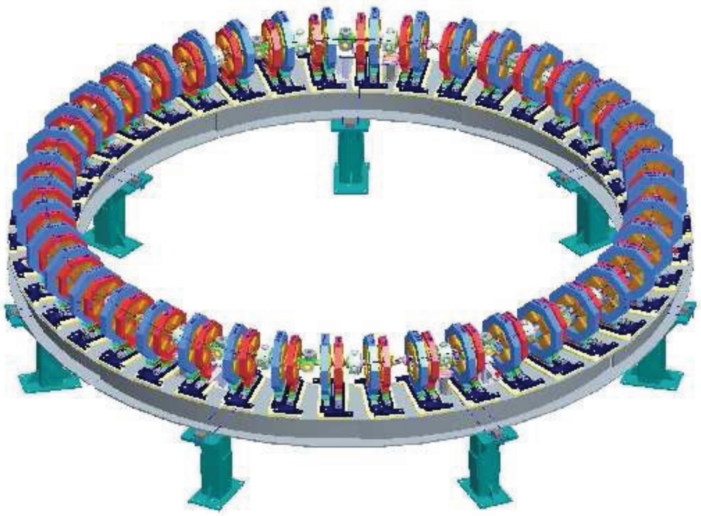
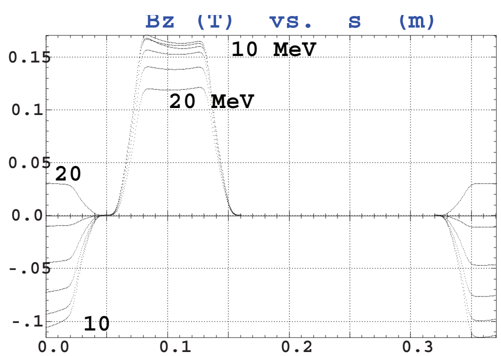
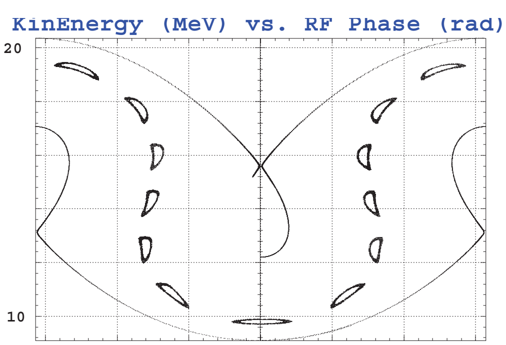




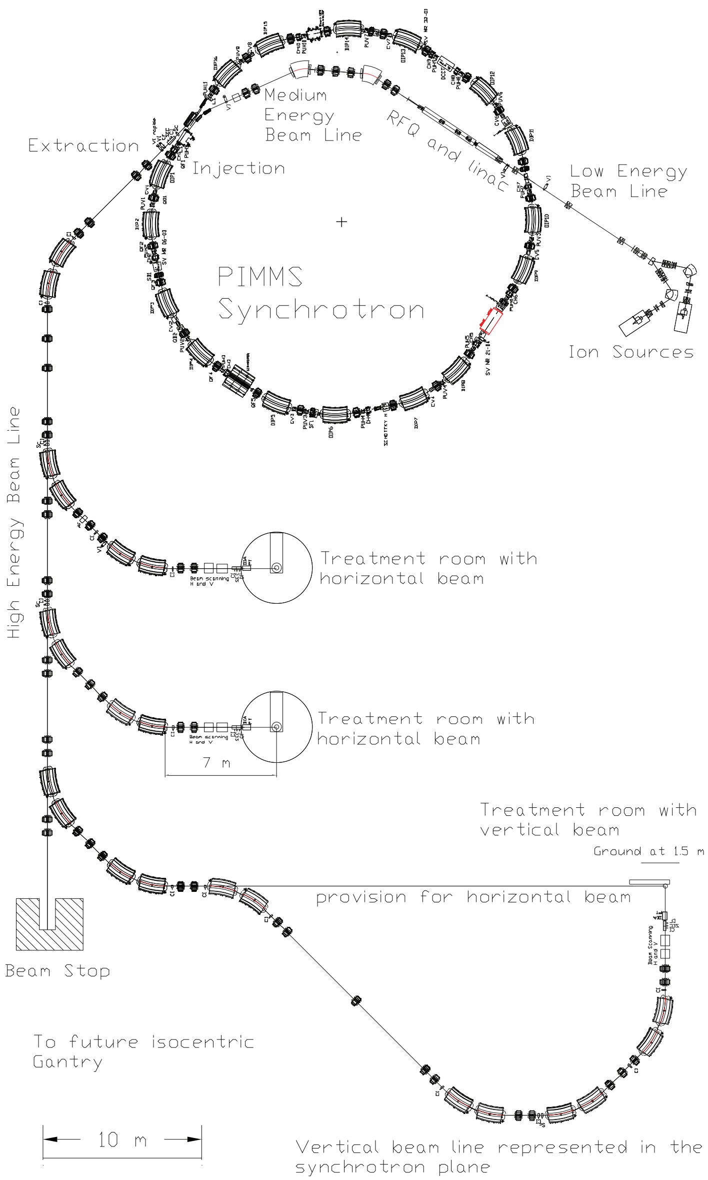


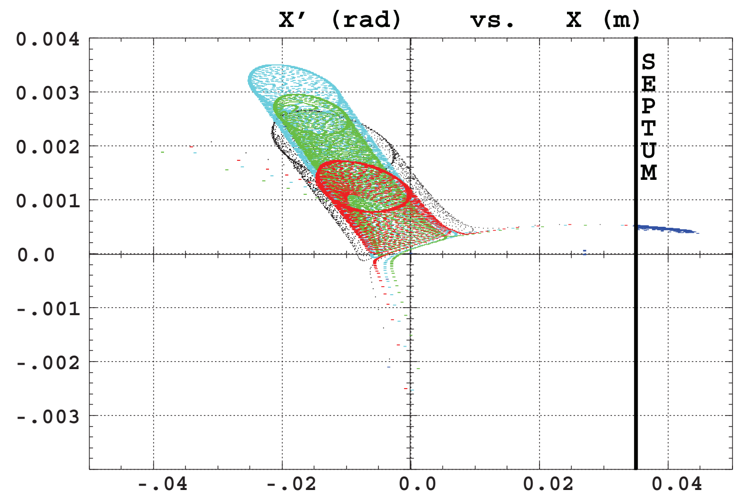
Zgoubi input data file - excerpt :

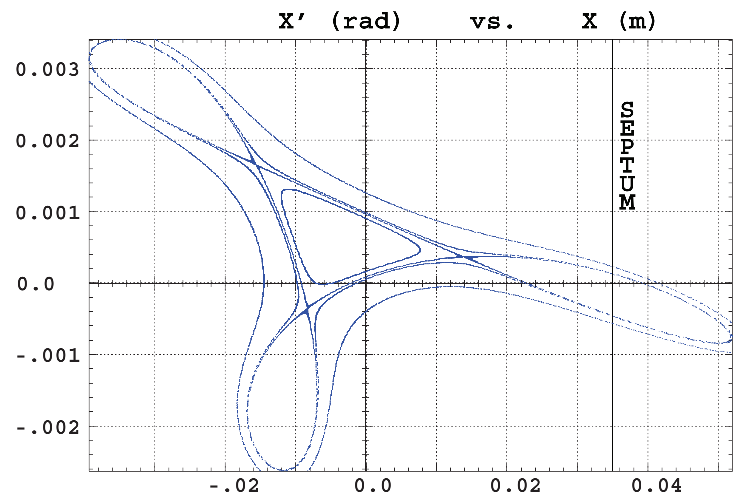
```
'MARKER' RingInj BegRing                    start of ring. Injection point
'MULTIPOL' QD                               start of first cell
0
7.5698 5.3 0. -2.49324 0 0 0 0 0 0 0 0
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 3.404834122312866 0.
'MARKER' BPM2 off                           BPM location
'DRIFT' sd
5.00
'MULTIPOL' QF
0
5.8782 3.7 0. 2.47708 0 0 0 0 0 0 0 0
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 0.7513707181808552 0.
'DRIFT' ld
8.
'CAVITE'                                    programmable RF cavity
7
0.736669 1.3552e9                           Orbit length, RF frequency
70e3 0.                                     Voltage, relative phase
'MARKER' BPM1 off                           BPM location
'CHANGREF'                                  cell orientation - wrt. next one
0. 0. -8.571428571429                       end of first cell
-----------
next 41 cells
-----------
'REBELOTE'                                  multiturn tracking
150 0.2 99
'END'
```

EXAMPLE (∼2000) – ACCURACY OF THE INTEGRATOR

- **Simulation of resonant slow extraction from a carbon synchrotron**
- **Main difficulties : (i) motion near separatrix, (ii) slow process, ∼0.1 second(s)** ⇒ $>> 10^5$ **turns tracking**.







3 THE ELECTRIFICATION OF ZGOUBI

... intervened in the early 1990s, motivated, as usual, by on-going R/D tasks.

• When both $\vec{e}$ and $\vec{b}$ are non-zero, the complete equation is solved,

$$\boxed{(B\rho)'\vec{u} + B\rho\,\vec{u}' = \vec{e}\,/\,v\; +\; \vec{u}\times\vec{b}}$$

One can then push the rigidity, with the same method of (truncated) Taylor series

$$(B\rho)(M_1) \approx (B\rho)(M_0) + (B\rho)'(M_0)\Delta s + ... + (B\rho)''''(M_0)\frac{\Delta s^4}{4!} \quad (3)$$

and the time of flight,

$$T(M_1) \approx T(M_0) + \frac{dT}{ds}(M_0)\,\Delta s + \frac{d^2T}{ds^2}(M_0)\,\frac{\Delta s^2}{2} + \frac{d^3T}{ds^3}(M_0)\,\frac{\Delta s^3}{3!} + \frac{d^4T}{ds^4}(M_0)\,\frac{\Delta s^4}{4!} \quad (4)$$

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• A list of the electrostatic elements :

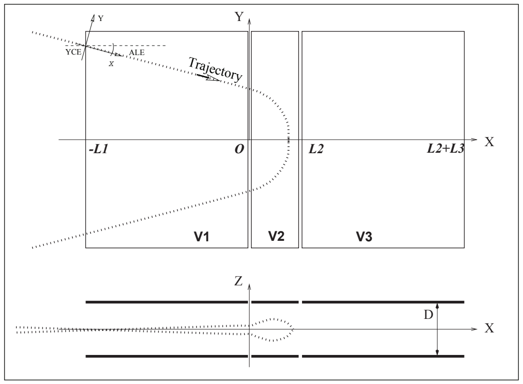
What you want to simulate :	*Keyword :*
Semi-analytical models :	
2-tube (bipotential) lens	EL2TUB
3-tube (unipotential) lens	UNIPOT
Decapole	ELMULT
Dipole	ELMULT
Dodecapole	ELMULT
Multipole	ELMULT
N-electrode mirror/lens, straight slits	ELMIR
N-electrode mirror/lens, circular slits	ELMIRC
Octupole	ELMULT
Quadrupole	ELMULT
R.F. (kick) cavity	CAVITE
Sextupole	ELMULT
Skewed multipoles	ELMULT
Field maps :	
1D, cylindrical symmetry	ELREVOL
2-D, no symmetry	MAP2D_E

• A list of the magneto-electrostatic elements :

What you want to simulate :	*Keyword :*
Semi-analytical models :	
Decapole	EBMULT
Dipole	EBMULT
Dodecapole	EBMULT
Multipole	EBMULT
Octupole	EBMULT
Quadrupole	EBMULT
Sextupole	EBMULT
Skew multipoles	EBMULT
Wien filter	SEPARA, WIENFILT

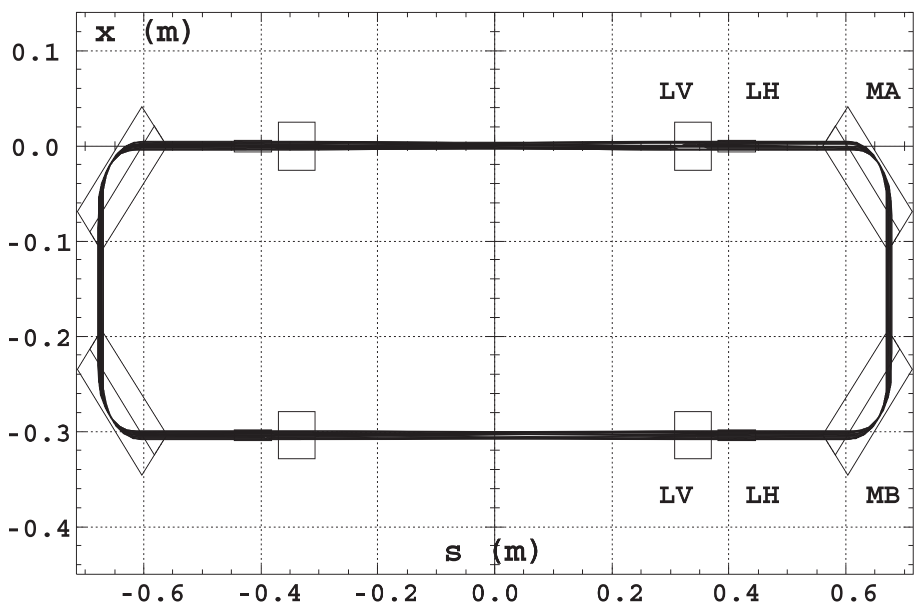
EXAMPLE (EARLY 2000s) - ELECTROSTATIC TIME-OF-FLIGHT RING SPECTROMETER

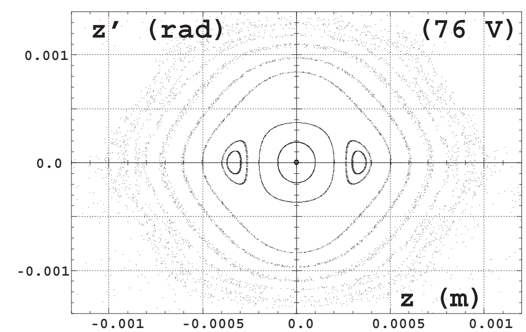
The simulation uses a single, highly non-linear element : 3-electrode parallel plate condenser 'ELMIR'

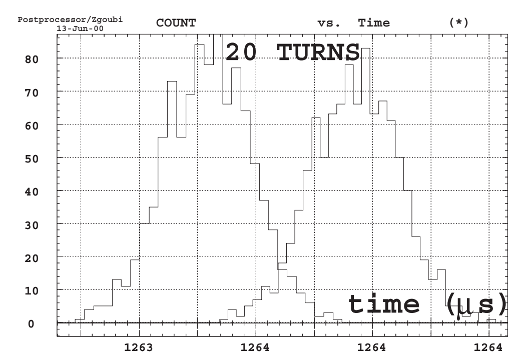


$$V(X,Z) = \sum_{i=2}^{3} \frac{V_i - V_{i-1}}{\pi} \arctan \frac{\sinh(\pi(X - X_{i-1})/D)}{\cos(\pi Z/D)}$$

Typical plate voltage 50-100 Volts.







... was installed in 1990 for a partial siberian snake project at the 3 GeV ring SATURNE, Saclay.

- **Equation of spin precession :**

$$\frac{d\vec{S}}{dt} = \frac{q}{m}\vec{S} \times \vec{\Omega}, \quad \textbf{with} \quad \vec{\Omega} = (1+\gamma G)\vec{b} + G(1-\gamma)\vec{b}_{//}$$

- **Normalize as earlier**

$$\boxed{\vec{S}\,' = \vec{S} \times \vec{\omega}}$$

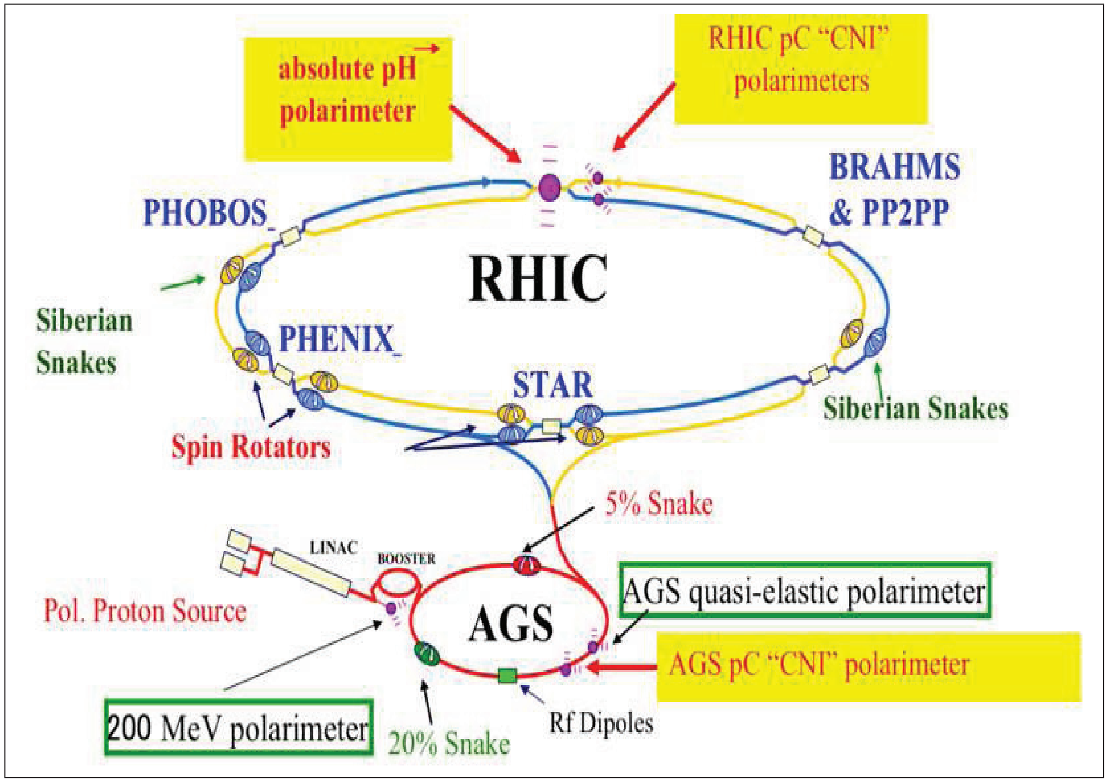
same form as $\vec{u}' = \vec{u} \times \vec{B}$!

It is solved using the outcomes of the particle ray-tracing.

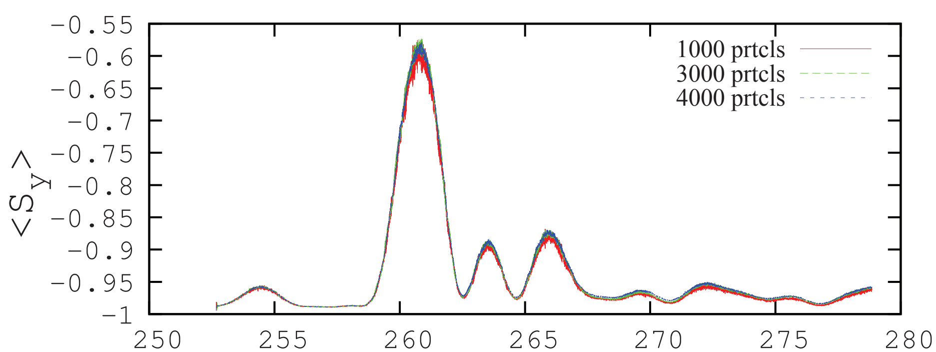
- **use again truncated Taylor expansion to push $\vec{S}$**

$$\vec{S}(M_1) \approx \vec{S}(M_0) + \frac{d\vec{S}}{ds}(M_0)\,\Delta s + \frac{d^2\vec{S}}{ds^2}(M_0)\frac{\Delta s^2}{2} + \frac{d^3\vec{S}}{ds^3}(M_0)\frac{\Delta s^3}{3!} + \frac{d^4\vec{S}}{ds^4}(M_0)\frac{\Delta s^4}{4!}$$

EXAMPLE (2009...) - ON-LINE MODEL OF THE AGS. RHIC STUDIES

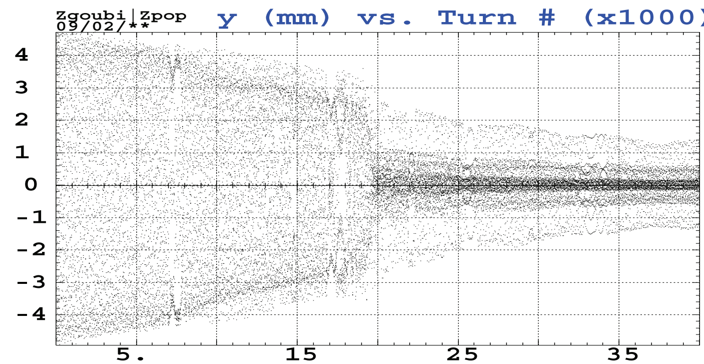


Polarization studies in RHIC - 10^5 turn runs :



Average polarization as a function of energy at traversal of the snake resonance $G\gamma = 231 + Q_y$.

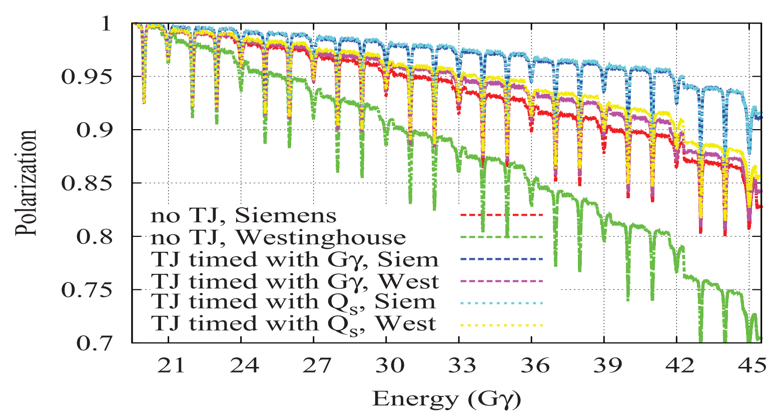
BD studies in the AGS with snakes :



Horizontal excursion from injection to transition energy. 5 particles.

∼40000 turns, 20 min. CPU.

Optimization of polarization transmission :



3000 particles tracking, 40000 turns. Exploring machine setting conditions.

5 SYNCHROTRON RADIATION - ENERGY LOSS

Was installed in $\sim$2000 for emittance increase studies along the linear collider BDS.

• The energy loss is calculated after each integration step Δs, in a classical manner, accounting for two random processes :

- probability of emission of a photon

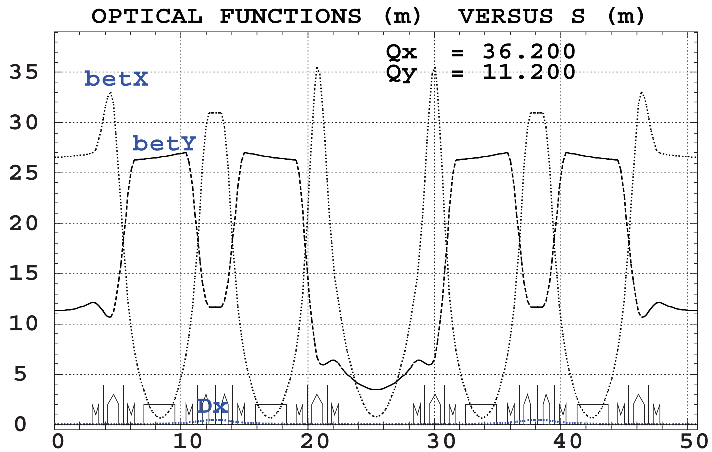
- probability of the photon energy

EXAMPLE (2009) – SYNCHROTRON RADIATION DAMPING IN RINGS

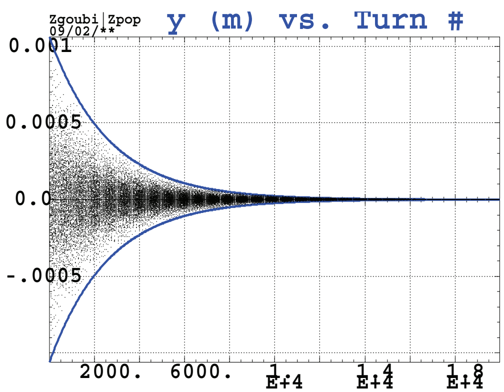
Consider ESRF Chasman-Green super-cell.

Interest : all-analytical understanding.

16 cells ring, 812.6 m, 64 bends.



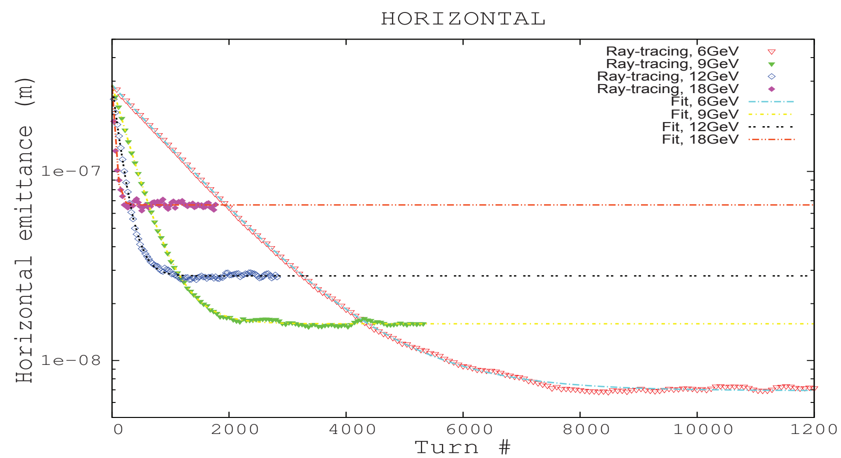
Principle :



Damping of vertical motion over 20000 turns (left), single particle is tracked. Its vertical invariant (right) decreases towards zero.

Emittance damping :

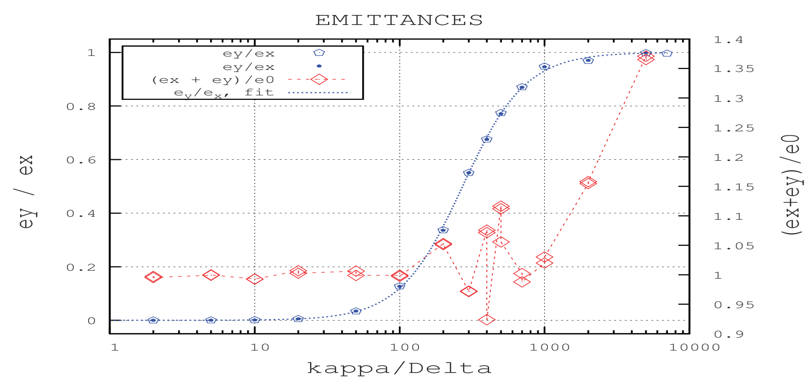
$$\epsilon(t) = \epsilon_0\, e^{-t/\tau} + \epsilon_{equil.}\left(1 - e^{-t/\tau}\right)$$



$\tau_x \approx 1300$ **turns.**

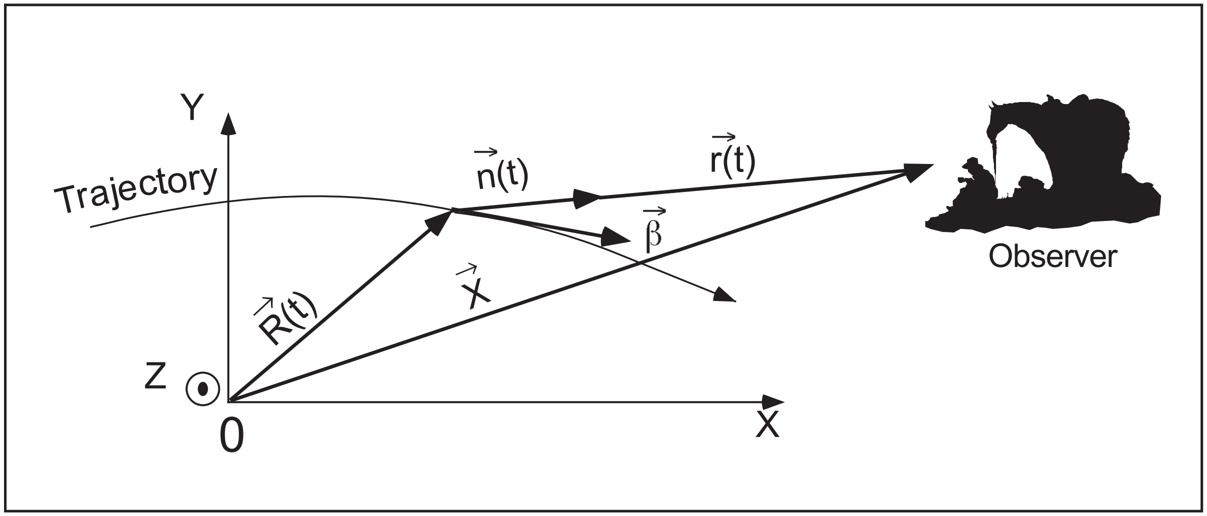
Coupling :

$$\frac{\epsilon_y}{\epsilon_x} = \frac{\kappa^2}{\kappa^2 + \Delta^2}, \qquad \epsilon_x + \epsilon_y = \epsilon_0.$$



 SYNCHROTRON RADIATION - SPECTRAL-ANGULAR DENSITY

- **Was installed in 1994 for the study of deleterous interference effects at the LEP beam diagnostics mini-wiggler.**



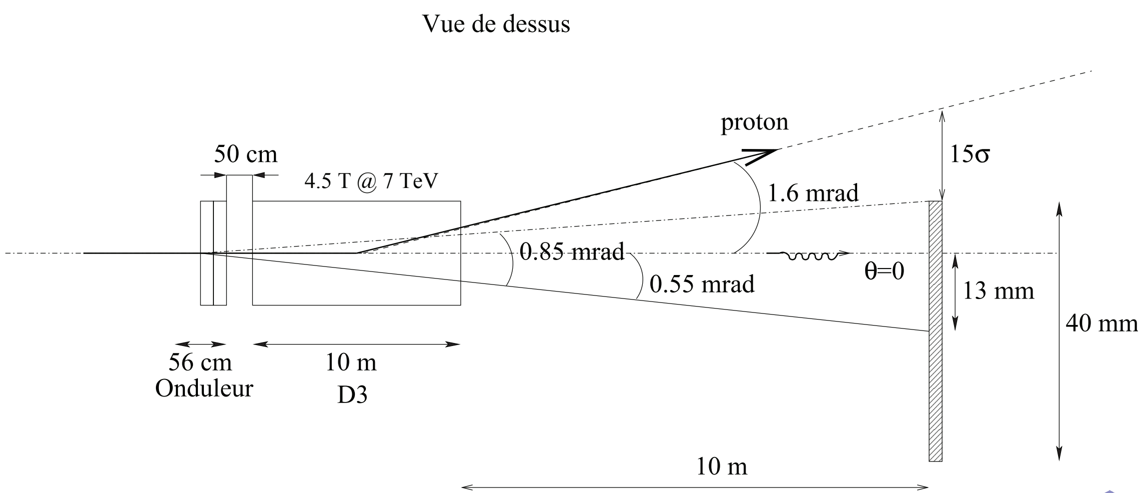
- **The ray-tracing provides the ingredients to compute**

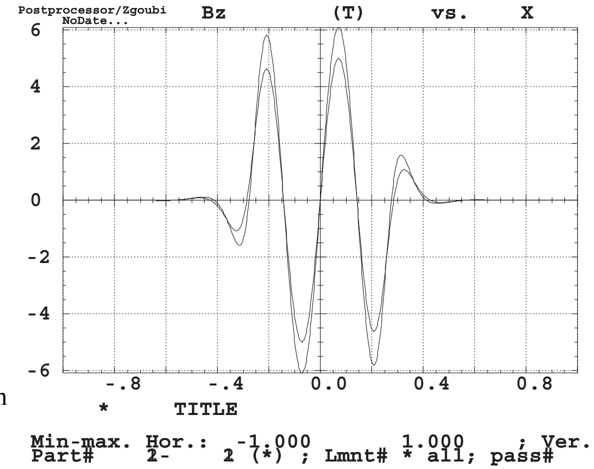
$$\vec{\mathcal{E}}(\vec{n},\tau) = \frac{q}{4\pi\varepsilon_0 c}\,\frac{\vec{n}(t)\times\left[\left(\vec{n}(t)-\vec{\beta}(t)\right)\times d\vec{\beta}/dt\right]}{r(t)\left(1-\vec{n}(t)\cdot\vec{\beta}(t)\right)^3}, \qquad \mathcal{B} = \vec{n}\times\vec{\mathcal{E}}/c$$

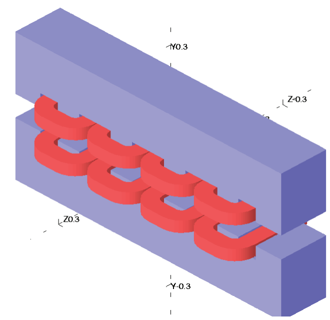
- **The electric field of the radiation is then Fourier transformed,**
  so yielding the spectral angular energy density :

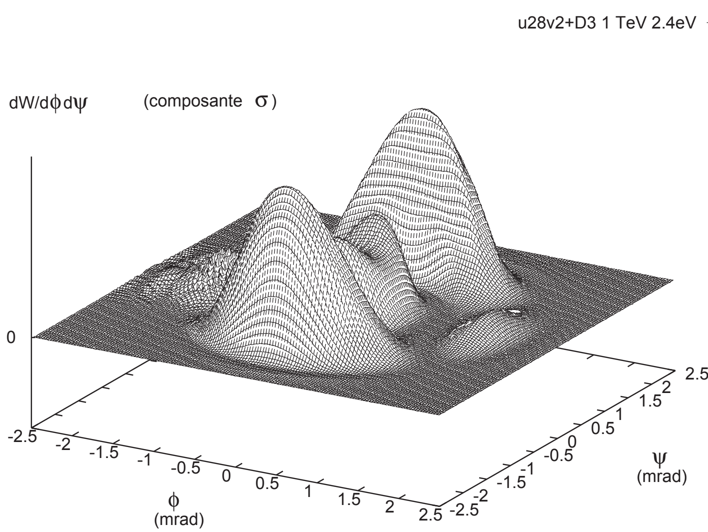
$$\partial^3 W/\partial\phi\,\partial\psi\,\partial\omega = 2r^2\left|FT_\omega\left(\vec{\mathcal{E}}(\tau)\right)\right|^2/\mu_0 c$$

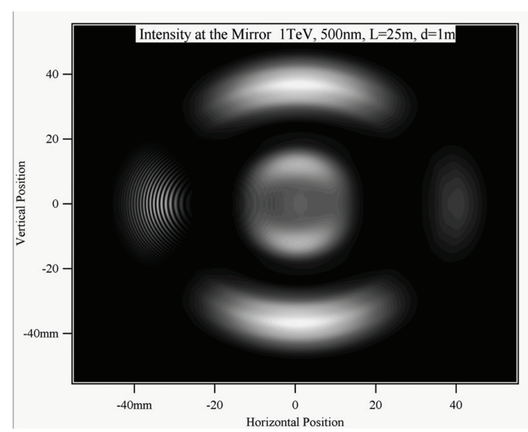
EXAMPLE (∼2000) – DESIGN OF DIAGNOSTICS INSTALLATIONS AT LHC











Intensity from 1 TeV protons in D3+U, λ = 500 nm. Zgoubi on the left, SRW on the right.

7 SPIN DIFFUSION

... a spin-off ! Comes for free

SPIN DYNAMICS
\+
STOCHASTIC ENERGY LOSS BY SR

$\Longrightarrow$ SPIN DIFFUSION

We are working on that, at the moment,

in relation with the eRHIC project R/D studies at BNL.

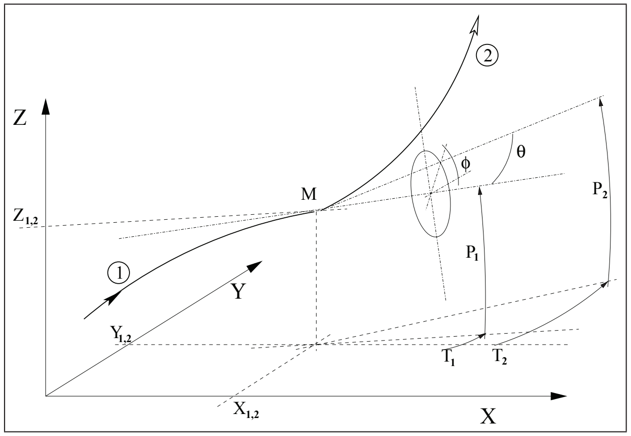
... Installed for eta meson spectrometry at SATURNE, Saclay, late 1980s.

- A classical Monte Carlo method.

- Used, e.g., in the Neutrino-Factory design studies :

$\pi \longrightarrow \mu + \nu$: muon collect channel studies

$\mu \longrightarrow \nu + e$: storage ring studies



9 THE FITTING PROCEDURE

Two methods installed, 1985, 2007.

An indispensable tool for
- preliminary adjustments (orbit, tunes ...)
- optimisations (higher order dynamics as DA, transmission efficiency ...)

FIT CONSTRAINTS :

Trajectory coordinates, at any location

A number of quantities deduced from trajectory coordinates, e.g. :
- first and higher order transport coefficients
- beam's α, β, emittances
- particle transmission efficiency,
- Spin coordinates
- etc.

In the case of periodic structures :
- closed orbits
- tunes, chromaticites, anharmonicities
- Spin closed orbit
- etc.

FIT VARIABLES : any data

Zgoubi input data file, EMMA :

```
'MARKER' RingInj BegRing                          start of ring. Injection point
'MULTIPOL' QD                                     start of first cell
0
7.56987 5.3 0. -2.493246 0 0 0 0 0 0 0 0
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 3.404834122312866 0.
'MARKER' BPM2 off                                 BPM location
'DRIFT' sd
5.00
'MULTIPOL' QF
0
5.87824 3.7 0. 2.477081 0 0 0 0 0 0 0 0
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 1.00 1.00 1.00 1.00 1.00 1. 1. 1. 1.
4 .1455 2.2670 -.6395 1.1558 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0.1
2 0. 0.7513707181808552 0.
'DRIFT' ld
8.
'CAVITE'                                          accelerating cavity
7
0.736669 1.3552e9                                 Orbit length, RF frequency
70e3 0.                                           Voltage, relative phase
'MARKER' BPM1 off                                 BPM location
'CHANGREF'                                        cell orientation - wrt. next one
0. 0. -8.571428571429                             end of first cell
'REBELOTE'                                        multiturn tracking
150 0.2 99
'END'
```

CONCLUSION (1)

• Given what we have seen, one may well imagine the following simulation, all integrated - in one Zgoubi run :

A high energy polarized muon cyclotron (FFAG) decay ring (à la "MuSTORM" (JB Lagrange, Monday))

• This is fully operational in Zgoubi. One will get :

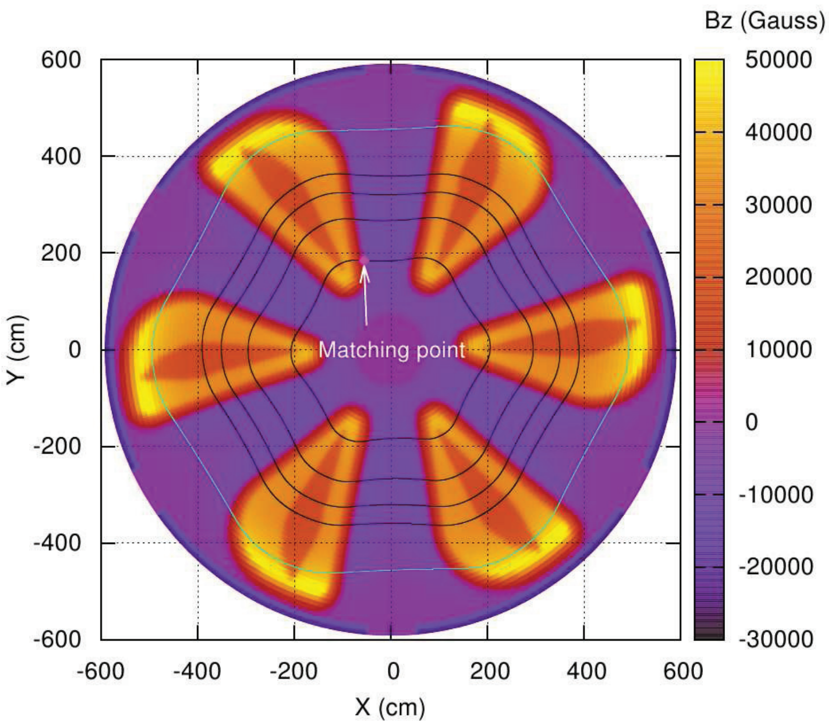
- muons tracks over their few-100 turns lifetime around the ring
- evolution of neutrino beam flux with time upon $\mu \longrightarrow \nu + e$ decay
- radiative pollution by the decay electrons - they are tracked as well, if requested
- muon beam polarization and its evolution with time
- and, why not, beam diagnostics using SR !

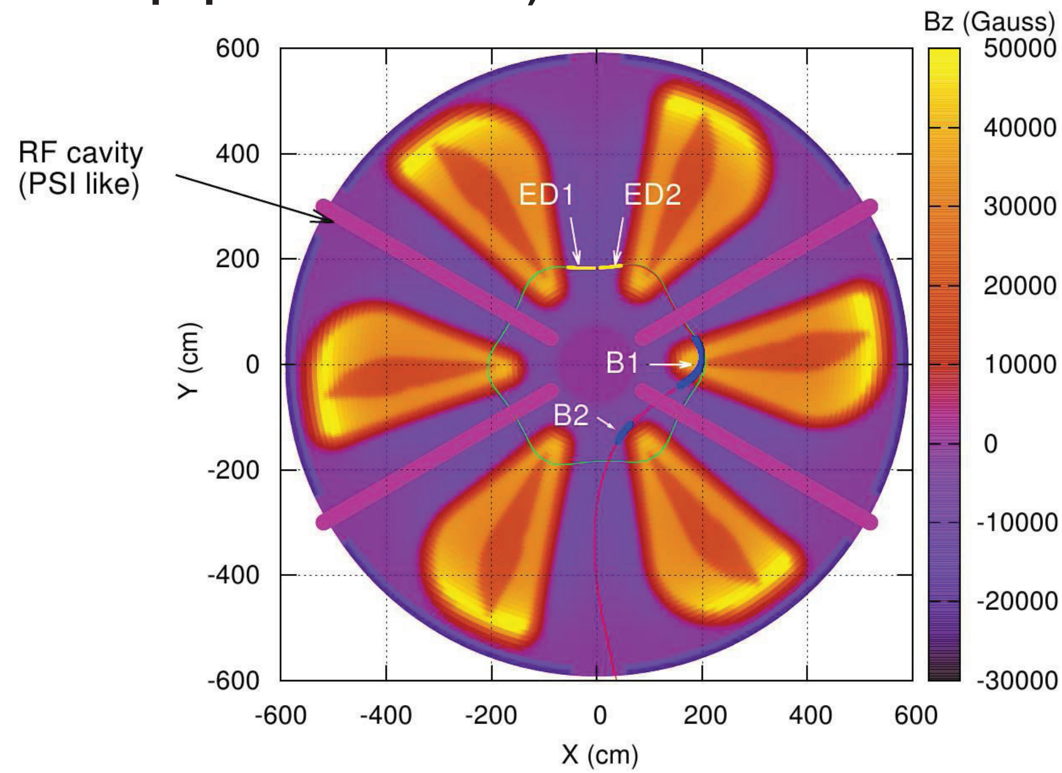
Estimated CPU time : 0.05 second/turn * 100 turns $\approx$ 5 seconds.

11 CONCLUSION (2)

... ZGOUBI ROUTINELY DOES CYCLOTRONS, AS WELL !

(several companion papers at CYC'13)





THANK YOU FOR YOUR ATTENTION