AN ALL-PURPOSE ACCELERATOR CODE, ZGOUBI*

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Abstract

A brief history of the ray-tracing code Zgoubi is given, illustrated with its numerous capabilities, up to the most recent 6-D tracking simulations in the largest accelerators.

INTRODUCTION

The ray-tracing code Zgoubi is being developed since the early 1970's for the design, development and operation of spectrometers, beam lines and circular accelerators. A Users' Guide is available [1] and provides extensive description of its methods and contents, whereas a voluminous documentation is available on web, *e.g.*, Ref. [2].

The code is a genuine compendium of numerical recipes allowing the simulation of most types and geometries of optical elements as encountered in accelerator assemblies. It provides built-in fitting procedures that make it a powerful design and optimization tool. It can account for synchrotron radiation, spin tracking, in-flight decay and other Monte Carlo based simulations. The high accuracy of the Zgoubi integrator allows efficient, long-term multi-turn tracking, in field maps and analytical models of fields. In particular, being based on stepwise integration Zgoubi allows taking full profit of modern high-accuracy magnet and RF system 4-D design codes (space + time), since it can directly use the field maps they produce.

A SHORT RETROSPECTIVE

1970s-1980s Period

Zgoubi and its integrator were first written for the design and development of the SPES-II spectrometer at the 3 GeV synchrotron SATURNE, Saclay [3]. The author inherited a copy of the code from Saby Valero at the "Theory Group" at SATURNE in 1985, for the purpose of a design collaboration with GSI regarding the KAOS spectrometer, still in operation nowadays [4]. This is also when the first version of the Users' Guide was written [5]. Zgoubi was at that time in use since a few years for the design of the high-resolution energy-loss spectrometer SPEG, still in operation at GANIL [6]. It was used as a beam line and spectrometer tool in a number of labs as CERN, JINR-Dubna, TRIUMF, etc.

SATURNE had then, second half of the 1980s, projects of a solenoidal partial snake (an idea that had emerged at the AGS, BNL) in complement with tune jump techniques then in use, to overcome depolarizing resonances [7]. The sophistication of the stepwise ray-tracing method was considered an appropriate candidate for an accurate evaluation of the technique, thus spin, together with periodic tracking, were installed in Zgoubi [8]. The code was later used in various spin studies in the following years, as the Neutrino Factory, super-B, ILC and, Section 4, RHIC.

1980s-early 2000s Period

Computer capabilities were in fast development, with prospects of CPU speed as well as memory capacities no longer being a concern in long-term, multi-particle beam and polarization transport simulations. Essentially, that period has seen the extensive development of Zgoubi for periodic machines, a daring and challenging technique in view of its excessive sophistication in regard to the paraxial machines that rings are, notwithstanding the culture of the guardian of truth "Hamiltonian integrators". The aim was to allow the use of the highest accuracy transport method: stepwise ray-tracing in realistic field models, This is addressed in the following two sections, more can be found in review and conference papers by the author, regarding, *e.g.*, LHC, Neutrino Factory muon rings, FFAGs, hadrontherapy and other electrostatic rings.

THE ZGOUBI METHOD

The Lorentz Equation

The *Lorentz* equation governs the motion of a particle of charge q, relativistic mass m and velocity $\vec{v}$ in electric and magnetic fields $\vec{e}$ and $\vec{b}$, and the *Thomas-BMT* equation which governs spin motion, write respectively (reference frame in Zgoubi defined in Fig. 1)

$$\frac{d(m\vec{v})}{dt} = q\,(\vec{e} + \vec{v}\times\vec{b})\,, \quad \frac{d\vec{S}}{dt} = \frac{q}{m}\,\vec{S}\times\vec{\omega}.$$

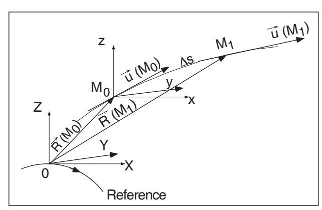

Figure 1: Position and velocity of a particle, pushed from location M_0 to location M_1 in Zgoubi frame.

Noting $()' = d()/ds$, $\vec{u} = \vec{v}/v$, $ds = v\,dt$, $\vec{u}\,' = d\vec{u}/ds$, $m\vec{v} = mv\vec{u} = q\,B\rho\,\vec{u}$, with $B\rho$ the rigidity of the particle, these equations can be rewritten in the reduced forms

$$(B\rho)'\,\vec{u} + B\rho\,\vec{u}\,' = \frac{\vec{e}}{v} + \vec{u}\times\vec{b}, \quad (B\rho)\,\vec{S}\,' = \vec{S}\times\vec{\omega} \quad (1)$$

with for the latter,

$$\vec{\omega} = (1+\gamma G)\vec{b} + G(1-\gamma)\vec{b}_{//} + \gamma(G + \tfrac{1}{1+\gamma})\tfrac{\vec{e}\times\vec{v}}{c^2},$$

G the gyromagnetic factor, γ the Lorentz relativistic factor. Both equations are solved using a truncated Taylor series

$$\vec{a}(M_1) \approx \vec{a}(M_0) + \vec{a}\,'(M_0)\,\Delta s + ... + \vec{a}^{(n)}(M_0)\,\frac{\Delta s^n}{n!} \quad (2)$$

where $\vec{a}$ stands for either the position $\vec{R}$ and velocity $\vec{u}$ which solves the motion, or for the spin $\vec{S}$. A scalar form of Eq. 2 is used to push the rigidity $B\rho(M)$ and time $T(M)$ in the presence of electric fields. The coefficients $a^{(n)} = d^n a/ds^n$ are obtained from the fields and their derivatives, provided using analytical models or field maps.

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With time and on-going projects, additional functionalities have been installed in Zgoubi, as follows:

Energy loss by synchrotron radiation was installed in the late 1990s for the purpose of ILC BDS dynamics studies [9]. It was further benchmarked for damping in rings in the frame of the Super-B project [10], and recently the e-RHIC electron-ion collider project at BNL [11], as a step ahead towards spin diffusion simulations. SR loss computation uses a Monte Carlo technique based on a Poisson law $p(k) = \lambda^{-k}\exp(-k)/k!$ for the photon emission and, for the probability of a photon energy $\epsilon = h\nu$, $\mathcal{P}(\epsilon/\epsilon_c) = \frac{3}{5\pi}\int_0^{\epsilon/\epsilon_c}\int_{\epsilon/\epsilon_c}^{\infty} K_{5/3}(x)dx$. k is the number of photons emitted over an arc Δs, $\lambda = 20er_0\beta^2 B\rho\Delta s/8\bar{h}\sqrt{3}$ the average/radian, $\epsilon_c = 2\pi\bar{h}\,3\gamma^3 c/2\rho$ the critical frequency, all quantities drawn from the ray-tracing ingredients above.

Spectral-angular SR density computation was installed for LEP studies in 1993 [12, 13], and later used for the design of the SR diagnostics station at LHC [14]. It is drawn from the electric field radiated by the particle,

$$\vec{\mathcal{E}}(\vec{n},\tau) = \frac{q}{4\pi\varepsilon_0 c}\,\frac{\vec{n}(t)\times\left[\left(\vec{n}(t)-\vec{\beta}(t)\right)\times\frac{d\vec{\beta}}{dt}\right]}{r(t)\left(1-\vec{n}(t)\cdot\vec{\beta}(t)\right)^3}$$

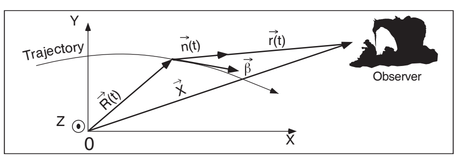

Figure 2: Vectors defining $\vec{\mathcal{E}}(\vec{n},\tau)$ at the observer.

In-flight decay was installed in the late 1980s for analysis of muon pollution from pion decay at the SATURNE spectrometer SPES-III focal plane [15], it was further developed in the mid 2000s for periodic machines and applied for studies in the Neutrino Factory decay rings [16].

Space-charge installation was undertaken by UK groups in the late 2000s for proton driver studies [17].

EXAMPLES

Acceleration of polarized protons at the AGS 9-D tracking (motion + spin), thousand-particle bunches, over the AGS cycle, are routinely performed for polarization transmission studies [18], Fig. 3. They address issues as tune jump timing settings, emittance growth effects, coupling. This is part of the capabilities of the live-data based "AGS on line model" using Zgoubi as an engine, including the 3-D OPERA field maps of the two helical snakes [19].

Crossing of snake resonances at RHIC $\vec{p}$-$\vec{p}$ collider Figure 4 shows typical simulations, 5×10^5 RHIC turns about, on NERSC CPU farms, tracking takes $\approx$ 35 hrs [20].

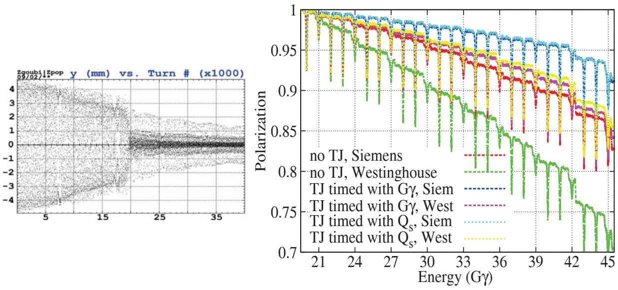

Figure 3: Left: horizontal excursion from injection to transition energy, 5 particles launched at 72 deg. betatron phase intervals on $\epsilon_x = \epsilon_y = 2.5\,\pi$ mm.mrd invariants, showing coupling and betatron resonance effects induced by the snakes. Right: polarization vs. energy depending on AGS jump quad timings and main magnet power supply used, 10^3 particles in a realistic 6-D bunch.

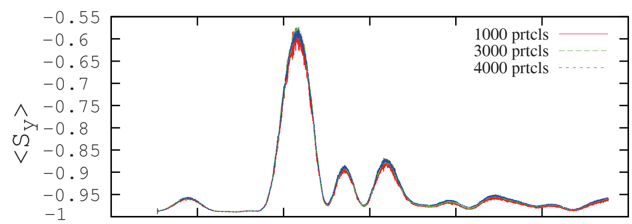

Figure 4: Average polarization (vertical axis, batches of 1000 to 4000 particles) as a function of energy (in units of $G\gamma$) at traversal of the resonance $G\gamma = 231 + Q_y$.

SR damping Extensive benchmarking against theoretical formulae based on a Chasman-Green test cell (ESRF lattice), including emittance coupling, has proven the excellent behavior of the code, two examples are given in Fig. 5 [11].

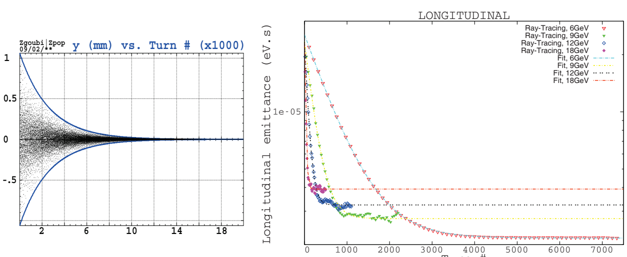

Figure 5: Left: damping of vertical motion to quasi-zero (no scattering) over a few damping times, uncoupled lattice, 6 GeV. Right: damping of the longitudinal emittance, numerical data are fitted with $\epsilon(t) = \epsilon_0\,e^{-t/\tau} + \epsilon_f\,\left(1 - e^{-t/\tau}\right)$, for 6, 9, 12 and 18 GeV.

High power cyclotron studies Cyclotron design is, surprisingly, a recent domain for Zgoubi, however very efficient in that task. An instance is shown in Fig. 6 [21]. Further, full 6-D acceleration up to extraction, dynamic aperture, beam path in tune diagram, etc, can be computed accurately, the limitation is essentially in the quality of the

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magnetic field map [22]. CPU-wise, tracking trials are much shorter than in many of the previous examples, given the comparatively smaller number of turns and ring size. An important piece is under development, space charge, undertaken in the recent past [17].

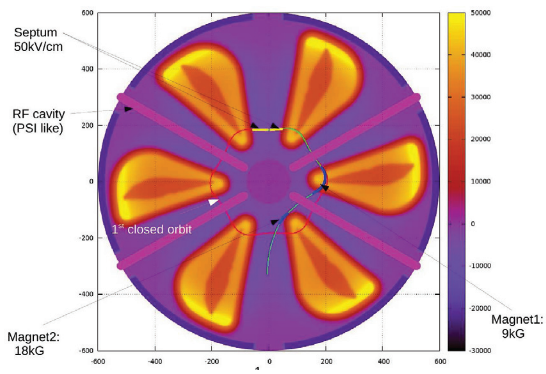

Figure 6: A design of the injection line into a molecular H2 separated sector cyclotron.

SR diagnostic at LHC The beam diagnostics installation in LHC IR4 is based on a super-conducting undulator. Elaborated angular light distributions were computed in designing it in the early 2000s, Fig. 7 [14].

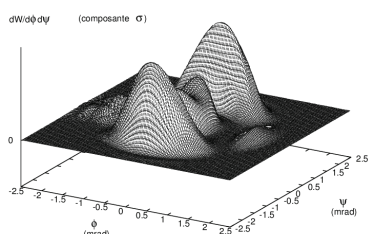

Figure 7: Interferential "Newton rings" from 1 TeV proton radiation. Two light sources interfere in this pattern: dipole edge and undulator, distant 1 m.

FFAG R/D Zgoubi has proven an accurate tool for 6-D acceleration in all possible types of FFAG rings, in the course of project R/D as the Neutrino Factory (fast acceleration of short-lived particles, linear FFAG, fixed RF frequency), hadrontherapy application (scaling lattice, including OPERA field maps or semi-analytical magnet models, pulsed RF) [23, 24, 25].

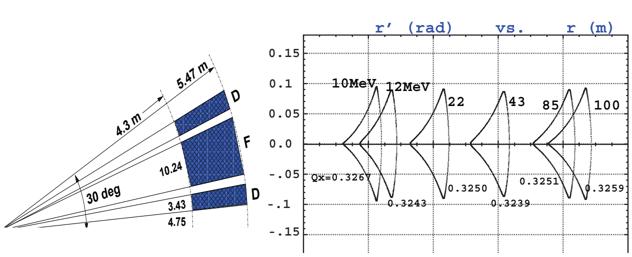

Figure 8: 150 MeV radial triplet FFAG, horizontal dynamic aperture and corresponding tunes at various energies. Not shown here: Zgoubi's semi-analytical FFAG triplet magnet model and the OPERA field maps give similar results [25].

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