6D COOLING IN PERIODIC LATTICES INCLUDING A PLANAR SNAKE*

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Abstract

A Muon Collider requires 6 dimensional (x, x’,y, y’, t, E) cooling down to a transverse emittance of 240 µm and a longitudinal emittance of the order of 2 mm [1]. Ionization cooling in absorbers, together with emittance exchange is used. Previously known lattices, using vacuum rf, are discussed. All use solenoid focusing combined with weak dipoles to generate dispersion. Three of the examples use bi-periodic focusing, known as SFOFO or RFOFO, in different geometries: a) rings [2]; slow helices (known as Guggenheims) [3]; and linear [4]. These lattices all require wedge shaped absorbers and work for only one muon sign at a time. The Helical FOFO Snake lattice [5], based on simple alternating solenoids, together with a weak rotating dipole, uses flat slab absorbers and works for both signs simultaneously. But this solution would require unreasonably high magnetic fields for cooling to the required transverse emittance.

A new lattice, the Planar RFOFO Snake [6] is then discussed. Like the Helical FOFO Snake, it uses slab absorbers and cools both signs simultaneously. But this Planar RFOFO lattice is based on bi-periodic solenoid focusing and allows cooling to lower emittances without excessive fields.

INTRODUCTION

This work is part of the Muon Accelerator Program (MAP) [7] that is studying Muon Colliders over a range of energies (approximately 125 GeV for a ’Higgs Factory’ as well as energies of several TeV. All of these require 6 dimensional (x, x’,y, y’, E, t) cooling down to a transverse emittance of approximately 240 µm, and a longitudinal emittance of the order of 2 mm (lower longitudinal emittances give undesirable space charge effects). Transverse ionization cooling is achieved by passing the muons though a low Z absorber, typically liquid hydrogen, in which both transverse and longitudinal momentum are lost. An rf cavity (only vacuum rf is considered in this paper) restores the lost longitudinal momentum, leaving the reductions of the transverse components. Multiple scattering leads to a minimum (equilibrium) emittance that is proportional to the betatron $\beta_\perp$ divided by the muon momentum. This $\beta_\perp$, and thus minimum emittance, can always be reduced by scaling all dimensions down, with the magnetic field scaled up ($B \propto 1/L$). It can also be decreased if the lattice focuses the muons to a lower $\beta_\perp$ at the absorber: something that cannot be done in a non-periodic lattice like the Helical Cooling Channel [8].

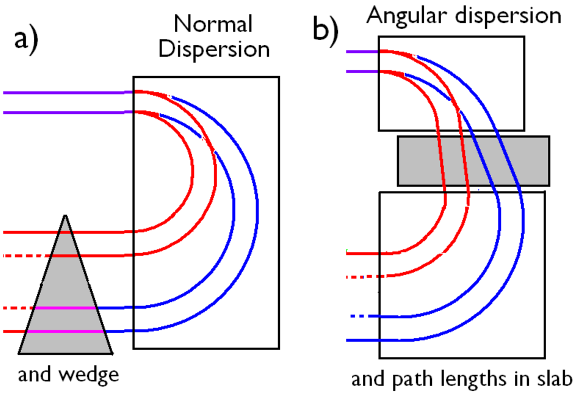

Figure 1: Emittance Exchange: a) using dispersion and wedge absorbers; b) using angular dispersion and slab absorbers.

For cooling in the longitudinal (Energy and time) dimensions, emittance exchange is required. The two methods applicable here are illustrated in Figure 1. In method a), a wedge absorber is placed in a region with dispersion so that the higher momenta see more energy loss than the lower. The same result can be achieved in a plane slab absorber, given angular dispersion, as shown in b). Emittance exchange can also be achieved in a non-periodic lattice like the Helical Cooling Channel [8] in which the absorbing material is everywhere and the higher momenta have longer paths.

Periodic lattices allow the possibility of lower transverse betatron lengths β at the absorbers, but have inevitable integer and half integer resonances that define the regions of momentum where they operate. The lattices types can be grouped by the region of phase advance used. Figure 2 illustrates the options.

Simple lattices consisting of equi-spaced alternating solenoids (known as FOFO lattices) operate above the single π resonance (see Figure 2a). Bi-periodic lattices [9], with pairs of coils of opposite polarities, are spaced by longer drifts, known as SFOFO (super FOFO) or RFOFO, (see Figure 2b). These lattice operate between the 2π and π resonances and have an attractively level β over their acceptance. There are also systems that look like FOFO or RFOFO lattices, but have small dipole fields that are different of some finite number of the basic FOFO or RFOFO cells. The effective phase advances now double, together with the number of resonances. Both cases considered here operate between the 3π and 4π stop bands (see Figure 2c).

Cooling channels are formed by sequences of lattice cells, arranged in differing geometries (see Figure 3). Examples of these differing channels will now be discussed in more detail.

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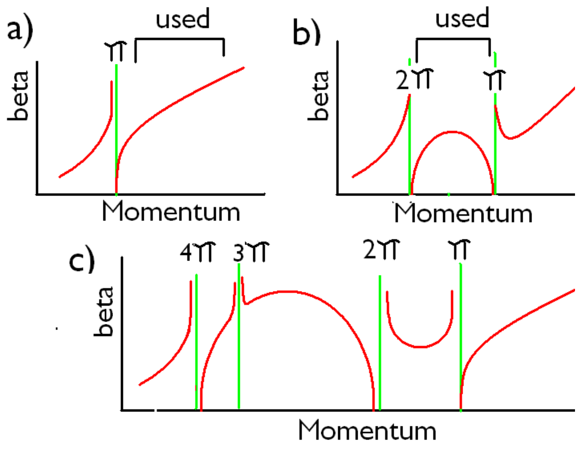

Figure 2: Lattice types: a) FOFO (Focus-focus) simply periodic alternating solenoids; b) RFOFO/SFOFO (super-FOFO) bi-periodic alternating solenoids; c) Higher tune lattices (Helical Snake & Planar Snake)

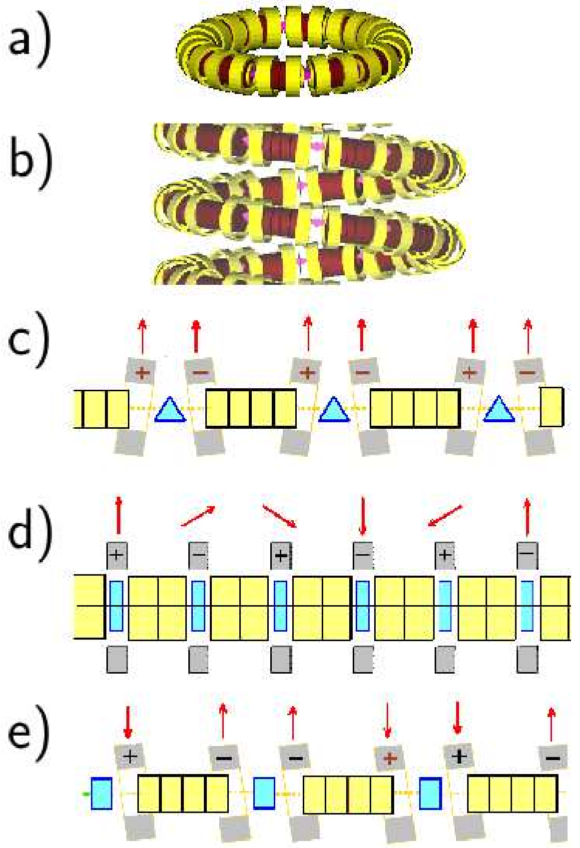

Figure 3: Geometries: a) RFOFO Ring; b) RFOFO Guggenheim; c) Rectilinear RFOFO; d) Helical FOFO Snake; e) Planar Snake.

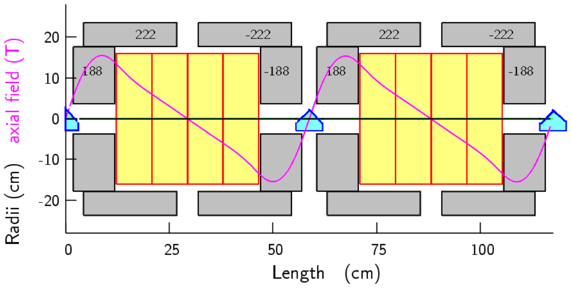

Figure 4: Cross section of a late stage RFOFO lattice showing the current densities in A/mm^2.

BI-PERIODIC RFOFO LATTICES

Three different geometries, using the same basic RFOFO concept, have been proposed. In each case, the solenoids are slightly tilted to generate upward dipole fields. In the first, shown in Figure 3a, the lattice is bent into a circle, with the curvature corresponding to that generated by the dipole components. Simulations have shown that suitable sequence of such rings, with multiple stages using different cell lengths, fields, and rf, can cool from the emittances available from the front end, down to the final requirements. However, injection into or extraction from such rings would be extremely difficult.

In the second case, represented in Figure 3b, the RFOFO cells are set on a gently upward or downward helix (as in the NY Guggenheim Museum and commonly referred to by that name). Simulations [3] have shown that their performance is almost exactly the same as that of rings of the same approximate bending radii. This case would appear to be practical for the early stages of 6D cooling, but would be increasingly difficult as the radii get smaller in the later stages. An added complication is that stray fields from one pitch can influence those before and after, and must be shielded or allowed for.

In the third case [4], as represented in Figure 3c, essentially the same cells from a ring or Guggenheim, including their coil tilts and resulting upward dipole fields, are laid out in a straight (rectilinear) geometry. The solenoid focusing is so strong, compared with the dipole deflections, that the closed orbits are merely displaced latterly, but continue down the now straight lattice. And the performance [4] is essentially the same as with rings or Guggenheim, but with greatly simplifying engineering.

Sequences of RFOFO Guggenheim lattices [10] (actually simulated as rings, but assumed to be representative of all cases) have shown to cool from those at the front end close to those, at the end. In all cases the absorbers are assumed to be wedges of liquid hydrogen with thin aluminum windows. The early stages employ 201 MHz rf and NbTi super-conducting solenoids. In the later stages, the solenoid fields and rf frequencies rise. At the end, the super-conductors are HTS YBCO and the frequencies are 805 MHz.

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Despite the success of such simulations, there remain significant challenges. In the last stage of either simulation, the cell cross sections are similar to that [10] shown in Figure 4. It is seen that the current densities are high (222 and 188 A/mm^2), and that the coils on either side of the absorber have opposed currents. The forces between these coils will be large and outward, and there is no space allowed for a structure to constrain them. The design of a wedge shaped liquid hydrogen absorber will also not be easy. In addition there is the general objection that such lattices can only cool one of the required muon signs. The entire sequence of such stages must be duplicated for the other sign.

HELICAL FOFO SNAKE

The Helical FOFO Snake [5], represented in Figure 2d, is based on a sequence of evenly spaced alternating solenoids, as in a FOFO. But the coil tilts, instead if all being in the same direction, rotate in azimuthal angle, giving closed orbits that perform approximation to a helix. Because of these changing dipole fields, the operating phase is higher than in the basic FOFO, lying between 3π and $2\pi_.$ The lattice has two very attractive characteristics: 1) it uses absorbers with plane parallel windows that would be much easier to build than the wedge shaped absorbers used in the RFOFO lattices; and 2) it works for both signs simultaneously. The lattice employs a different mechanism for its emittance exchange: that represented in Figure 1b. The required large angular dispersion is obtained by the resonant excitation of the helical motion induced by the tilted solenoids. Its performance in early stages has been shown to be comparable to that in an RFOFO lattice.

Unfortunately, it has one significant disadvantage. The absorbers must be located where the $\beta_\perp$ is at their maxima (instead of being at beta minima in the RFOFOs). As a result the product of required magnetic field times βs is higher than in any of the RFOFOs. The lattice is thus not suitable for late stages of 6D cooling.

PLANAR RFOFO SNAKE

The initial objective, never achieved, was a lattice, assumed to use wedges and conventional dispersion, that looked much like Figure 4 but in which the coils on either side of the absorber would have the same polarity. Such a lattice has field reversals only in the center of the rf, half as many reversals as the FOFO (it was described as a 'half flip' lattices). Instead, and surprisingly, it proved possible to generate strong angular dispersion at the absorbers, by merely tilting all coils in the same direction. The resulting dipole periodicity is almost exactly like the solenoids, but reduced in amplitude by the small tilt angles. The angular dispersion is highly non-linear: low over most of the momentum range, but very high at or near the momentum of the 2π resonance. Figure 6 shows the separate dispersions in x, and y as a function of length. High dispersion (≈ 30 cm) is achieved, but not at the absorbers. In contrast, however, the x dispersion at the absorbers is rapidly changing, indication strong angular dispersion.

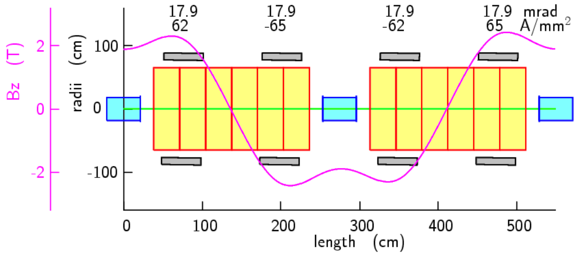

Figure 5: Section of an early stage Planar RFOFO Snake.

Unfortunately, the addition of the alternating dipole fields, though small, excite the new 3π resonance that falls between the 4π and 2π resonances that, without the dipoles would have defined an operating momentum aperture between the then 2π and π resonances in the basic RFOFO lattice. Transmission through this new resonance is poor, even with sub-degree tilts, and so the operating momentum band is reduced as shown in Figure 2c.

This angular dispersion caused emittance exchange, and thus 6D cooling, in absorber slabs as illustrated in Figure 3e. And since the beam centers are displaced from the axis by a small fraction of the apertures (e.g. 4 cm out of 18 cm for the early stage lattice), these apertures can be set on the axis, and there is nothing that favors one sign or the other. Cooling can take place for both signs simultaneously. As in a true RFOFO, the betas at the absorbers (see Figure 7) are much lower (35 cm) than in the transport between them (≈ 120 cm).

Early Stage 6D Cooling Lattice

The concept was first simulated in a lattice suitable for early 6D cooling, with parameters given in Table 1 1. Figure 5 shows a cross section of one complete cell. All solenoid coils are tilted in the same direction by 17.9 mrad. The rf is at 201 MHz operating at 17 MV/m axial gradient, and a phase of approximately 30 degrees.

Figure 8 shows the emittances and transmission for this lattice. It shows good cooling in both transverse and longitudinal emittance, with the rate of muon loss, after initial losses, being dominated by the inevitable decays. The results plotted in Figure 8 were obtained using ICOOL [11] by running in a mode in which fields are defined along an axis, straight in this case, and the program derives the fields at other locations by a 5th order extrapolation. Note that the lattice has since been simulated [6] with ICOOL using a realistic field map, and also with G4BL [12] using the same map. The results are very similar but slightly better in these simulations.

Late Stage 6D Cooling Lattice

The concept has also been tested in a lattice designed for late stage 6D cooling, with the transverse emittance cooled to the required 240 µm, and a longitudinal emittance close

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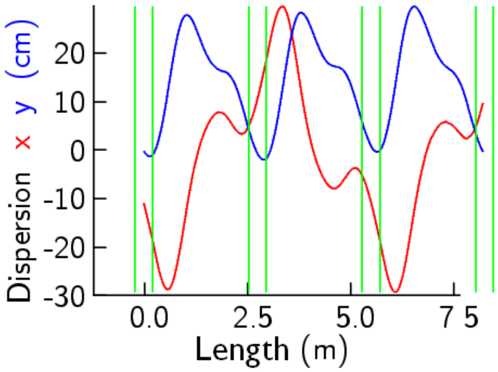

Figure 6: Dispersion of the cooled beam as a function of length for the early stage lattice. The green lines define the location of the absorbers.

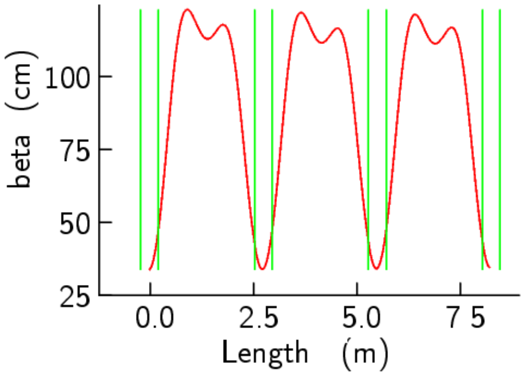

Figure 7: Beam betas as a function of length. The green lines define the location of the absorbers.

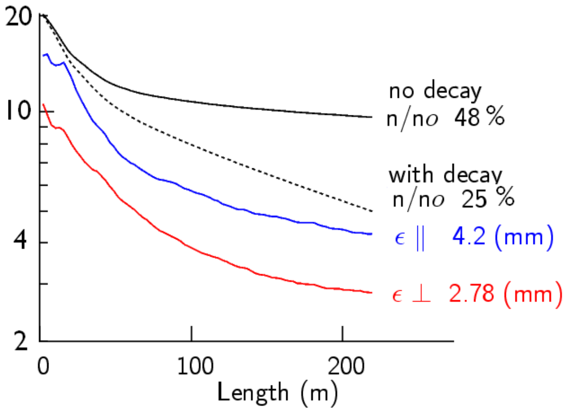

Figure 8: Emittances and transmission of early stage planar snake.

Table 1: Parameters of Early 6D Cooling Lattice

start m	dl m	rad m	dr m	tilt rad	I/A A/mm^2
0.500	0.500	0.770	0.110	0.017	62.22
1.750	0.500	0.770	0.110	0.017	-65.45
3.250	0.500	0.770	0.110	0.017	-62.22
4.500	0.500	0.770	0.110	0.017	65.45

	material	length cm	radius cm
Half absorber	Liquid H_2	21.3	18
Absorber window	Aluminum	0.05	18
Gap	Vacuum	17.15	50
6 rf cavities	Vacuum	33	64
Gap	Vacuum	17.15	50
Absorber window	Aluminum	0.05	18
Half absorber	Liquid H_2	21.3	18

to 2 mm. This longitudinal emittance is only a factor of 2 less than and is easily achieved by raising the rf frequency to 805 MHz, which in turn should increase the operating gradient from 17 to 35 MV/m. This change reduces the relative fraction of length needed for the rf and give more space for solenoid coils.

In order to reach this transverse emittance from ($\approx$ 2.8 mm to 0.21 mm), the local β at the absorbers has to be reduced (from 30 cm to 2.1 cm). This reduction, without changing the essential beam dynamics, is achieved by scaling the cell lengths (from 275 to 38.5 cm), and raising the peak magnetic fields ($\propto 1/L$) from 2.1 T to 24 T.

If the coil cross sections were scaled by the same factor, the current densities ($\propto 1/L^2$) would be far too high, so the coil cross sections are made relatively much larger, and the space gained by reducing the length of rf, is also used for another coil.

Figure 9 shows a cross section of one complete cell. The parameters given in Table 2. All solenoid coils are tilted in the same direction by 12 mrad. The rf is at 805 MHz operating at 35 MV/m axial gradient, and a phase of approximately 15 degrees. Note that the coils on either side of each absorber have the same sign. Thus, the forces between them are inward, where an essentially solid plate can easily with stand them. Detailed parameters can be found in Table 2.

Figure 10 shows the ICOOL simulation of this late stage lattice. Cooling to the required transverse emittance of 0.24 mm is achieved with the longitudinal emittance being maintained at 2 mm as also required. The transmission is poor, presumably because of the limitted momentum acceptance. It is hoped that further optimization will help.

CONCLUSION

Simulations of the Guggenheim concept for 6 dimensional ionization cooling has given the required parameters, but there remain engineering challenges and two complete

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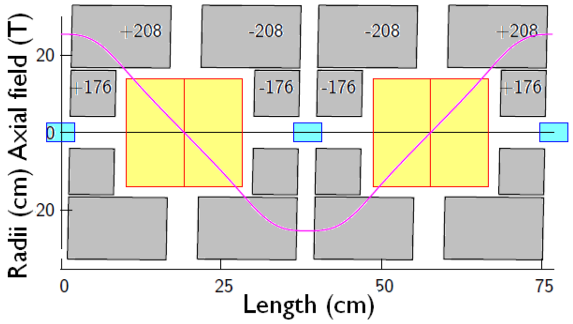

Figure 9: Section of a late stage Planar RFOFO Snake.

Table 2: Parameters of Late 6D Cooling Lattice

start	dl	rad	dr	tilt	I/A
m	m	m	m	mrad	A/mm^2
0.014	0.070	0.042	0.119	12.0	176.47
0.014	0.154	0.168	0.161	12.0	208.11
0.217	0.154	0.168	0.161	12.0	-208.11
0.301	0.070	0.042	0.119	12.0	-176.47
0.399	0.070	0.042	0.119	12.0	-176.47
0.399	0.154	0.168	0.161	12.0	-208.11
0.602	0.154	0.168	0.161	12.0	208.11
0.686	0.070	0.042	0.119	12.0	176.47

	material	length	radius
		cm	cm
Half absorber	Liquid H_2	2.2	2.5
Absorber window	Aluminum	0.01	2.5
Gap	Vacuum	8.04	5
rf cavity	Vacuum	9.0	14
rf cavity	Vacuum	9.0	14
Gap	Vacuum	8.04	5
Absorber window	Aluminum	0.01	2.5
Half absorber	Liquid H_2	2.2	2.5

systems are needed for the two particle signs. The Helical FOFO Snake can cool both signs but not to the required final emittance.

The Planar RFOFO Snake Lattice lattice was conceived to reduce current densities and forces in the late stages of the cooling, but was tested first in an early stage. Large dispersions (20–35 cm) were obtained with small tilts (0.5–1 deg.) arising from the 2π resonance at the high momentum end of the accepted range, but these dispersions were small at the absorber. The angular dispersion at the absorbers however, though non linear, was large enough to give emittance exchange and 6D cooling of both signs with flat absorbers.

This method of 6D cooling is similar to that in a Helical FOFO Snake [5], but is here generated by an alternating dipole fields in a single direction, resulting conceptually in planar snaking orbits. The dipoles in the Helical Snake, in contrast, rotate in their azimuthal directions. The planar RFOFO snake also allows a beta at the absorbers much higher than elsewhere.

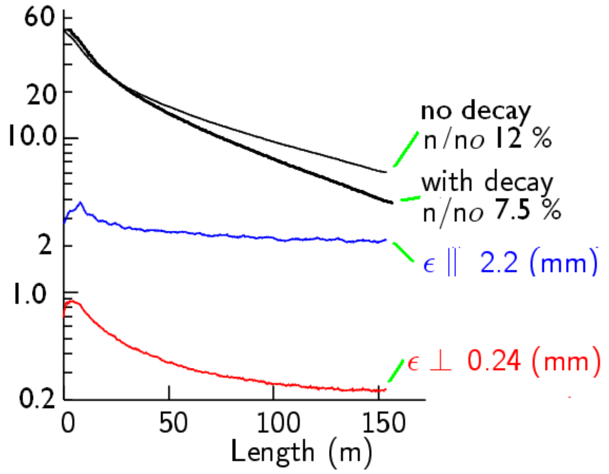

Figure 10: Emittances and transmission of late stage planar snake.

This result has been confirmed using field maps in both ICOOL and G4BeamLine. Example lattice designs have been constructed for both an early stage of cooling, and one approximating a last stage that reached the specified emittances with current densities consistent with YBCO HTS conductors. The forces between coils being inward, that should be manageable

REFERENCES

[1] R.B. Palmer; Muon Collider, Proc. AHIPA09.

[2] R.C. Fernow, et al., Muon Cooling in the RFOFO Ring Cooler, Proc. of PAC 2003, Portland, OR, p. 2002 (2003).

[3] R.B. Palmer, et al., Phys. Rev. ST Acc. Beams 8 (2005) 061003; P. Snopok and G. Hanson, IJMMA 24, p. 987 (2009).

[4] V. Balbekov, Investigation and Simulation of Muon Cooling Rings with Tilted Solenoids, Proc. of PAC 2003, Portland, OR, p. 2017 (2003).

[5] Y. Alexahin, AIP Conf.Proc. 1222 (2010) 313-318.

[6] R.B. Palmer, 6D Cooling in Periodic Lattices, June 6, 2013, `http://www.cap.bnl.gov/AAG/GroupMeetings/` J.S. Berg, et al., A Planar Snake Muon Ionization Cooling Lattice, Submitted to NA-PAC13, Pasadena, Sept. 2013.

[7] `http://map.fnal.gov`

[8] Y. Derbenev and R.P. Johnson, PRSTAB 8, 041002(2005).

[9] A. Sessler, private communication (1999).

[10] D. Stratakis, et al., A tapered six-dimensional cooling lattice for a Muon Collider, Proc. of 2013 IPAC, Shanghai, China, p. 1547 (2013).

[11] `http://pubweb.bnl.gov/users/fernow/www/icool/readme.html`

[12] T. Roberts, et al., Proc. EPAC08, WEPP120, Genoa, Italy.

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