TRANSVERSE-LONGITUDINAL COUPLING BY SPACE CHARGE IN CYCLOTRONS

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Abstract

Based on a linear space charge model and on the results of PIC-simulations with OPAL , we analyze the conditions under which space charge forces support bunch compactness in high intensity cyclotrons and/or FFAGs. We compare the simulated emittance increase and halo formation for different matched and mismatched particle distributions injected into a separate sector cyclotron with different phase curves.

INTRODUCTION

Isochronous cyclotrons have no longitudinal focusing by their operation principle. Hence the phase width (bunch length) of the beam is constant for low intensity machines and tends to increase in the presence of space charge forces leading to a limitation of the maximal beam current. The usual strategy to overcome this limitation is to raise the phase acceptance and reduce the energy spread by the installation of one or more flat-top cavities. Flat-top operation raises the intensity limit, but at the cost of a reduction of the average energy gain and the installation of additional expensive hardware.

The PSI Injector II cyclotron is an example of an alternative mode of high intensity operation for cyclotrons. The installation of two bunchers in the injection line increases the longitudinal space charge force and allows for operation in the space charge dominated regime as predicted in Ref. [1, 2]. The former flat-top cavities are now operated as additional accelerating cavities. In this mode the bunch length remains strikingly short even at high beam currents (see for instance Ref. [3, 4, 5, 6, 7]).

Numerous papers have been published that (predicted or) described this effect [1, 2, 8, 9], but due to the complexity and non-linearity of the phenomenon the debate has not finished yet, i.e. the exact conditions to reach this regime are only approximately known and the properties of a beam (i.e. the optimal distribution in phase space at a given intensity) that matches the requirements of space charge dominated operation are not precisely understood.

In preceeding papers we described a method to compute the parameters of matched beams for the space charge dominated regime [10, 11, 12]. The method is based on a linear approximation, strictly valid only for particles close to the center of the bunch. Therefore the applicability of this approach and the underlying model is questionable.

Here we present simulation results obtained with the parallel space charge solver of OPAL -cycl that enables to study the behavior of high intensity beams with a state-of-the-art PIC code [13, 14]. The results show that the predictions of the linear (matching-) model are surprisingly powerful. In fact, the linear model does not only allow to compute the parameters of a matched beam with minimal halo production and minimal emittance increase, it also allows to understand the sensitivity of the bunches with respect to the phase slip. With some additional arguments the model also allows to derive a consistent picture of how the effects are related to the “negative mass instability”.

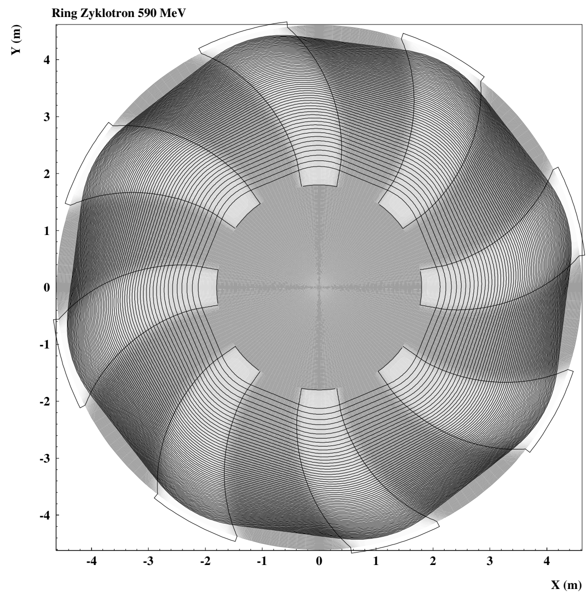

Figure 1: Top view of the idealized ring machine with some equilibrium orbits.

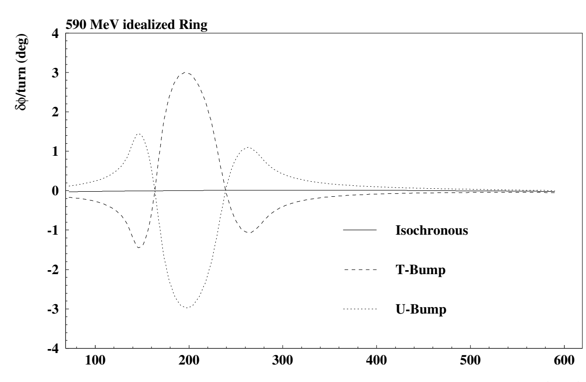

Figure 2: Phase shift per turn vs. radius of the idealized ring machine with bump “U” and bump “T”.

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SPACE CHARGE DOMINATED ACCELERATION

Focusing Frequency

Using the transverse horizontal coordinates x and x' and the longitudinal coordinates y, δ with $\delta \equiv \frac{\Delta p}{p}$, particle motion is focused (i.e. oscillatory), if the horizontal space charge term K_x and the derivative of the relative field error $\varepsilon = B/B_{iso} - 1$ fulfill the inequality [10]:

$$K_x > \frac{1}{r}\,\frac{d\varepsilon}{dr}\,, \tag{1}$$

where B_{iso} is the isochronous field so that ε is related to the phase slip per turn $\frac{d\phi}{dn}$ by

$$\varepsilon \approx -\frac{1}{2\,\pi\,N_h}\,\frac{d\phi}{dn}\,. \tag{2}$$

The relation to the slip factor $\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$ is obtained

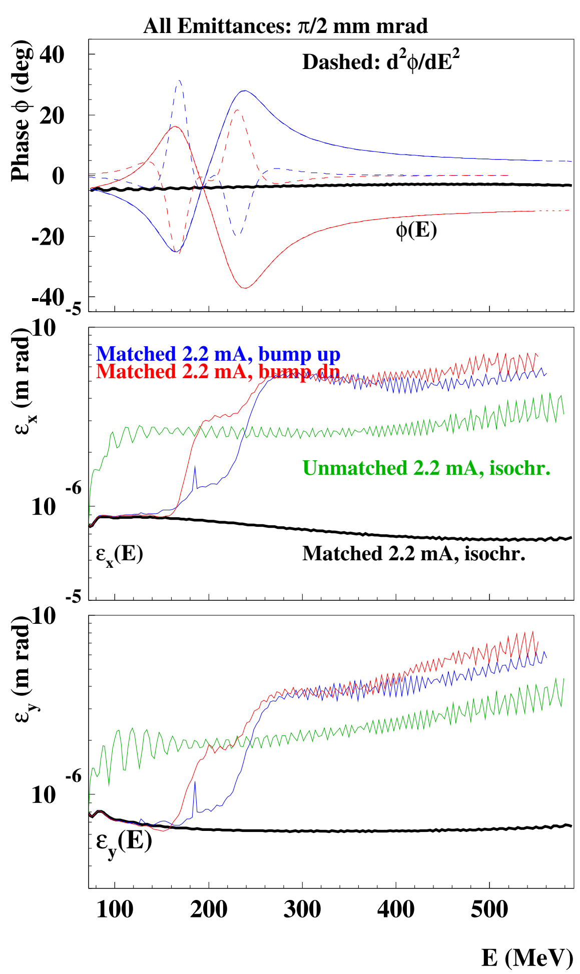

Figure 3: Phases, second derivative, horizontal and longitudinal emittance vs. energy of matched bunches in idealized ring machine with bump "U" (red) and bump "T" (blue). The emittances of a bunch with a mismatched σ-matrix are shown in green.

from the definition of the transition gamma $\gamma_t^2 \equiv \frac{r}{p}\,\frac{dp}{dr}$, the equation for the average field $B(r) = B_0\,\frac{1+\varepsilon(r)}{\sqrt{1-\frac{r^2}{a^2}}}$ and from $p = r\,q\,B(r)$:

$$\eta = -r\,\frac{d\varepsilon}{dr}\,. \tag{3}$$

The derivative $\frac{d\varepsilon}{dr}$ can be computed using $\frac{dE}{dn} = V\,\cos\phi$ and $\frac{dE}{dr} \approx E_0\,\gamma^3\,\frac{r}{a^2}$ so that:

$$\frac{1}{r}\,\frac{d\varepsilon}{dr} \approx -\frac{V\,E_0\,\gamma^3}{2\,\pi\,N_h\,a^2}\left(\frac{d^2\phi}{dE^2}\,\cos\phi - \left(\frac{d\phi}{dE}\right)^2\,\sin\phi\right)\,. \tag{4}$$

Isochronous cyclotrons are always operated "at transition" by definition, but small field deviations ε which might be positive at one radius (energy) and negative at another, can cause one or several passages of the "transition".

From Eq. 1 and Eq. 3 we conclude that the motion of a particle in a bunch is longitudinally focused (i.e. *stable*) above transition (for $\eta > 0$ or $\frac{d\varepsilon}{dr} < 0$, resp.) and unfocused (i.e. *divergent* and therefore *unstable*) for $\frac{d\varepsilon}{dr} > r\,K_x$ and $\eta < -r^2\,K_x$.

Negative Mass Instability

The term "negative mass instability" (NMI) was introduced by Nielsen, Sessler and Symon [15, 16] to describe the behavior of coasting beams in accelerators with a constant longitudinal charge distribution. Pozdeyev *et al* used this term to describe the results of their ingenious experiment *small isochronous ring* (SIR) [17]. According to theory and experiment the NMI appears *above* transition, i.e. for $\eta > 0$.

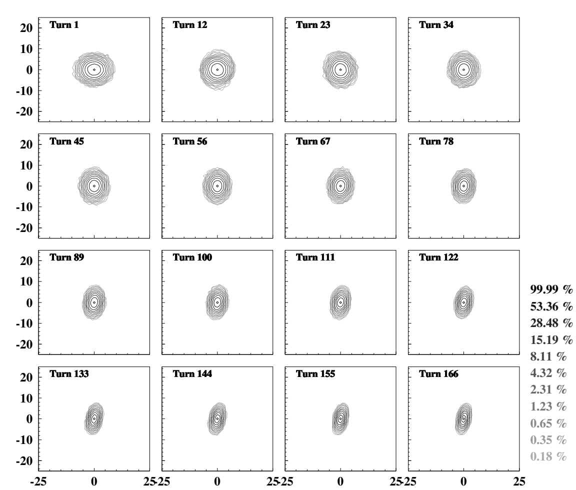

Figure 4: Topview snapshots of matched bunches in form of contour plots for the flat phase. The horizontal axis corresponds to the longitudinal, the vertical axis to the transversal beam size in mm.

This is in some contrast to the conditions of stability as expressed in by Eq. 1 and Eq. 3, especially as the linear model used to derive above equations has be shown to be symplectically similar to a Hamiltonian $\mathcal{H}$ of the form [11]

$$\mathcal{H} = \beta\, x^2 + x'^2 - \gamma^2\, K_y\, y^2 - \Gamma\, \delta^2\,, \qquad (5)$$

with (positive) parameters β and Γ. Eq. 5 quite obviously describes a system which has negative effective mass in the y-direction (and canonical momentum δ). Hence the physical situation seems to be identical and it should be possible to find a description which is compatible with both models.

RESULTS OF OPAL SIMULATIONS

The Idealized Ring Machine

In order to clarify the situation we started several OPAL simulations [13, 14] of bunched beams with an idealized isochronous separate sector cyclotron. The parameters of this idealized machine have been chosen close to the PSI ring machine [18] (see Fig. 1). In fact, we verified that a beam matched to our ideal machine, is also well-matched to the PSI ring machine. We prepared 3 field maps, one being strictly isochronous, one with additional iron forming a positive and one with removed iron forming a negative field bump (see Fig. 2). We then injected matched and mismatched [10] multivariate Gaussian particle distributions [19] with 10^5 particles and tracked them for about 170 turns from 72 MeV to about 590 MeV, i.e. from injection to extraction energy of the PSI ring machine.

From Eq. 4 one expects a loss of focusing (i.e. instability), if $\frac{d^2\phi}{dE^2}$ is sufficiently negative. This is in reasonable agreement with the results as shown in Fig. 3. Topview snapshots of matched bunches for all 3 phase curves are shown in Fig. 4, Fig. 5 and Fig. 6. The results for a magnetic field map with flat phase curve (Fig. 4) illustrate the ideal situation for a high intensity cyclotron. The bunch stays "round", developes no visible halo and the emittance increase is negligible. With $\varepsilon = 0$ and a strong enough space charge force the stability condition (Eq. 1) is fulfilled.

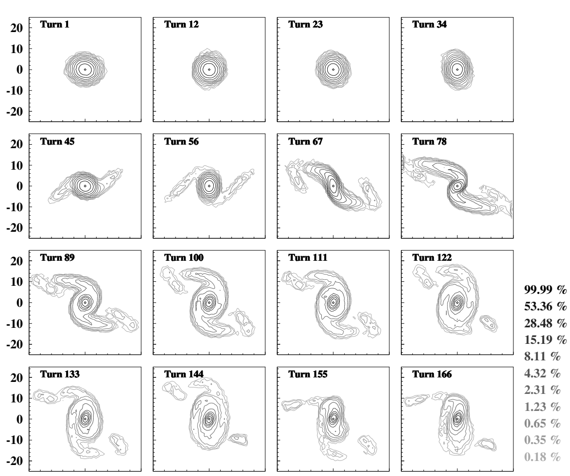

Figure 5: Topview snapshots of matched bunches in form of contour plots as in Fig. 4 for the phase curve with bump "T".

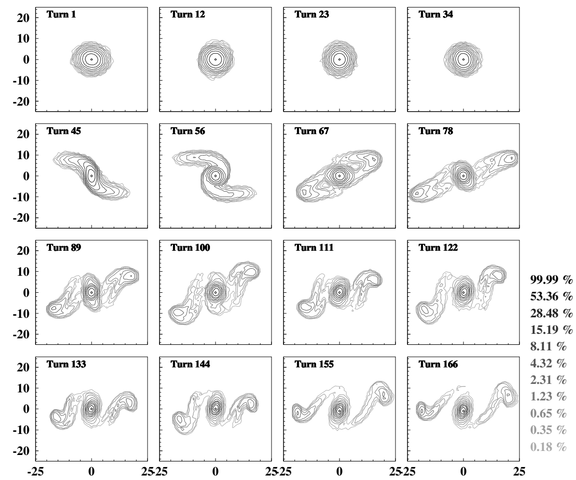

Figure 6: Topview snapshots of matched bunches in form of contour plots as in Fig. 4 for the phase curve with bump "U".

The situation is quite different, if the phase curve has sufficiently strong excursions. As shown in Fig. 3, there is an emittance increase for both signs of $\frac{d^2\phi}{dE^2}$, but it is significantly stronger for $\frac{d^2\phi}{dE^2} < 0$ and exceeds even that of the unmatched beam. The results nicely agree with the expectations of the linear model - we found good bunch stability for matched bunches with a flat phase curve. However this is the regime, where the NMI should occur and it therefore remains unclear what the NMI really means.

Mismatched or Unbalanced Bunches

The answer is related to the results of simulations using matched beams with increasingly unequal emittances. We start with a matched distribution which is only slightly unbalanced, i.e. the (eigen-) emittances are not equal, but have a ratio of $\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = \frac{2}{3} : 1 : \frac{3}{2}$. While in Fig. 7, the ratio of the eigenvalues of $\sigma\,\gamma_0$ is sufficiently small for the bunch to stay compact, it is large enough in Fig. 8 to lead to significant emittance increase and halo formation. In cyclotrons with electrostatic extraction (like the PSI ring) this is a very undesirable feature which increases activation and limits the maximal production beam current. The lesson to be learned from these results is quite clear: The linear model can be applied, if the emittances of all degrees of freedom are sufficiently close to each other. But if the beam has (for instance) a very large extent in the longitudinal direction, then it does not remain stable - even in case of a flat phase and linearily matched parameters.

There is a simple (but qualitative) argument that might explain this behavior: It is well-known that a particle distribution is in dynamic equilibrium, if the phase space density ρ is a function of the Hamiltonian only, i.e. if $\rho = \rho(\mathcal{H})$.

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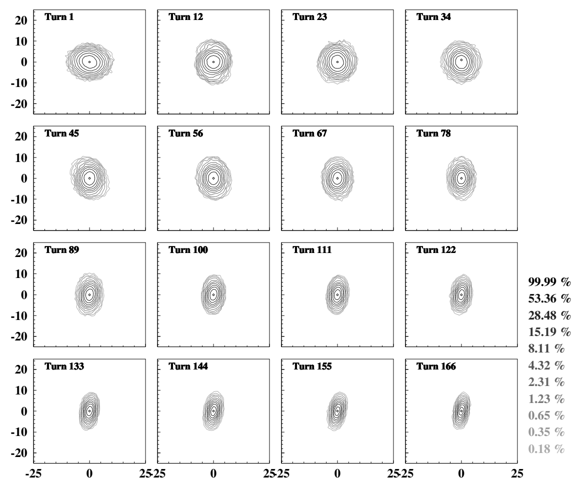

Figure 7: Topview snapshots as in Fig. 4 for matched bunches for a flat phase curve but asymmetric eigenemittances of the starting distribution. Here: $\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = \frac{2}{3} : 1 : \frac{3}{2}$. The axis show the beam size in mm. At turn number 12, a slight spiraling is visible, but it is suppressed or smeared out later on.

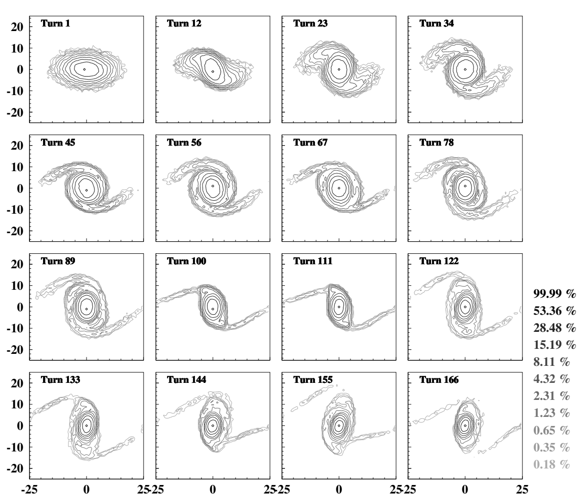

Figure 8: Topview snapshots of matched bunches as in Fig. 4 for a flat phase curve but asymmetric eigenemittances of the starting distribution. Here: $\varepsilon_1 : \varepsilon_2 : \varepsilon_3 = \frac{1}{4} : 1 : 4$. The axis show the beam size in mm. Even though the initial bunch is matched with respect to the linear model, non-linearities caused by the asymmetrie of the distribution lead to increased halo-formation.

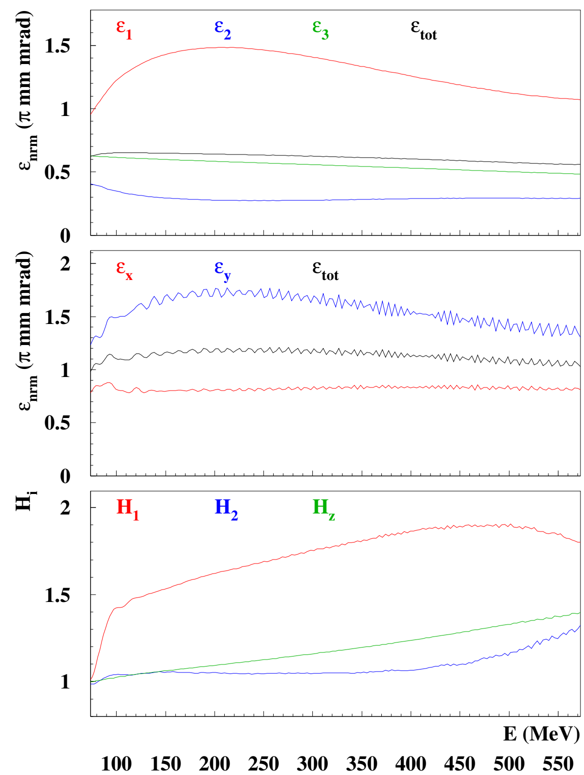

Figure 9: Eigenemittances, geometric emittances and halo parameters [20] for the distribution of Fig. 7.

This can for instance be a Boltzmann distribution

$$\rho \propto \exp\left(-\mathcal{H}/kT\right), \tag{6}$$

which is in the case of a positive definite quadratic Hamiltonian identical to a Gaussian distribution. But if we have negative inertia terms as in Eq. 5, then the integral $\int \rho\, dq\, dp$ does not exist. The ensemble can lower its energy by emittance exchange between the longitudinal and the transversal degree of freedom. In case of sufficient linearity, such (non-symplectic) processes are suppressed - especially if the phase space density in both coordinates is equal. One could say that the distribution is meta-stable.

In case of strong mismatching and/or strong emittance unbalance, the non-linearity of the space charge force destroys this meta-stability and the emittance is increased by filamentation. The filamentation found at SIR is of a special type, different from the well-known behavior of two transverse degrees of freedom in the presence of non-linearities: the beam breaks up into a number of separate longitudinal fragments [17]. This behavior can be (qualitatively) explained by the fact that longitudinal focusing has no absolute zero. Since it is self-focusing, the center of motion is the bunch center and not a specific (longitudinal) position defined by a reference orbit. The bunches shown in Ref. [17] are not only mismatched, they are also strongly unbalanced. It is this combination that leads to beam break up.

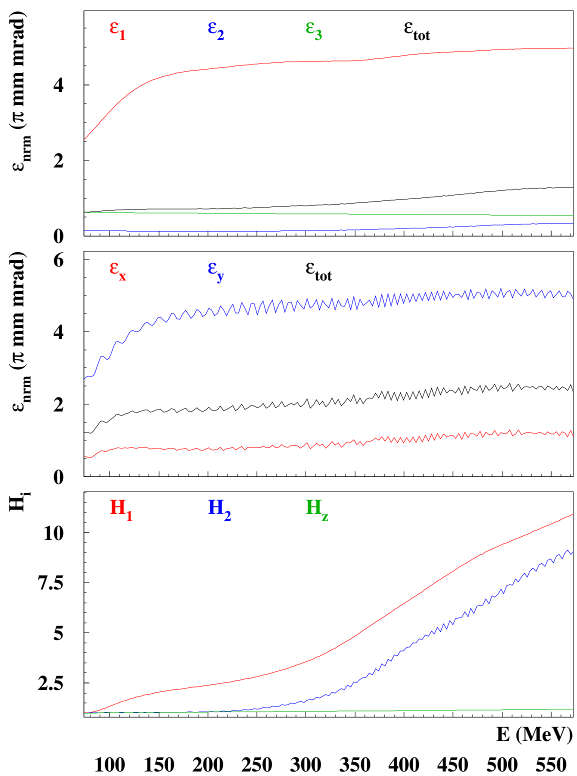

Figure 10: Eigenemittances, geometric emittances and halo parameters [20] for the distribution of Fig. 8.

The fact that the beam breakup appears only above transition, i.e. in the presence of longitudinal focusing, can then be related to the circumstance that (mis-) matching requires focusing. Only focused degrees of freedom allow to compute (and to define) matched distributions. Hence the "instability" can only appear, if longitudinal focusing is present, i.e. at or above transition.

SUMMARY

We tested the linear model of transverse-longitudinal coupling as derived in Ref. [10]. Based on this model and the theory of symplectic transformations (Ref. [11, 12]) we computed matched Gaussian particle distributions (see Ref. [19]) as input distributions for OPAL [13, 14] simulations of high intensity beams in an ideal ring cyclotron similar to the PSI ring machine [18]. We evaluated emittances (and halo-parameters) of the bunches and analyzed the dependence on the phase slip of the cyclotron and on the symmetrie of the initial distribution.

The results suggest to re-interpret the negative mass instability [15, 16, 17] as a special type of mismatching. The simulations showed that matched and balanced particle distributions can be interpreted as meta-stable states and that this meta-stability is destorted by a non-flat phase, where the dominating term is proportional to the second derivative of the phase.

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