Noise suppression in relativistic electron beams

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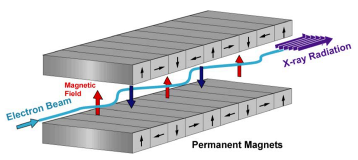
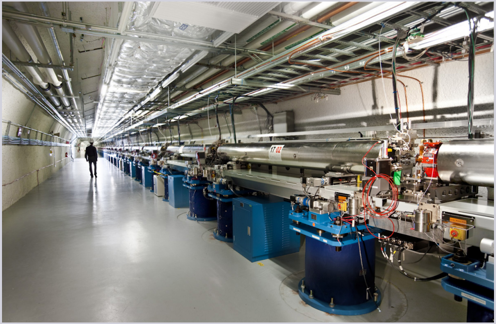
Outline of the talk

- Introduction
- Approaches to noise suppression in relativistic beams
- Noise suppression via Coulomb interaction
- Using wakefield interactions for noise suppression
- Undulator+chicane and two undulators+chicane setup
- Effect of energy noise in FEL start-up
- Conclusions

X-ray FELs

In an x-ray free electron laser a relativistic beam with a small emittance and a small energy spread is sent through a long undulator. Due to intrinsic beam instability it gets micro-bunched and generates intense radiation.

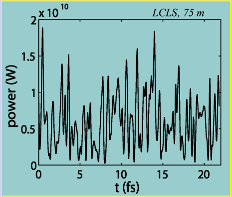




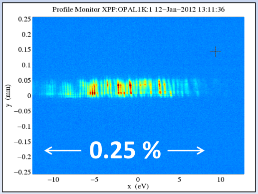
LCLS undulator at SLAC

SASE vs seeded FEL

The SASE (Self Amplified Spontaneous Emission) radiation starts from initial shot noise in the beam, with the resulting radiation having an excellent transverse coherence but a rather poor temporal one.



Simulated power vs time for LCLS



Measured SASE spectrum at LCLS

SASE vs seeded FEL

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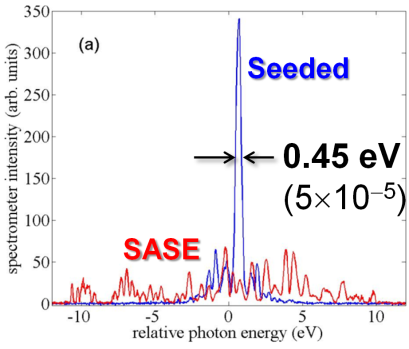


Amman et al., Nature Photon., (2012)

In a seeded FEL an initial narrow-band seed is introduced in the beam, which is then amplified through the undulator. This improves the longitudinal coherence of x-rays.

Introduction

Situations when initial noise in the beam may adversely affect FEL operation:

- In seeded FELs shot noise competes with external modulations of the beam being amplified in the process of the seeding. Suppressing the noise relaxes requirements for the seed power.
- Microbunching instability is believed to start from the shot noise. If not controlled, it leads to the degradation of the FEL performance and blinds OTR diagnostics.
- Suppressing shot noise could also allow controlling instabilities and increasing efficiency in coherent electron cooling of relativistic beams (Litvinenko, 2009)

To what level the noise can be suppressed, and what are the challenges?

Brief history

Suppression of long wavelength shot noise was observed in microwave tubes as early as the 1950s.

In the last few years, several groups have independently proposed suppressing shot noise at short wavelengths in relativistic electron beams (Gover & Dyunin–2009, Nause et al.–2010, Litvinenko–2009, Ratner et al.–2011).

First experimental observation of shot noise suppression at sub-micron wavelengths were recently reported by Ratner & Stupakov–2012 and Gover et al.–2012.

Shot noise and plasma oscillations

There are several approaches to suppress shot noise in the beam.
Gover & Dyunin–2009 studied density fluctuations with plasma frequency. If a cold beam is prepared in such initial state that there is only density fluctuations, but there is no velocity fluctuation, after a quarter of the plasma period it will be fully converted into velocity fluctuation.

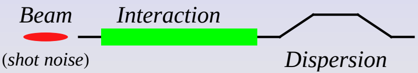
$$\frac{\omega_p}{c} = \left(\frac{4\pi}{\gamma^3 S}\frac{I}{I_A}\right)^{1/2}$$

where I is the beam current, $I_A = mc^3/e = 17.5$ kA is the Alfven current, and S is the transverse cross section area of the beam.

An example: beam energy 1 GeV, $I = 1$ kA, $S = 100\ \mu m \times 100\ \mu m$, $\Rightarrow$ $\pi c/2\omega_p \approx 16$ m. For lower currents and higher beam energies using this method requires more space which makes it less attractive.

Approaches to suppress the shot noise

In another approach a relatively short interaction region is used followed by a dispersive element:

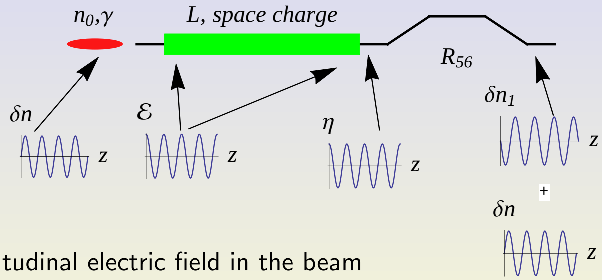


It is assumed that particles are frozen at their longitudinal positions through the interaction region. They are shifted longitudinally as the result of passage through the dispersive element R_{56}. This approach has a promise of more compact setup.

Litvinenko–2009 considered two wigglers and a chicane. Radiation from the first wiggler is amplified in an optical amplifier and then recombines with the beam in the second chicane. The interaction with this radiation would lead to the noise suppression in the beam.

Coulomb interaction in drift space

1D Coulomb interaction (space charge), $\sqrt{S} \gg \gamma/k$, where $k = 2\pi/\lambda$ and λ is the wavelength of interest, S—transverse beam area. 1D beam density $n = n_0 + \delta n(z)$. Assume $\delta n(z) = \delta n_0 \sin kz$, $\lambda \ll \sigma_z$.



$\mathcal{E}$ is the longitudinal electric field in the beam

$\eta = \Delta E/E_0$

Density perturbation δn_1 should be added to the initial one δn, and can compensate it.

Coulomb interaction—calculations

$\mathcal{E}$ is the longitudinal electric field in the beam, $n = n_0 + \delta n(z)$ is the number of particle per unit length

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{4\pi e}{S}\delta n_0 \sin kz \quad \to \quad \mathcal{E}(z) = -\frac{4\pi e}{Sk}\delta n_0 \cos kz.$$

Over the length of the drift L this field changes the energy of a particle located at z by

$$\eta(z) = \frac{e\mathcal{E}(z)L}{E_0} = -\frac{4\pi e^2 L}{SkE_0}\delta n_0 \cos kz$$

The chicane shifts each particle by

$$\Delta z = R_{56}\eta(z)$$

and create an additional density perturbation (cold beam model)

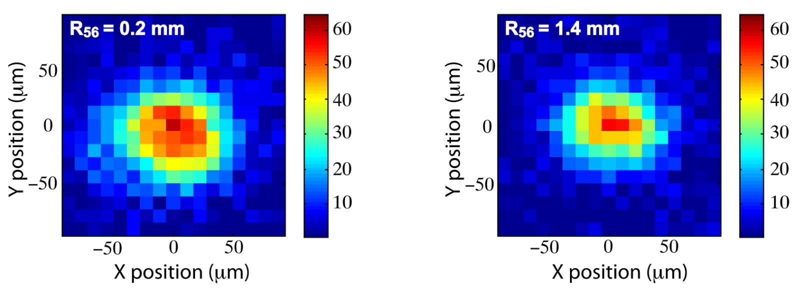
$$\delta n_1 = -n_0 \frac{\partial \Delta z}{\partial z} = -n_0 R_{56} \frac{4\pi e^2 L}{SE_0}\delta n_0 \sin kz$$

Coulomb interaction in drift space

The initial density is compensated if [Ratner et al.–2011]

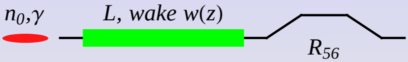
$$\frac{4\pi e^2 L}{S E_0} n_0 R_{56} = 1$$

Numerical example: beam energy 1 GeV, $I = 1$ kA, $S = 100$ μm $\times 100$ μm, $L \approx 5$ m $\rightarrow$ $R_{56} = 5.2$ μm. Note suppression at all frequencies. This was experimentally demonstrated in Ratner&Stupakov–2012. Beam image without noise suppression (left) and with (right).



More general (than space charge) interaction

Assume interaction in the beam via a wake function $w(z)$



Particles change energy due to the wake

$$\Delta E(z) = e^2 \int_{-\infty}^{\infty} w(z - z')\delta n(z')$$

Associated with the wake is the longitudinal impedance $Z(k)$ defined through the Fourier transform of the wake,

$$Z(k) = -\frac{1}{c}\hat{w}_k$$

The method: assume initial fluctuations of the beam distribution function $\delta f_i(z, \eta)$ due to shot noise $\rightarrow$ compute $\delta n(z)$ and the electric field $\mathcal{E}$ $\rightarrow$ compute $\eta(z)$ $\rightarrow$ compute $\delta f_f(z, \eta)$ after the chicane and find $|\delta n_k|^2$. We take into account the energy spread of the beam σ_η.

More general (than space charge) interaction

Ratio of the final to initial density fluctuations

$$F(k) \equiv \frac{|\delta n_k|^2}{2\pi n_0} = 1 - 2T(k)\mathrm{Im}\,Q(k) + |Q(k)|^2 T(k)$$

where

$$Q(k) = R_{56} n_0 \frac{r_e c}{\gamma} k Z(k), \qquad T(k) = e^{-(k R_{56} \sigma_\eta)^2}$$

Depending on the sign of $\mathrm{Im}\,Q$ and values of Q and T, the noise can be either reduced or increased. Coulomb interaction is obtained in the limit $T = 1$ (cold beam) and $Z_{sc} = 4\pi i L/Skc$.

The full noise suppression ($F = 0$) requires purely imaginary Q with $\mathrm{Im}\,Q = 1$, and $T = 1$ (cold beam). Assuming a small energy spread, such that $k^2 R_{56}^2 \sigma_\eta^2 \ll 1$, and $\mathrm{Re}\,Z = 0$

$$\min F \approx k^2 R_{56}^2 \sigma_\eta^2 \quad \text{when} \quad \mathrm{Im}\,Q = 1$$

For a beam with finite energy spread only a *partial* noise suppression can be achieved.

Suppression shot noise through interaction and dispersion

The previous numerical example (for space charge): beam energy 1 GeV, $I = 1$ kA, $S = 100\ \mu$m $\times 100\ \mu$m, $\sigma_\eta = 10^{-4}$, $L \approx 5$ m. We obtain $F \approx 0.11$ at the wavelength of 10 nm. Note that increasing the interaction length in this example would eventually violate the requirement $L \ll \pi c/2\omega_p$.

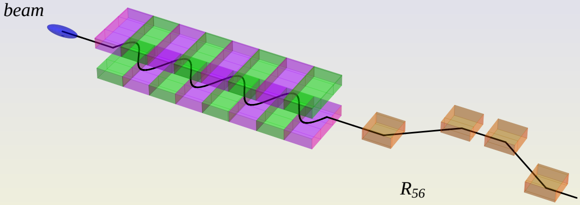
To suppress more we need smaller R_{56}, hence larger Z, with Z purely imaginary

$$\min F = \left(\frac{I_A}{I}\frac{\sigma_\eta \gamma}{\operatorname{Im} Z(k) c}\right)^2$$

Because $R_{56} \propto 1/I$, the optimal suppression can be achieved in a local region.

1D undulator interaction

What structures can have an imaginary impedance (at short wavelengths) larger than the space charge?



Electrons passing through an undulator interact with each other through emitted electromagnetic field (similar to CSR wake). 1D model is valid if $S \gg L_u/k$.

1D undulator interaction

Consider a helical undulator with N_u periods and the undulator parameter $K = eB/mc^2k_u$. 1D wake oscillates with the undulator radiation wavelength, $\lambda_0 = \lambda_u(1+K^2)/2\gamma^2$:

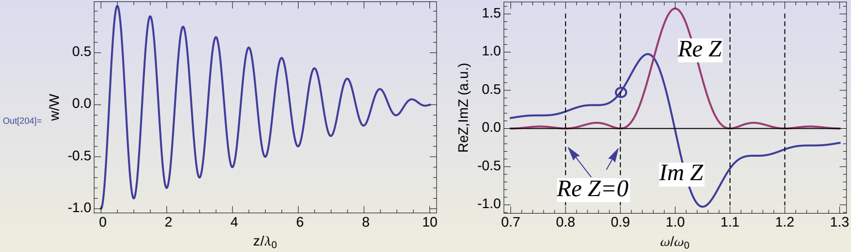
$$w_u(z) = \begin{cases} -W[1 - z/(N_u\lambda_0)] \cos k_0 z, & 0 < z < N_u\lambda_0, \\ 0, & \text{otherwise}, \end{cases}$$

where $\lambda_0 = 2\pi/k_0$ is the wavelength of the undulator radiation,

$$W = 8\pi \frac{N_u \lambda_0 \gamma^2}{S} \frac{K^2}{(1+K^2)^2} \,.$$

1D undulator interaction

Plot of the wake and impedance for $N_u = 10$.



The maximal imaginary part of Z_u is at $\omega/\omega_0 = 1 \pm N_u^{-1}$

$$\operatorname{Im} Z_u \approx \pm \frac{W N_u}{2ck_0}.$$

1D undulator interaction

In the limit $K \gtrsim 1$, the undulator impedance is $N_u/2$ times larger than Z_{sc}. But the real part of Z_u vanishes only at particular values of ω.

Assuming for simplicity $K = 1$ we obtain for the noise factor

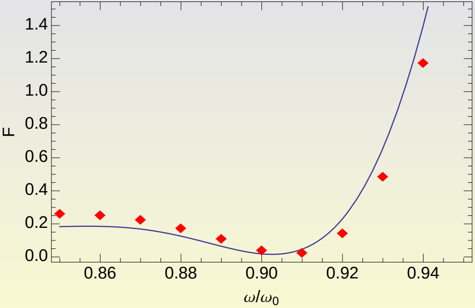
$$\min F = \left(\frac{I_A}{I} \frac{2S\sigma_\eta}{N_u^2 \lambda_0^2 \gamma} \right)^2$$

As a numerical example we consider the case: beam energy 1 GeV, $I = 1$ kA, $S = 100$ µm $\times 100$ µm, $\sigma_\eta = 10^{-4}$, $\lambda = 10$ nm, $N_u = 30$. These parameters give $\min F = 0.04$.

Increasing N_u would narrow the suppression band and lead to the FEL effects in the undulator.

Simulation

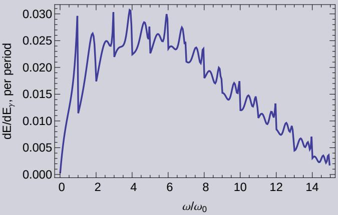
10^5 particles interact through the 1D undulator wakefield, $\Delta\eta_i = (e^2/\gamma mc^2)\sum_j w(z_i - z_j)$, and then shifted $\Delta z_i = R_{56}\eta_i$. The final bunching factor b_f is calculated as a function of frequency ω, $F = |b_f(\omega)|^2/b_0^2$.



Line-charge beam in undulator

Decreasing S? This leads to 3D undulator wake. Line-charge beam in the undulator: $S \ll L_u/k$.

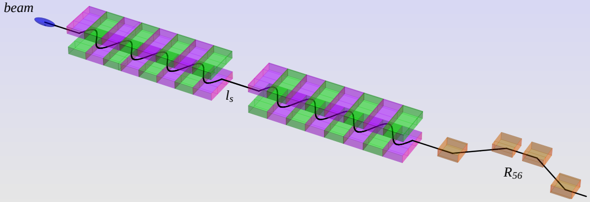
The impedance is found from $\mathrm{Re}\,Z = (\pi/e^2) d\mathcal{W}/d\omega$, where $d\mathcal{W}/d\omega$ is the energy radiated by the electron in unit frequency interval. The imaginary part of the impedance can be found with the help of the Kramers-Kronig relation.



Undulator spectrum for $K = 4$

The problem, however, is that for a strong undulator $d\mathcal{W}/d\omega$ and hence $\mathrm{Re}\,Z$ do not vanish.

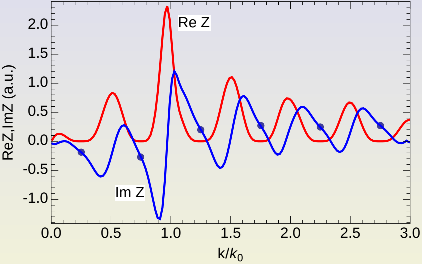
Two undulators



Two undulators are separated by distance l_s. Interference between the two undulator radiation introduces a phase shift factor $\phi = kl_s/2\gamma^2$. It adds a factor $[1 + \cos(\phi)]^2$ to the intensity of the total radiation, which vanishes for $kl_s/2\gamma^2 = \pi(n + 1/2)$, where n is an integer, $n = 0, 1, 2, \ldots$.

Two undulators

Calculated functions $\mathrm{Re}\,Z$ and $\mathrm{Im}\,Z$ for the case of two undulators with $N_u = 10$ periods,



$K = 1$ and $\phi = 4\pi$.

Two undulators

For the noise suppression we have

$$\min F = \left(\frac{I_A}{I} \frac{\sigma_\eta \gamma}{c \operatorname{Im} Z} \right)^2$$

Formally this result does not depend on frequency (as long as $\operatorname{Im} Z$ is provided at the frequency of interest by a suitable undulators), but the applicability condition of line-charge model $S \ll L_u/k$ is very strict for small wavelength.

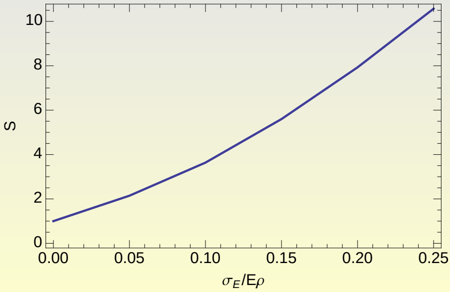
Numerical example: beam energy 1 GeV, $I = 1$ kA, $\sigma_\eta = 10^{-4} \Rightarrow \min F_n = 4 \times 10^{-3}$. This result requires a small beam radius, and hence small beam emittance. For $\lambda = 30$ nm the beam radius should be smaller than 12 μm.

Effect of energy noise on FEL startup

Taking into account the energy spread in the beam shows that there is a contribution from energy fluctuations as well. When the density fluctuations are maximally suppressed the thermal noise adds considerably to the FEL startup intensity

$$\min F = k^2 R_{56}^2 \sigma_\eta^2 \cdot S\left(kR_{56}\sigma_\eta, \frac{\sigma_\eta}{\rho}\right)$$

Plot of function S for $kR_{56}\sigma_\eta = 0.1$.



Conclusion

- Control of shot noise at short wavelengths in relativistic beams would allow for important improvements in radiation properties of FELs. It is currently an active part of research in the FEL community.
- There are several approaches to the problem; in this talk we focused on interaction+dispersion section scheme. The interaction can be provided by space charge, but interaction in an undulator is much stronger, although localized in narrow-band regions.
- A more efficient two-undulator setup was presented.
- It seems that practically achievable levels of suppression for realistic beam parameters do not go below ~ 0.1.