Simulations of stochastic cooling of antiprotons in the collector ring CR

C. Dimopoulou


TUIOB02 @ COOL'11
Alushta, Ukraine, September 2011
Required performance of CR stochastic cooling

- Short bunch of hot secondary beam from production target into the CR
- After bunch rotation and adiabatic debunching the $\delta p/p$ is low enough to apply stochastic cooling
- Fast 3D stochastic cooling required to profit from production rate of secondary beams

<table>
<thead>
<tr>
<th></th>
<th>Antiprotons 3 GeV, $10^8$ ions</th>
<th>Rare isotopes 740 MeV/u, $10^9$ ions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta p/p$ (rms)</td>
<td>$\varepsilon_{h,v}$ (rms)</td>
</tr>
<tr>
<td></td>
<td>$\pi$ mm mrad</td>
<td>$\pi$ mm mrad</td>
</tr>
<tr>
<td>Before cooling</td>
<td>0.35 %</td>
<td>45</td>
</tr>
<tr>
<td>After cooling</td>
<td>0.05 % (*)</td>
<td>1.25 (<em>) (</em>)</td>
</tr>
<tr>
<td>Phase space reduction</td>
<td>$9 \times 10^3$</td>
<td>$1 \times 10^6$</td>
</tr>
<tr>
<td>Cooling down time</td>
<td>$\leq 9$ s</td>
<td>$\leq 1$ s</td>
</tr>
<tr>
<td>Cycle time</td>
<td>10 s</td>
<td>1.5 s</td>
</tr>
</tbody>
</table>

(*) 20% lower (if possible) for HESR as accumulator ring (instead of RESR)

C. Dimopoulou, COOL'11
Overview of the CR stochastic cooling systems

<table>
<thead>
<tr>
<th>Systems in frequency band 1-2 GHz</th>
<th>Pickup</th>
<th>Kicker</th>
<th>pbars</th>
<th>RIBs</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>PH</td>
<td>KH</td>
<td>hor.</td>
<td>hor.</td>
<td>final stage</td>
<td>difference PU</td>
</tr>
<tr>
<td>PV</td>
<td>KV</td>
<td>vert.</td>
<td>vert.</td>
<td>final stage</td>
<td>difference PU</td>
</tr>
<tr>
<td>PH+PV</td>
<td>KH+KV</td>
<td>long.</td>
<td>long.</td>
<td>final stage</td>
<td>Sum PU + notch filter</td>
</tr>
<tr>
<td>PP</td>
<td>KH</td>
<td>-----</td>
<td>hor.+long.</td>
<td>first stage</td>
<td>Palmer: difference PU at high D</td>
</tr>
<tr>
<td>PP</td>
<td>KV</td>
<td>-----</td>
<td>vert.</td>
<td>first stage</td>
<td>difference PU</td>
</tr>
</tbody>
</table>

System in frequency band 2-4 GHz (future option)

| P2-4   | K2-4 | long. | ------ | Sum PU + notch filter |

Main issue for pbars: increase ratio Schottky signal \(\propto Q^2\) thermal noise

C. Dimopoulou, COOL'11
Principle of betatron cooling & basic ingredients

Phase advance PU-K ≈ 90°

High amplification needed, electronic gain ~ 10^7 (140 dB)

"rms" theory (analytical model)
\[ \Psi_{\perp}(J,t) = \partial N/\partial J \]

\[ -\frac{1}{\epsilon} \frac{d\epsilon}{dt} = \frac{1}{\tau_{\perp}} = \frac{2W}{N} \left[ 2gB - g^2(M+U) \right] \]

System gain g = PU response x Electronic gain x K response ~ 10^{-2}

Coherent term= cooling force x undesired mixing (PU→K)

Diffusion= heating from Schottky noise (desired mixing (K→PU)) + from thermal noise
Principle of betatron cooling & basic ingredients

Phase advance PU-K $\approx 90^0$

High amplification needed, electronic gain $\sim 10^7$ (140 dB)

$$- \frac{1}{\varepsilon} \frac{d\varepsilon}{dt} = \frac{1}{\tau_\perp} = \frac{2W}{N} \left[ 2gB - g^2(M+U) \right]$$

- Good cooling for overlapping Schottky bands i.e. $M=1$ and low ratio thermal noise/Schottky signal $U$
- To cool all the particles within the initial momentum distribution $B \geq 0$
- $B$ and $M$ depend in a contradictory way on the spread $\Delta T/T = -\Delta f/f \sim -\eta_{\text{ring/pk}} \Delta p/p$ of the beam particles, they vary during momentum cooling
- In reality: choose $\eta_{\text{ring/pk}}$ for a compromise between $B$ and $M$

C. Dimopoulou, COOL'11
**Principle of momentum cooling with notch filter**

well-separated Schottky bands $M>1$

$B \geq 0$ for increased undesired mixing

very small $|\eta| \approx 1\%$

i.e. ring almost @ $\gamma_{tr}$

**Fokker-Planck equation (solved with CERN code)**

$$\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial E} \left[ -F \Psi + \left( D_s \Psi + D_n \right) \frac{\partial \Psi}{\partial E} \right],$$

$$\Psi(E,t) = \frac{\partial N}{\partial E}$$

Coherent term = cooling force x undesired mixing ($PU \rightarrow K$)

System gain $G = PU$ response x Filter response x Electronic gain x K response

The response of the notch filter

$$H_{\text{notch}} = \frac{i}{2} \left( 1 - e^{-i 2\pi f_0 t} \right) = -\sin \left( \pi m \eta \delta p \frac{\partial p}{p} \right) e^{-i \pi \eta \frac{\partial p}{p}}$$

- provides the cooling force,
- induces extra undesired mixing

C. Dimopoulou, COOL'11
Features and developments for the 1-2 GHz system

- PU/Kicker tank consists of 2 plates (up+down or left+right) with 64 electrodes/plate
  PH/KH=PV/KV rotated by 90°
- Plunging of PU electrodes i.e. moving closer to beam during cooling
- No plunging of KI electrodes

Slotline PU electrodes at 20-30 K
Cryogenic low-noise preamplifiers at 80 K
(open option of preamplifiers in UHV at 20 K)
Kickers at 300 K
- Optical notch filter (< 40 dB deep notches within 1-2 GHz )

Effective noise temperature at preamplifier input $T_{\text{eff}} = 73$ K
Longitudinal PU/K impedance, sensitivity, PU plunging

Circuit convention:

\[ Z_k = \frac{U^2_{\text{rms}}}{P_k}, \quad Z_p = \frac{P_p}{I^2_{\text{rms}}}, \quad Z_k = 4 \cdot Z_p \]

\[
\sqrt{Z_k(f, y)} \approx \sqrt{Z_k(f_c)} \cdot S(y) \cdot S(f)
\]

\[
\sqrt{Z_p(f, y)} \approx \sqrt{Z_p(f_c)} \cdot S(y) \cdot S(f)
\]

HFSS simulations, absolute values:

- \( Z_p(f_c) = 11.25 \, \Omega \) at \( y_{PU} = \pm 60 \, \text{mm} \)
- \( Z_p(f_c) = 37.75 \, \Omega \) at \( y_{PU} = \pm 20 \, \text{mm} \)

Plunging of PU electrodes:
factor 1.8 in sensitivity (3.4 in \( Z_p \))
from \( y_{PU} = \pm 60 \, \text{mm} \rightarrow \pm 20 \, \text{mm} \)

Relative measurements on prototype PU:

Simplify:

\[ S(y) \approx 1 + \text{slope} \cdot y \]
## Input parameters & requirements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR Circumference</td>
<td>221.45 m</td>
</tr>
<tr>
<td>3 GeV antiprotons</td>
<td></td>
</tr>
<tr>
<td>β=0.9712, γ=4.197, rev. frequency f₀=1.315 MHz</td>
<td></td>
</tr>
<tr>
<td>Ring slip factor η, slip factor PU-K ηpk</td>
<td>-0.011, -0.033</td>
</tr>
<tr>
<td>Distance PU-K/circumference</td>
<td>0.378</td>
</tr>
<tr>
<td>Beam intensity</td>
<td></td>
</tr>
<tr>
<td>Initial rms momentum spread</td>
<td>10⁸</td>
</tr>
<tr>
<td>Initial rms emittance εᵩᵥ</td>
<td>3.5 × 10⁻³, Gaussian/parabolic</td>
</tr>
<tr>
<td></td>
<td>45 π mm mrad</td>
</tr>
<tr>
<td>System bandwidth</td>
<td>1-2 GHz</td>
</tr>
<tr>
<td>Number of PU, K (longitudinal cooling)</td>
<td>128, 128</td>
</tr>
<tr>
<td>Number of PU, K (transverse cooling)</td>
<td>64, 64</td>
</tr>
<tr>
<td>PU, Kicker impedance at midband 1.5 GHz</td>
<td></td>
</tr>
<tr>
<td>PU/K sensitivity S(y)=1+slope* y</td>
<td></td>
</tr>
<tr>
<td>PU/K sensitivity vs. frequency S(f)</td>
<td></td>
</tr>
<tr>
<td>Effective temperature for thermal noise</td>
<td>73 K</td>
</tr>
<tr>
<td>ideal, infinitely deep notch filter + 90° phase shifter</td>
<td></td>
</tr>
<tr>
<td>Total installed power at kickers (limited by funding, can be upgraded)</td>
<td>4.8 kW</td>
</tr>
</tbody>
</table>

**Goal:** Cool longitudinally from σₚ/p= 3.5 × 10⁻³ → 4 × 10⁻⁴ in 9 s  
Simultaneous transverse cooling from εᵩᵥ = 45 → ≈ 1 π mm mrad

C. Dimopoulou, COOL'11
Momentum cooling: Cooling force and diffusion

\( G_{||} = 150 \text{ dB} \left(3.2 \times 10^7\right); \ t=10 \text{ s} \)

Coherent term:
- linear notch filter response around \( \Delta p/p=0 \rightarrow \) cooling force
- momentum acceptance of system (undesired mixing \(\geq 0\)) > total initial \(\Delta p/p\)
  \(\rightarrow\) Cooling of all particles

Schottky noise dominates \(\rightarrow\) long. cooling time \(\sim N\)

Notch filter cuts thermal noise around all harmonics

C. Dimopoulou, COOL’11
Momentum cooling: Feedback by the beam

Feedback by the beam included:

\[ G(m, E) \to \frac{G(m, E)}{1 - S(m, E, t)} \]

\[ S(m, E, t) = \sqrt{n_p n_k Z_p(m)Z_k(m) G(m, E) \cdot BTF(m, E, t)} \]

\[ B(m, E, t) = -\frac{e f_0^2}{m} \left[ \frac{\pi}{|\kappa|} \frac{d\Psi}{dE} + \frac{i}{\kappa} \int_{-\infty}^{+\infty} \frac{d\Psi}{E^* - E} \frac{dE^*}{dE} \right] \]

\[ G_{||} = 150 \text{ dB (3.2 } 10^7\text{)}; \ t=10 \text{ s} \]
Optimization:
For a given signal/noise ratio there is a gain so as to reach the desired $\sigma_p/p$ in the desired time. Lower gain leads to lower $\sigma_p/p$ but cooling takes longer.

For ultimate $\sigma_p/p$: increase signal/noise by plunging the PU electrodes during cooling

Total cw power in bandwidth at kicker: $P_{\text{max}} = P_s(t=0) + P_n$

Schottky $P_s(t) = \frac{1}{2} \left(2e f_0 \right)^2 n_p \sum_m \sum_E Z_p(m) |G(E,m)|^2 \cdot \Psi(E,t)$, decreases as $\sigma_p/p$ shrinks

Filtered thermal $P_n \approx \frac{1}{4} W K T_{\text{eff}} G_{||}^2$

Required installed power = $4P_{\text{max}}$ (to account for signal fluctuations)

C. Dimopoulou, COOL'11
Betatron cooling rate: details

\[- \frac{1}{\varepsilon} \frac{d\varepsilon}{dt} = \frac{1}{\tau} = \frac{2W}{N} \left[ 2gB \left| \sin(\mu_{pk}) \right| - g^2 (M + U) \right] \]

\[B(t) \approx \cos \left( 2\pi m_e x_{pk} \eta_{pk} \frac{\Delta p}{p}(t) \right)\]

\[M(t) \approx \frac{1}{m_c \eta} \frac{\Delta p}{p}(t)\]

\[U(t) = \frac{k_B T_{eff}}{N e^2 f_0 \beta_p \text{slope}^2 n_p} \left[ \frac{f_0}{2W} \sum_{m=-\infty}^{+\infty} \frac{1}{Z_p(m)} \right] \frac{1}{\varepsilon(t)}\]

Simultaneous notch filter momentum cooling ON
Ansatz from Fokker-Planck results at \(G_|| = 150\) dB

\[\dot{\phi}(t) = \left( \frac{\phi}{p} \right)_{ini} e^{-\frac{t}{\tau_{long}}}\]

Interplay between betatron & momentum cooling

Optimum gain \(g_{opt}(t) = \frac{B(t) \left| \sin(\mu_{pk}) \right|}{M(t) + U(t)}\)

C. Dimopoulou, COOL'11
Beyond power limits...cw $P_{\text{max}} = 950$ W!

For precise treatment, feedback by the beam must be included

Initially:

$U = 1.2$, $M = 11$ ! and grows...

$\rightarrow$ M dominates the heating at all $t$: $M \approx 10$ U

in principle, need long cooling at very low gain (plunging helps only at the end)

- Reached $\varepsilon_h = 4 \pi$ mm mrad in 9 s
- Beyond power limits...cw $P_{\text{max}} = 950$ W!
- For precise treatment, feedback by the beam must be included
Conclusions I

- Pbar filter momentum cooling from $\sigma_p/p = 3.5 \times 10^{-3} \rightarrow 4 \times 10^{-4}$ in 9 s is possible in the 1-2 GHz band:
  - with a gain around 150 dB ($3.2 \times 10^7$),
  - required max. installed power $\sim 2.6$ kW (cw $\sim 0.7$ kW),
  - assuming unplunged PU electrodes (conservative case), plunging expected to help reaching lower $\sigma_p/p$,
  - feedback by the beam not negligible but loop stable.

- The design $\eta = -0.011$ of CR is optimum for both 1-2 and 2-4 GHz bands (undesired mixing)

C. Dimopoulou, COOL'11
Conclusions II

- Preliminary results show that betatron cooling is possible
  - with separately optimized simultaneous filter momentum cooling (150 dB, ~2.6 kW),
  - down to $\varepsilon_{\text{rms}} \sim 4 \pi \text{ mm mrad}$ within 9 s,
  - with an electronic gain at midband around 140 dB ($10^7$),
  - with max. required installed power ~ 4 kW (cw ~1 kW) per plane h/v i.e. beyond the foreseen available power,
  - assuming unplunged electrodes.

- As expected, betatron cooling suffers from large desired mixing $M$ (required by filter momentum cooling) dominating the diffusion at all $t$.
  → Way out: slow-down momentum cooling in the beginning

C. Dimopoulou, COOL'11
Outlook

- Include **feedback by the beam** into betatron cooling model

- **Time-optimization** of momentum and betatron cooling **together**, distribution of available power accordingly, e.g.,
  - Initially, slower filter cooling to help the betatron cooling, then inversely to reach ultimate emittances and momentum spread.
  - Apply initially time-of-flight and later notch filter momentum cooling, with simultaneous betatron cooling.

- Include **plunging of PU electrodes**, expected to reduce diffusion by factors 4-9, especially transversally

- Additional **filter momentum cooling in the 2-4 GHz band**, study **handshake** between 2 bands

C. Dimopoulou, COOL'11