



# Helical Cooling Channels for Muon Colliders

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Muons, Inc. (<http://www.muonsinc.com/>)

Helical Cooling Channels, based on the same helical dipole Siberian Snake magnets used for spin control in synchrotrons and storage rings, are now proposed for almost all stages of muon beam cooling that are required for high luminosity muon colliders. We review the status of the theory, simulations, and technology development for the capture, phase rotation, 6-D ionization cooling, parametric-resonance ionization cooling, and reverse emittance exchange sections of one of the candidate scenarios for a high-luminosity, high-energy muon collider.

Please visit "Papers and Reports" and  
"LEMC Workshop" at  
<http://www.muonsinc.com/>



# Overview

Slava Derbenev's theory of the Helical Cooling Channel (HCC) is being exploited to provide a basis for the ionization cooling and emittance manipulation techniques needed for intense muon beams for energy frontier colliders.

- Since the last COOL meetings, a Muon Collider has become the most attractive path for an energy-frontier machine in the USA and is now seen on official Fermilab and DOE documents. (i.e. E of ILC too low, CLIC too power hungry)
- I will show steps that we now see as needed to provide muon beams for a high-luminosity, high energy ( $>10^{34}$ ,  $\sim 3$ -5 TeV) Muon Collider that could fit on the Fermilab site.
- I will describe the HCC aspects of the steps in more detail, concentrating on the newest technology developments



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## 5 Steps for Muon Production & Cooling

- 1) A 4 MW proton beam hits a target in a 20 T solenoid field which tapers to  $\sim 2$  T
- 2) Pions decay to muons and are phase rotated and captured into a string of bunches in a straight solenoid configuration.
- 3) Muons enter successive HCC segments each with smaller dimensions (allowing higher magnetic fields and higher RF frequency), as the beam is cooled in 6d using  $H_2$ -filled RF cavities.
  - Adjustment of slip factor affords larger RF buckets for better matching
  - $\sim 6$  orders of magnitude 6-d emittance reduction in 300 m long HCC
  - Recent beam tests of a  $H_2$ -pressurized RF cavity are encouraging
  - Helical Solenoid invention for HCC being tested in models with HTS
- 5) Parametric Resonance Ionization Cooling is used for more transverse emittance reduction (10x in each plane) Derbenev!
  - Twin-Helix HCC with correlated optics a new invention ( " & Morozov)
- 6) Reverse Emittance exchange with wedge absorbers reduces transverse emittance while longitudinal grows (to hourglass limit)  
(HCC theory also used for transitions between steps)





# Compact Muon Collider Fit in Fermilab campus

**Muon Collider: 20??**

Cost: Unknown

Energy: 0.5 ~ 4 TeV

Components: 10,000

**CLIC: 20??**

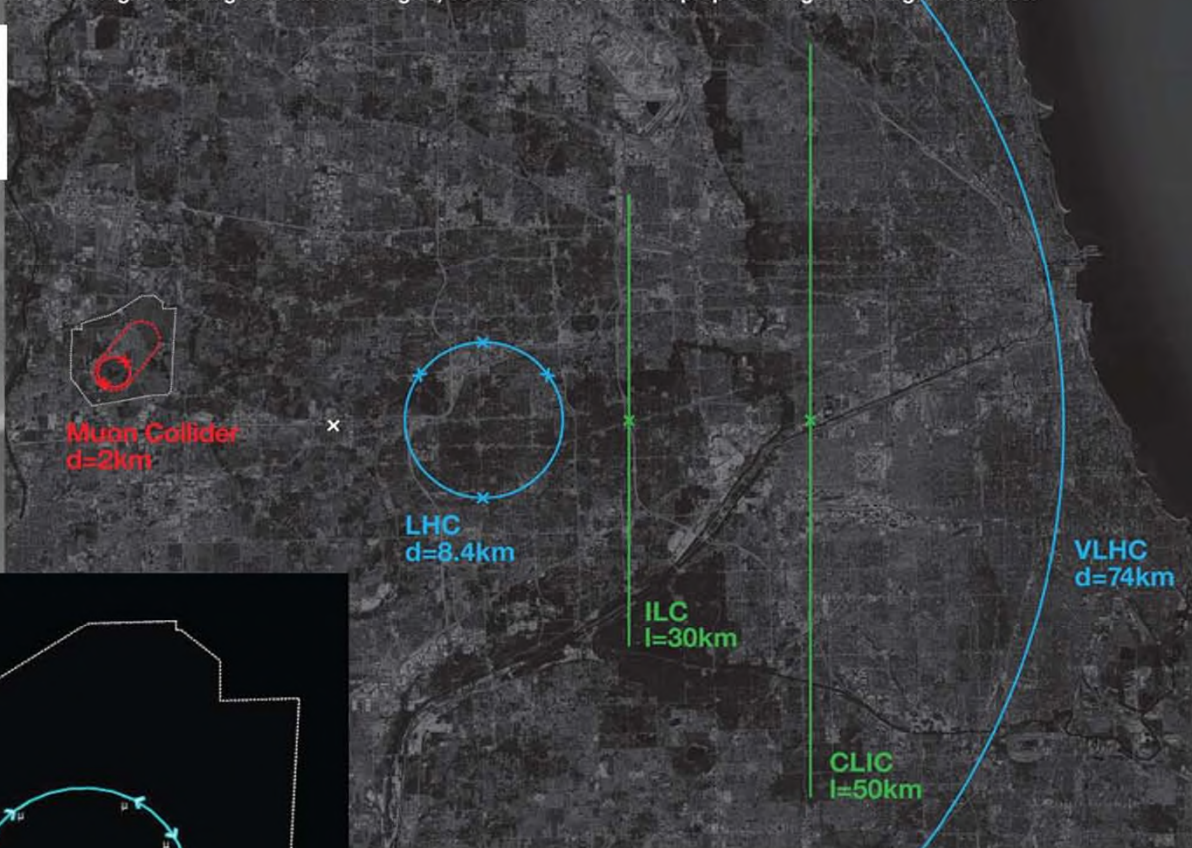
Cost: Estimate due in 2010

Energy: 0.5 ~ 3 TeV

Components: 260,000

## Comparison of Particle Colliders

To reach higher and higher collision energies, scientists have built and proposed larger and larger machines.



## Muon Collider Conceptual Layout

### Project X

Accelerate hydrogen ions to 8 GeV using SRF technology.

### Compressor Ring

Reduce size of beam.

### Target

Collisions lead to muons with energy of about 200 MeV.

### Muon Cooling

Reduce the transverse motion of the muons and create a tight beam.

### Initial Acceleration

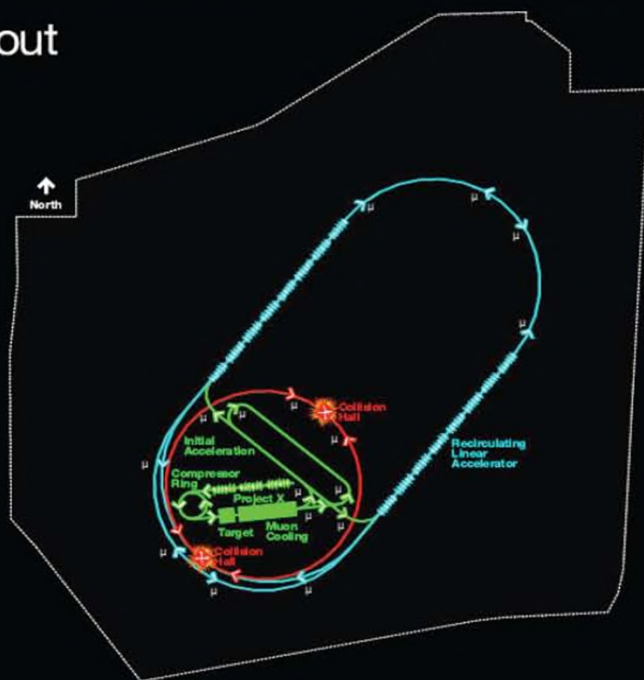
In a dozen turns, accelerate muons to 20 GeV.

### Recirculating Linear Accelerator

In a number of turns, accelerate muons up to 2 TeV using SRF technology.

### Collider Ring

Located 100 meters underground. Muons live long enough to make about 1000 turns.



**LHC: 2009-present**

Cost: US \$4.6 billion

Energy: 14 TeV

Components: 11,000

**VLHC: 20???**

Cost: Unknown

Energy: 40 ~ 200 TeV

**ILC: 20??**

Cost: US \$8 billion in 2007

Energy: 0.5 TeV

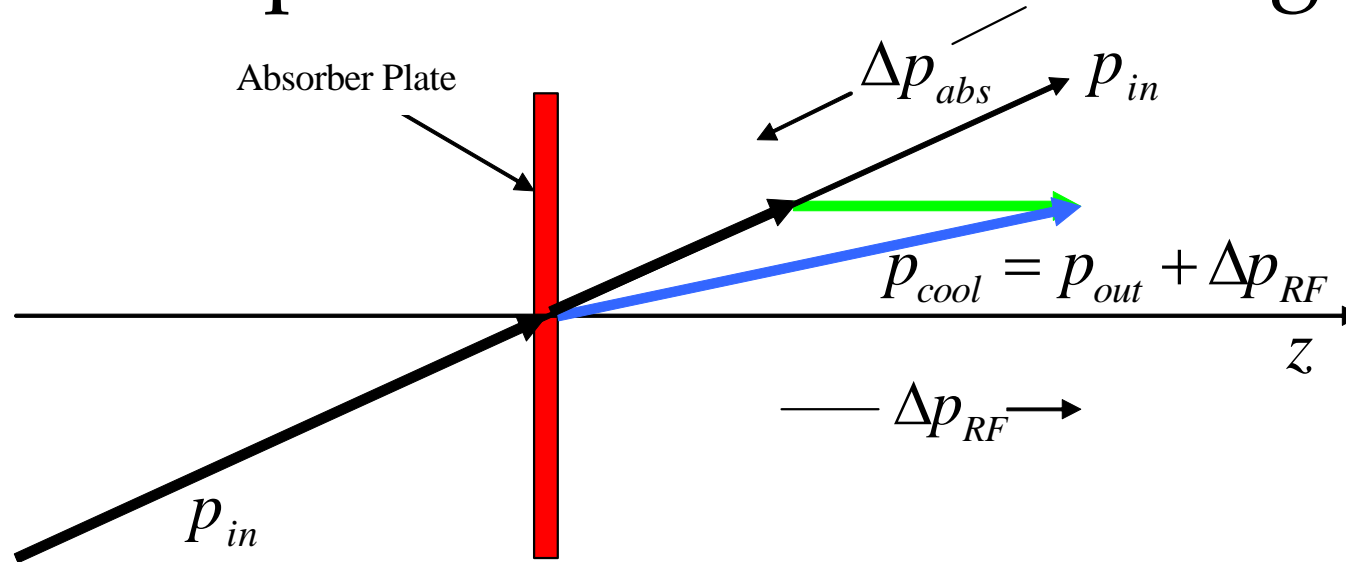
Components: 38,000

Numbers are taken from  
*Nature* **462**, 260-261 (2009)



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# Principle of Ionization Cooling



- Each particle loses momentum by ionizing a low-Z absorber
- Only the longitudinal momentum is restored by RF cavities
- The angular divergence is reduced until limited by multiple scattering
- Successive applications of this principle with clever variations leads to small emittances for many applications
- Early work: Budker, Ado & Balbekov, Skrinsky & Parkhomchuk, Neuffer

# Transverse Emittance IC

- The equation describing the rate of cooling is a balance between cooling (first term) and heating (second term):

$$\frac{d\varepsilon_n}{ds} = -\frac{1}{\beta^2} \frac{dE_\mu}{ds} \frac{\varepsilon_n}{E_\mu} + \frac{1}{\beta^3} \frac{\beta_\perp (0.014)^2}{2E_\mu m_\mu X_0}$$

Bethe-Bloch

Moliere (with low Z mods)

- Here  $\varepsilon_n$  is the normalized emittance,  $E_\mu$  is the muon energy in GeV,  $dE_\mu/ds$  and  $X_0$  are the energy loss and radiation length of the absorber medium,  $\beta_\perp$  is the transverse beta-function of the magnetic channel, and  $\beta$  is the particle velocity.





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# Wedges or Continuous Energy Absorber for Emittance Exchange and 6d Cooling

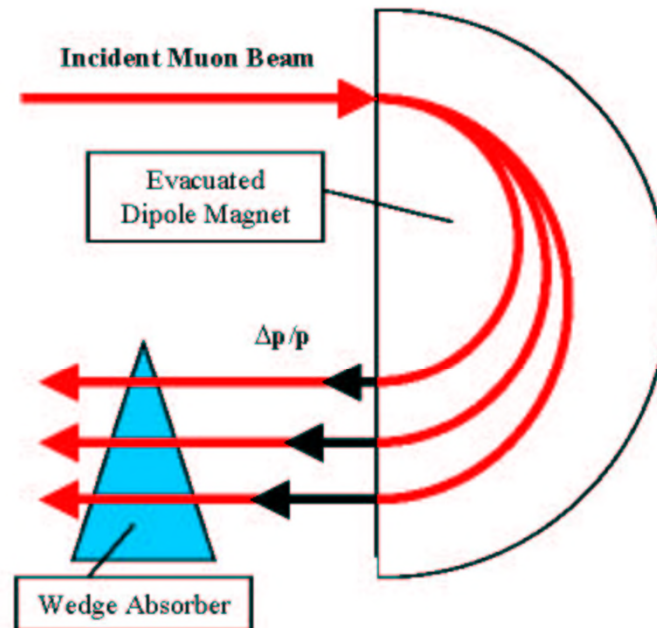


Figure 1. Use of a Wedge Absorber for Emittance Exchange

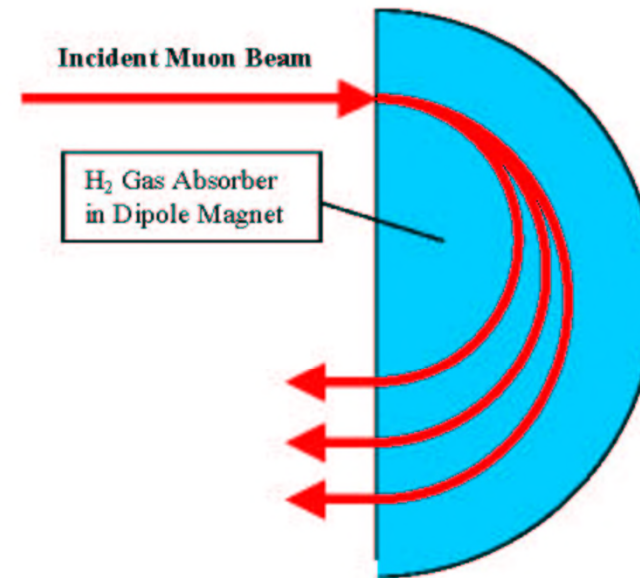


Figure 2. Use of Continuous Gaseous Absorber for Emittance Exchange

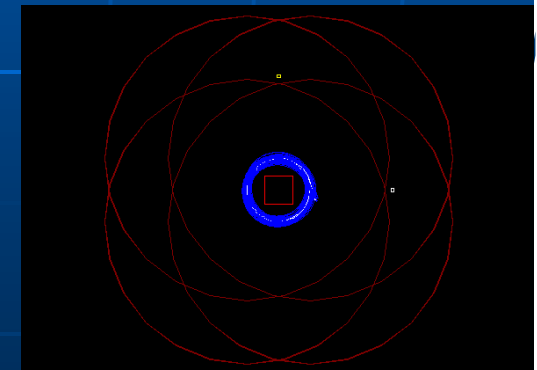
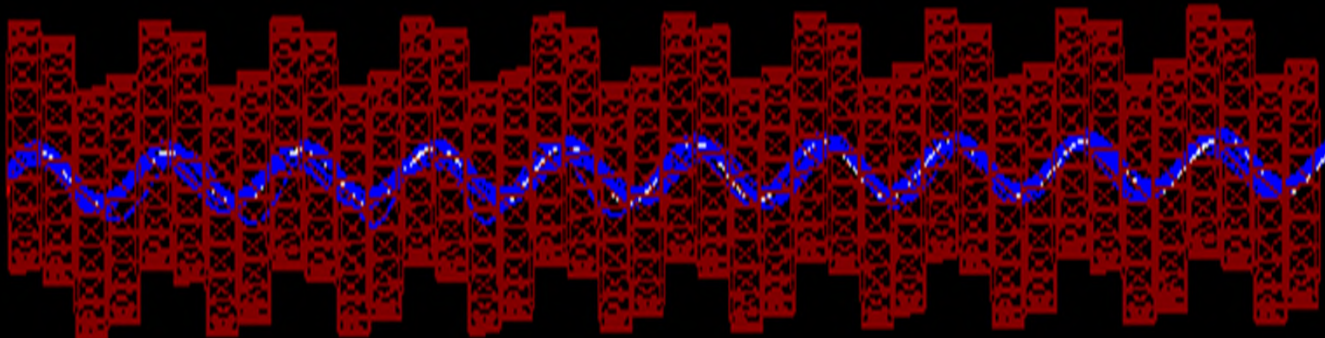
Ionization Cooling is only transverse. To get 6D cooling, emittance exchange between transverse and longitudinal coordinates is needed.



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## 6-Dimensional Cooling in a Continuous Absorber

- Helical cooling channel (HCC)
  - Continuous absorber for emittance exchange
  - Solenoid, transverse helical dipole and quadrupole fields
  - Helical dipoles known from Siberian Snakes
  - z- and time-independent Hamiltonian
  - Derbenev & Johnson, Theory of HCC, April/05 PRST-AB
    - <http://www.muonsinc.com/reports/PRSTAB-HCCtheory.pdf>





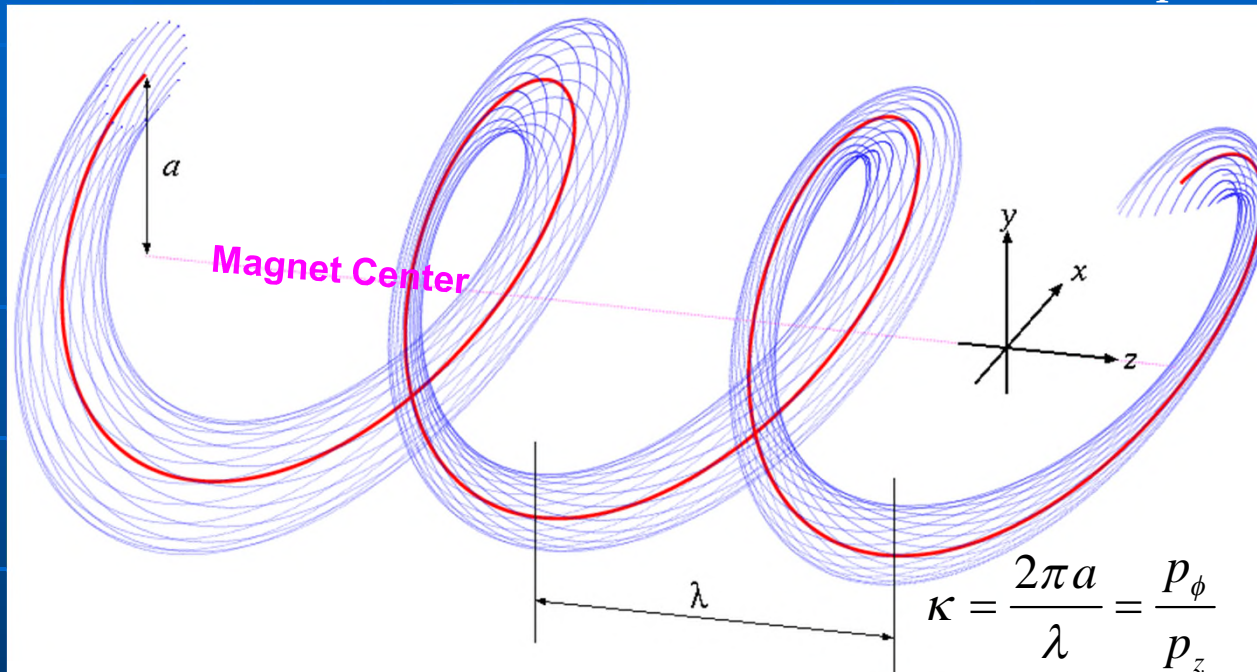


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# Particle Motion in a Helical Magnet

Combined function magnet (invisible in this picture)

Solenoid + Helical dipole + Helical Quadrupole



Red: Reference orbit

Blue: Beam envelope

Dispersive component makes longer path length for higher momentum particles and shorter path length for lower momentum particles.

Opposing radial forces

$$F_{h-dipole} \approx p_z \times B_{\perp}; \quad b \equiv B_{\perp}$$

$$F_{solenoid} \approx -p_{\perp} \times B_z; \quad B \equiv B_z$$

Transforming to the frame of the rotating helical dipole leads to a time and  $z$  – independent Hamiltonian

$b'$  added for stability and acceptance



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# Some Important Relationships

Hamiltonian Solution  $p(a) = \frac{\sqrt{1+\kappa^2}}{k} \left[ B - \frac{1+\kappa^2}{\kappa} b \right] \quad k = 2\pi/\lambda \quad \kappa = ka$

Equal cooling decrements  $q \equiv \frac{k_c}{k} - 1 = \beta \sqrt{\frac{1+\kappa^2}{3-\beta^2}} \quad k_c = B\sqrt{1+\kappa^2}/p$

Longitudinal cooling only  $\hat{D} \equiv \frac{p}{a} \frac{da}{dp} = 2 \frac{1+\kappa^2}{\kappa^2} \quad q = 0$

~Momentum slip factor  $\eta = \frac{d}{d\gamma} \frac{\sqrt{1+\kappa^2}}{\beta} = \frac{\sqrt{1+\kappa^2}}{\gamma \beta^3} \left( \frac{\kappa^2}{1+\kappa^2} \hat{D} - \frac{1}{\gamma^2} \right) \quad \frac{\kappa^2}{1+\kappa^2} \hat{D} \sim \frac{1}{\gamma_{transition}^2}$



## Matching Between Step 2 Capture and Step 3 HCC

$$A_{bucket} \cong \frac{16}{w_{rf}} \sqrt{\frac{eV'_{max} \lambda_{RF} m_{\mu} c^2}{2\pi |\eta_H|}} \left[ \frac{1 - \sin(\phi_s)}{1 + \sin(\phi_s)} \right] \quad (1)$$

where

- the term in brackets is an approximation for the moving-bucket factor
- $w_{rf}$  is the RF frequency in radians/second
- $V'_{max}$  is the maximum E-field voltage gradient
- $\lambda_{rf}$  is the RF wavelength
- $m_{\mu}$  is the mass of the muon
- $\phi_s$  is the synchronous particle RF phase, and  $\eta_H$  is the slip factor, derived in [1] for an HCC as:



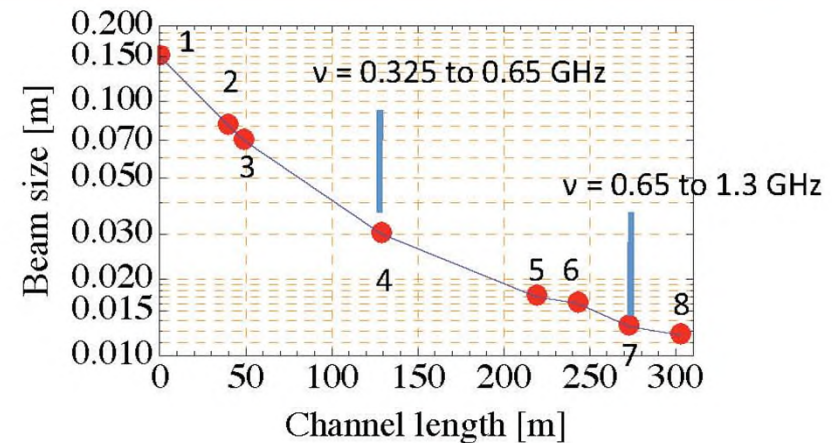
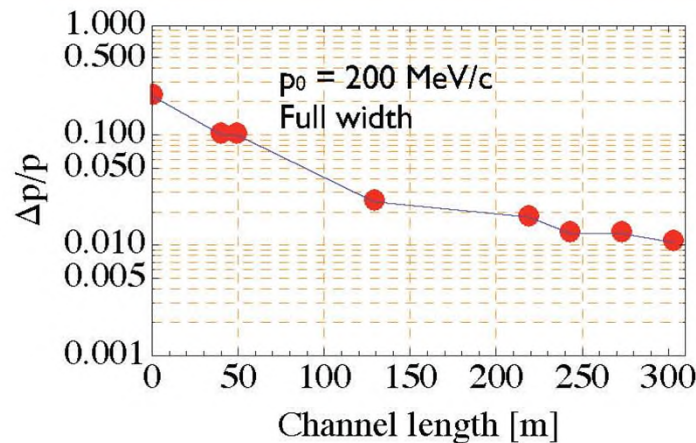
# Step 3: 6d Ionization Cooling w pressurized RF Cavities

## Simulation parameters & results



Simulation has been made with analytical EM field expression in G4beamline

	$z$	$b$	$b'$	$bz$	$\lambda$	$N$	$\epsilon_T$	$\epsilon_L$	$\epsilon_{6D}$	$\epsilon$
unit	m	T	T/m	T	m	GHz	mm rad	mm	mm <sup>3</sup>	Transmission
1	0						20.4	42.8	12900	
2	40	1.3	-0.5	-4.2	1.0	0.325	5.97	19.7	415.9	0.92
3	49	1.4	-0.6	-4.8	0.9	0.325	4.01	15.0	10.8	0.86
4	129	1.7	-0.8	-5.2	0.8	0.325	1.02	4.8	3.2	0.73
5	219	2.6	-2.0	-8.5	0.5	0.65	0.58	2.1	2.0	0.66
6	243	3.2	-3.1	-9.8	0.4	0.65	0.42	1.3	0.14	0.64
7	273	4.3	-5.6	-14.1	0.3	0.65	0.32	1.0	0.08	0.62
8	303	4.3	-5.6	-14.1	0.3	1.3	0.34	1.1	0.07	0.60



### Other important parameters:

RF  $E_{\text{peak}} = 27 \text{ MV/m} \Rightarrow$  Peak RF power = 10 to less than 0.2 MW / m

60  $\mu\text{m}$  Be window at RF entrance

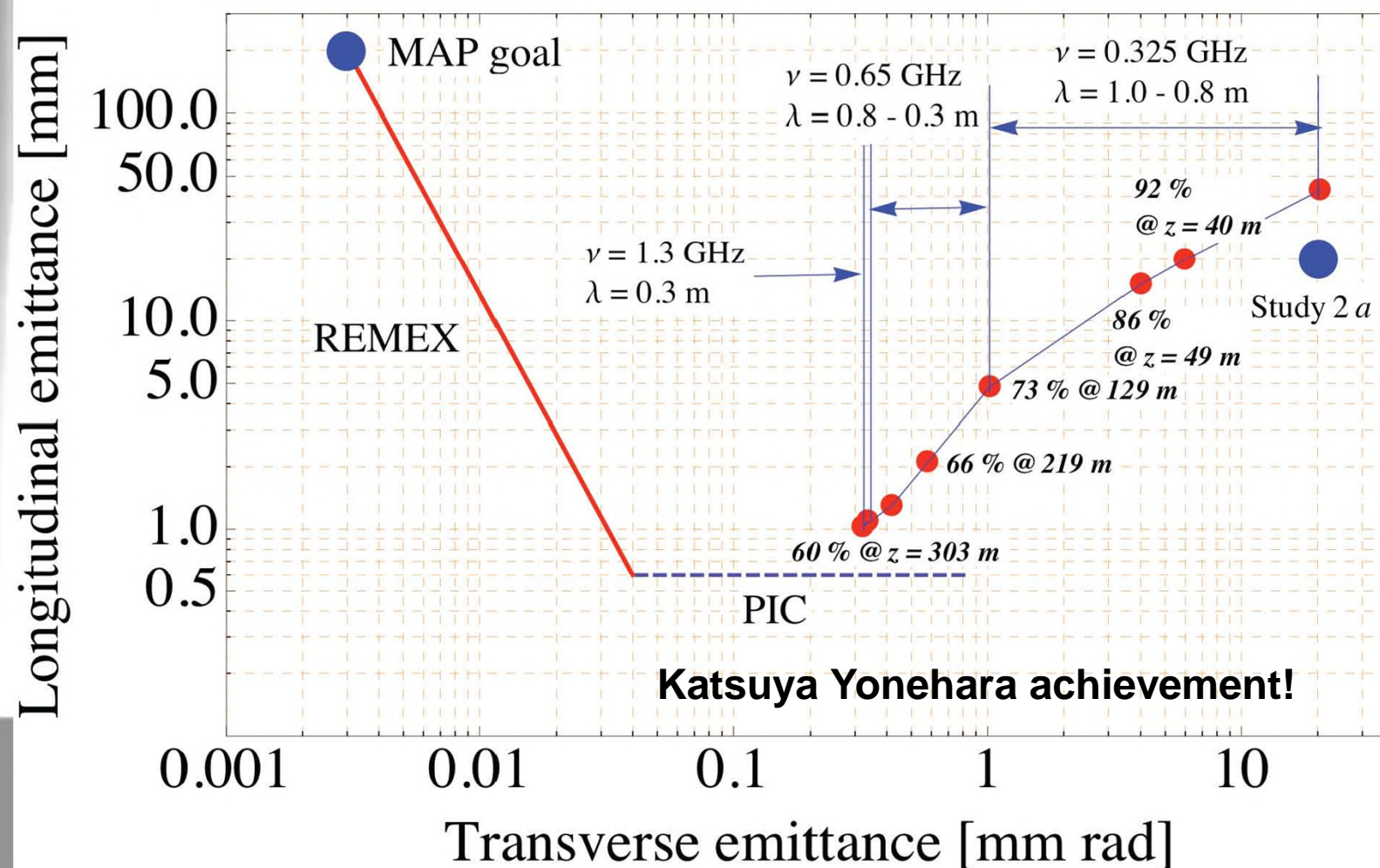
GH2 pressure = 160 atm at 300 K

(optimization is on going,  
not covered in this talk)



# Emittance Evolution in Homogeneous GH2 Filled HCC

- $10^6$  of 6D cooling factor is needed for MC
- HCC demonstrated 6D cooling factor  $> 10^5$  with 60 % transmission efficiency
- Additional cooling is needed (Parametric Ionization Cooling etc.)





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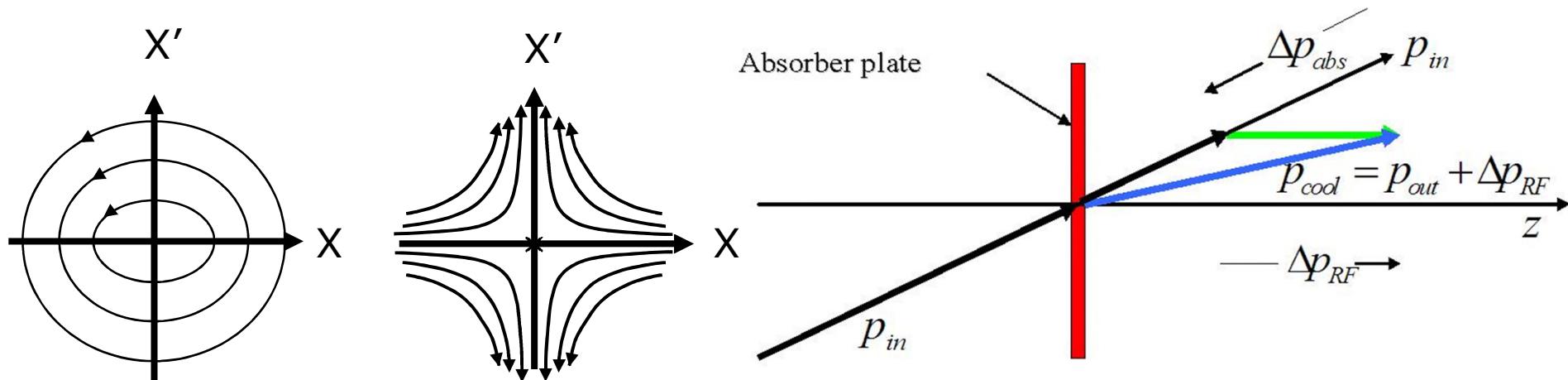
## Step 4: Parametric-resonance Ionization Cooling

Excite  $\frac{1}{2}$  integer parametric resonance (in Linac or ring)

- Like vertical rigid pendulum or  $\frac{1}{2}$ -integer extraction
- Elliptical phase space motion becomes hyperbolic
- Use  $xx' = \text{const}$  to reduce  $x$ , increase  $x'$
- Use IC to reduce  $x'$

Detuning issues being addressed (chromatic and spherical aberrations, space-charge tune spread). Simulations underway.

Smaller beams from 6D HCC cooling essential for this to work!





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# PIC using a Twin-Helix

- PIC uses a half-integer parametric resonance.
- Twin-Helix is superposition of 2 opposite helicity HCCs (similar to getting linear polarization from two circularly polarized light beams)
- correlated h and v betatron periods -> simultaneous focusing in both planes.
- Energy absorber plate & energy-restoring RF cavity at beam focal point
- IC limits the angular spread, parametric resonance reduces beam spot size.
- The achievable normalized equilibrium transverse emittance is given by [2]

$$\varepsilon_{\perp}^n = \frac{\sqrt{3}}{4\beta} (Z+1) \frac{m_e}{m_{\mu}} w, \quad (1)$$

- $w$  is the average absorber thickness in the beam direction. Compared to conventional IC, the this emittance is reduced by at least an order of magnitude by: ,

$$\frac{\pi}{\sqrt{3}} \frac{w}{\lambda_x} = \frac{\pi}{2\sqrt{3}} \frac{\gamma'_{acc}}{\gamma'_{abs}}, \quad (2)$$

- where  $\lambda_x$  is the period of the horizontal betatron oscillations and  $\gamma'_{acc}$  and  $\gamma'_{abs}$  are the RF acceleration and intrinsic absorber energy loss rates, respectively.

# PIC using Twin-Helix (cont.)

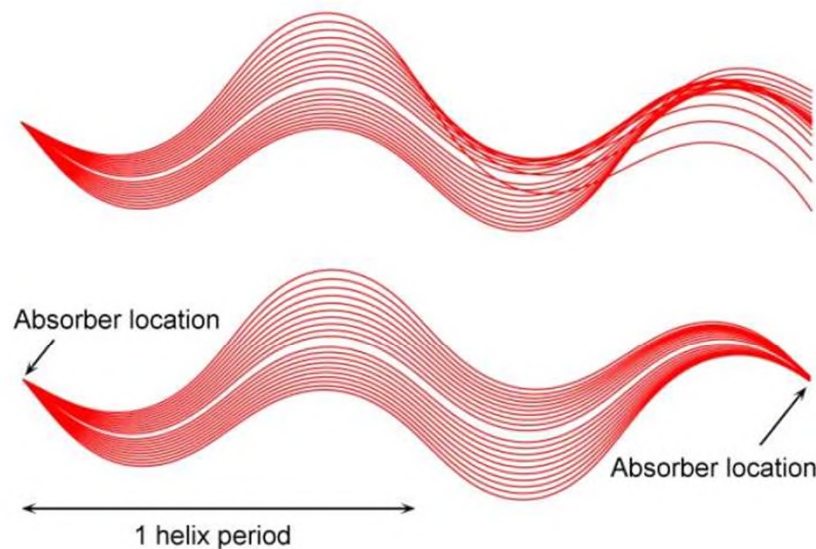


Figure 3:  $\pm 200$  mrad  $\mu^-$  tracks distributed over  $\pm 200$  mrad in the horizontal plane before (top) and after (bottom) compensation of the horizontal spherical aberration.

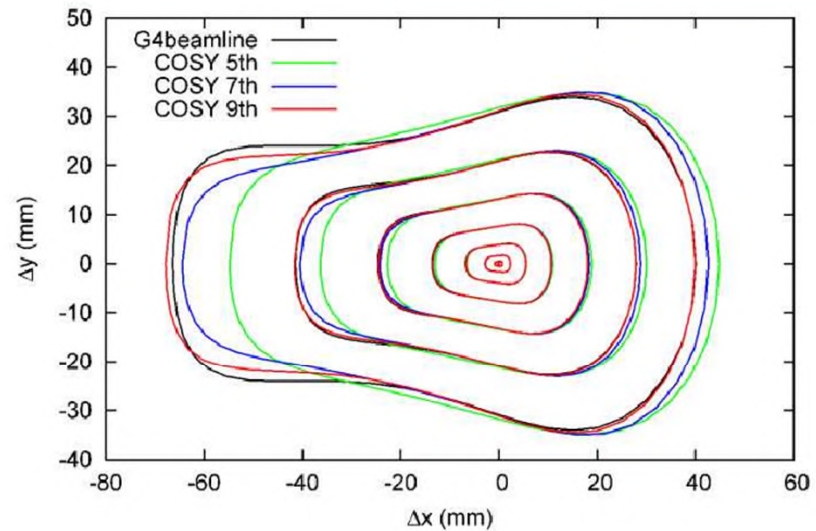


Figure 5: Beam smear due to spherical aberrations after 2 helix periods. G4beamline simulation is compared to various-order COSY Infinity calculations.

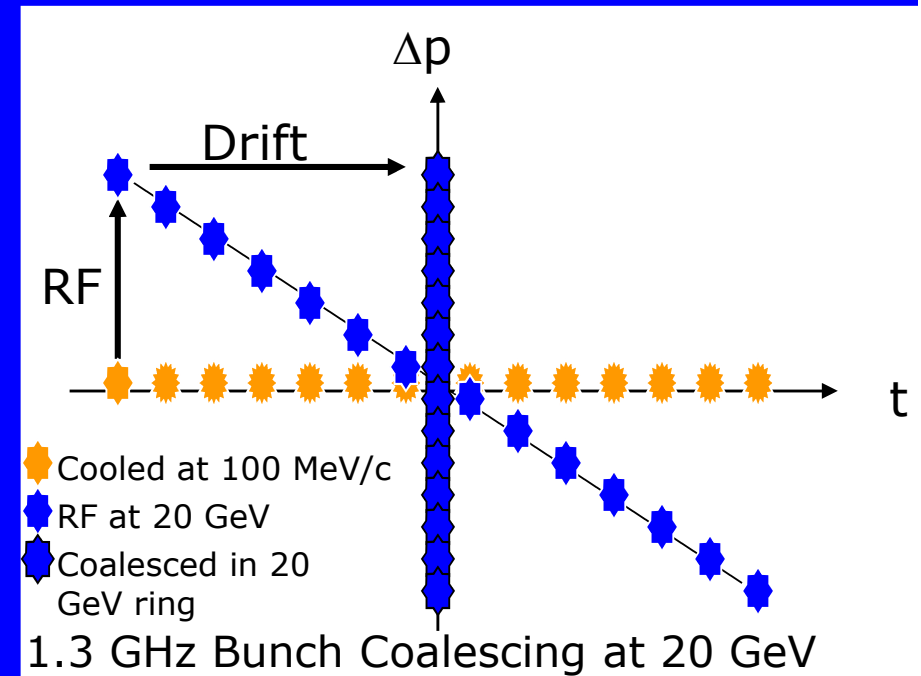
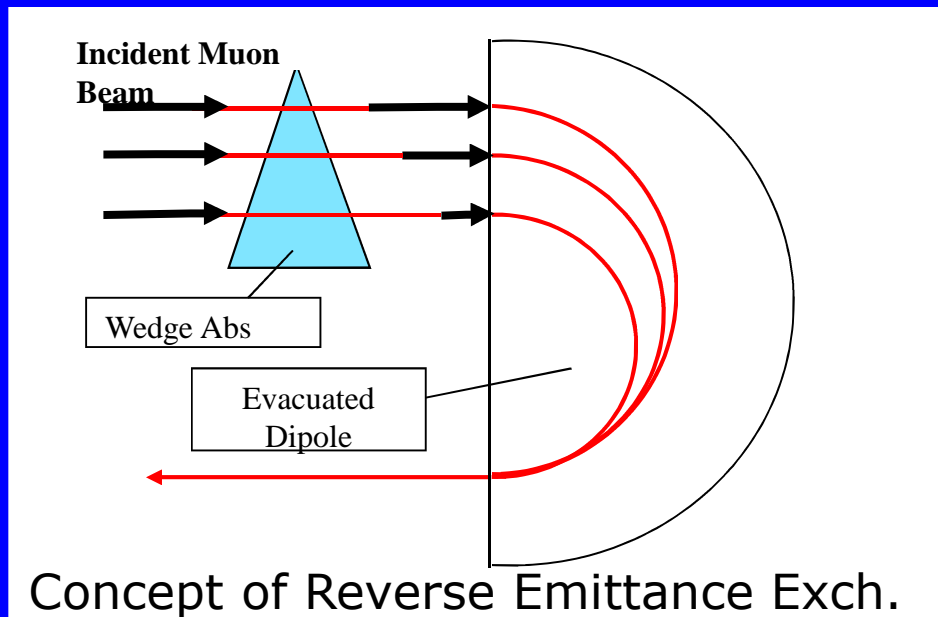




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## Step 5 - REMEX Reverse Emittance Exchange, Coalescing

- $p(\text{cooling})=100\text{ MeV}/c$ ,  $p(\text{colliding})=2.5\text{ TeV}/c \Rightarrow$  room in  $\Delta p/p$  space
- Shrink the transverse dimensions of a muon beam to increase the luminosity of a muon collider using wedge absorbers
- Allow bunch length to increase to size of low beta
- Low energy space charge, beam loading, wake fields problems avoided
- 20 GeV Bunch coalescing in a ring Neutrino factory and muon collider now have a common path



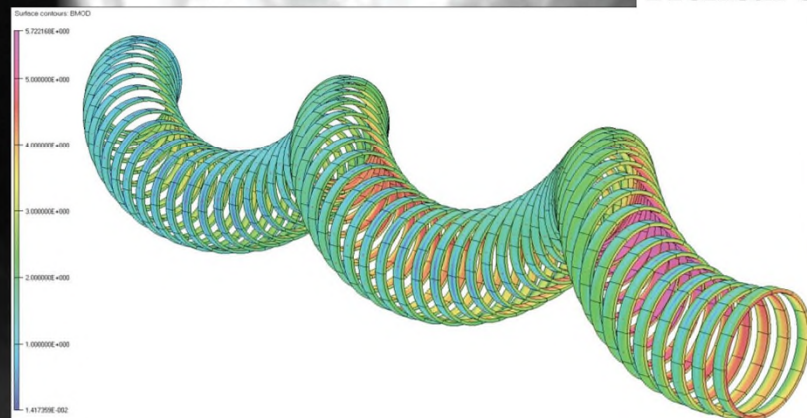


# Hardware Development

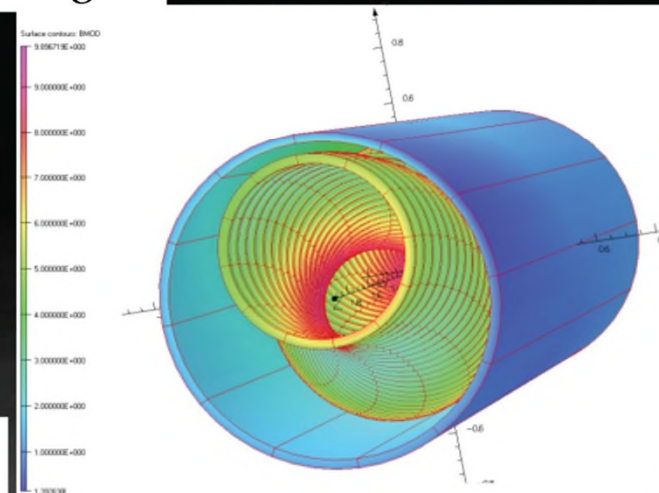
- Helical Solenoid invention to get HCC fields
  - 4-coil NbTi HS model tested (1<sup>st</sup> 6d HCC segment)
  - 6-coil YBCO HS model tested (last 6d HCC segment)
- High Pressure H<sub>2</sub> Cavity development
  - Test cell shows no HV max dependence on external B
  - First beam tests show agreement with models
    - No RF breakdown
    - Ionization electrons move far enough to heat H<sub>2</sub> – reduce Q
    - Mitigated with 0.01% SF<sub>6</sub> dopant – will allow MC application

# Helical Magnet

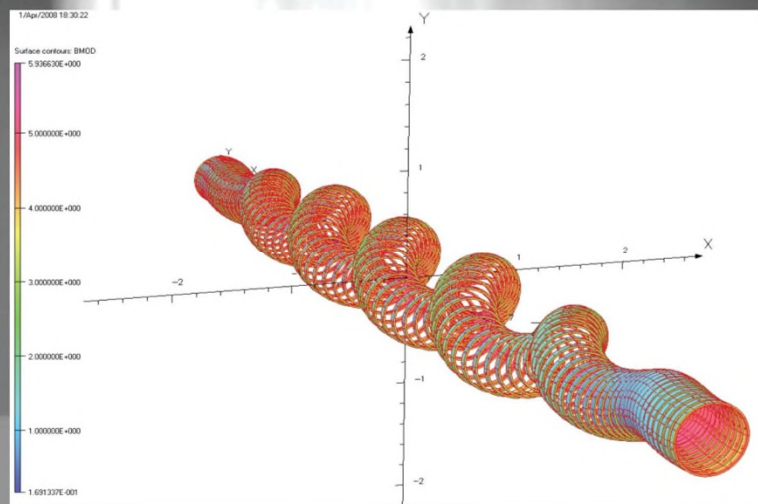
## Helical solenoid magnet



- Coil center follows on the helical reference track
- It generates proper helical dipole + field gradient



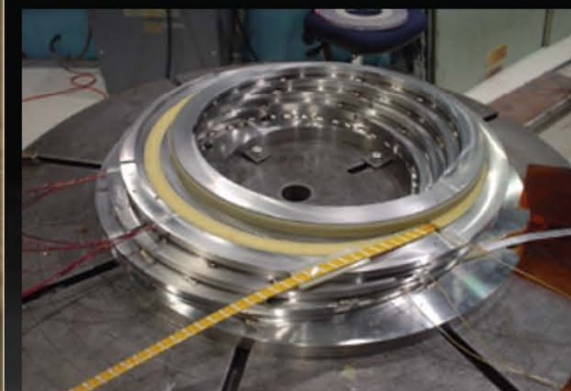
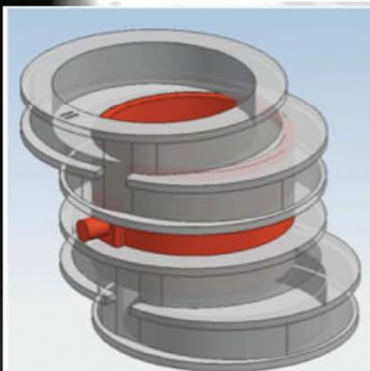
- By adding a solenoid coil, it tunes all three field components ( $b$ ,  $b'$ ,  $b_z$ )



- By modulating the coil position, it can make a beam adapter to connect between straight and helical magnet sections.
- Helical solenoid magnet generates more uniform field than analytical field.
- It means that **helical solenoid magnet has larger acceptance than analytical one.**

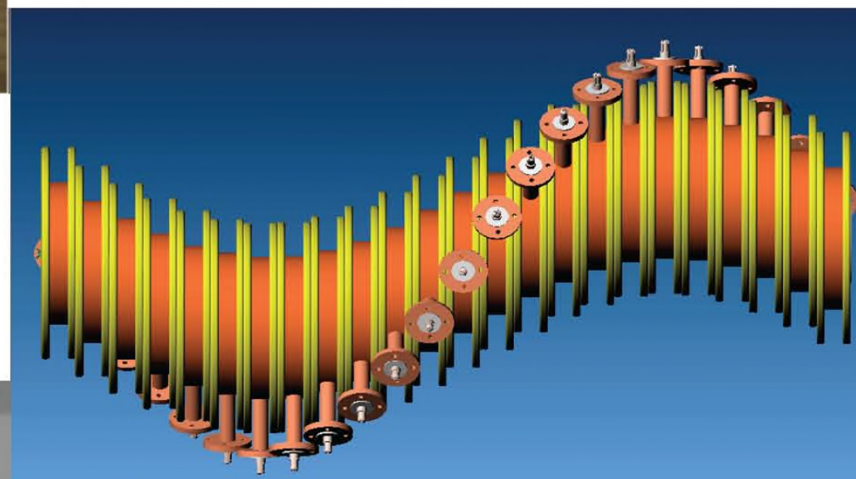


# Incorporate RF Cavity into Helical Magnet



- 4-coil helical solenoid magnet to study support structure, splice ground insulation, field quality test, etc...

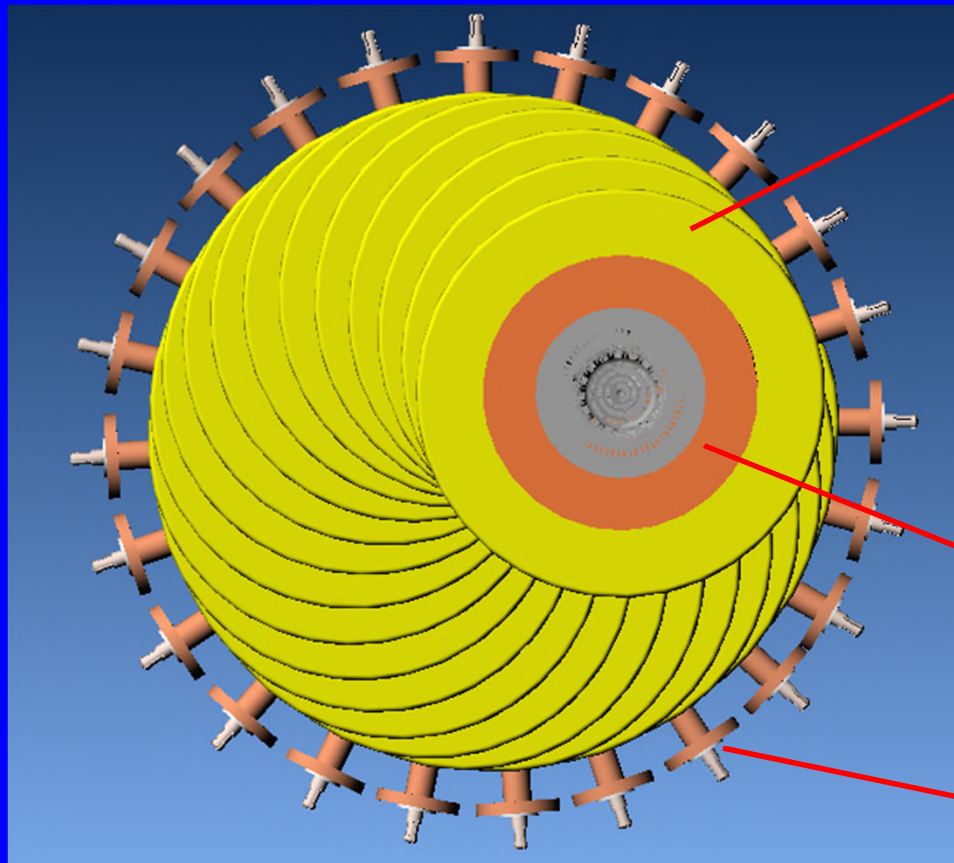
- Plastic model to demonstrate integrating RF into helical magnet
- Segment RF cavity and helical solenoid coil
- Red: RF cavity
- White: Helical magnet



- CAD drawing to show one helical period



# This is a 24 cavity per period 400 MHz design



Magnet  
Coil

- Approximate  
Diameter at the  
coax flange is 40"

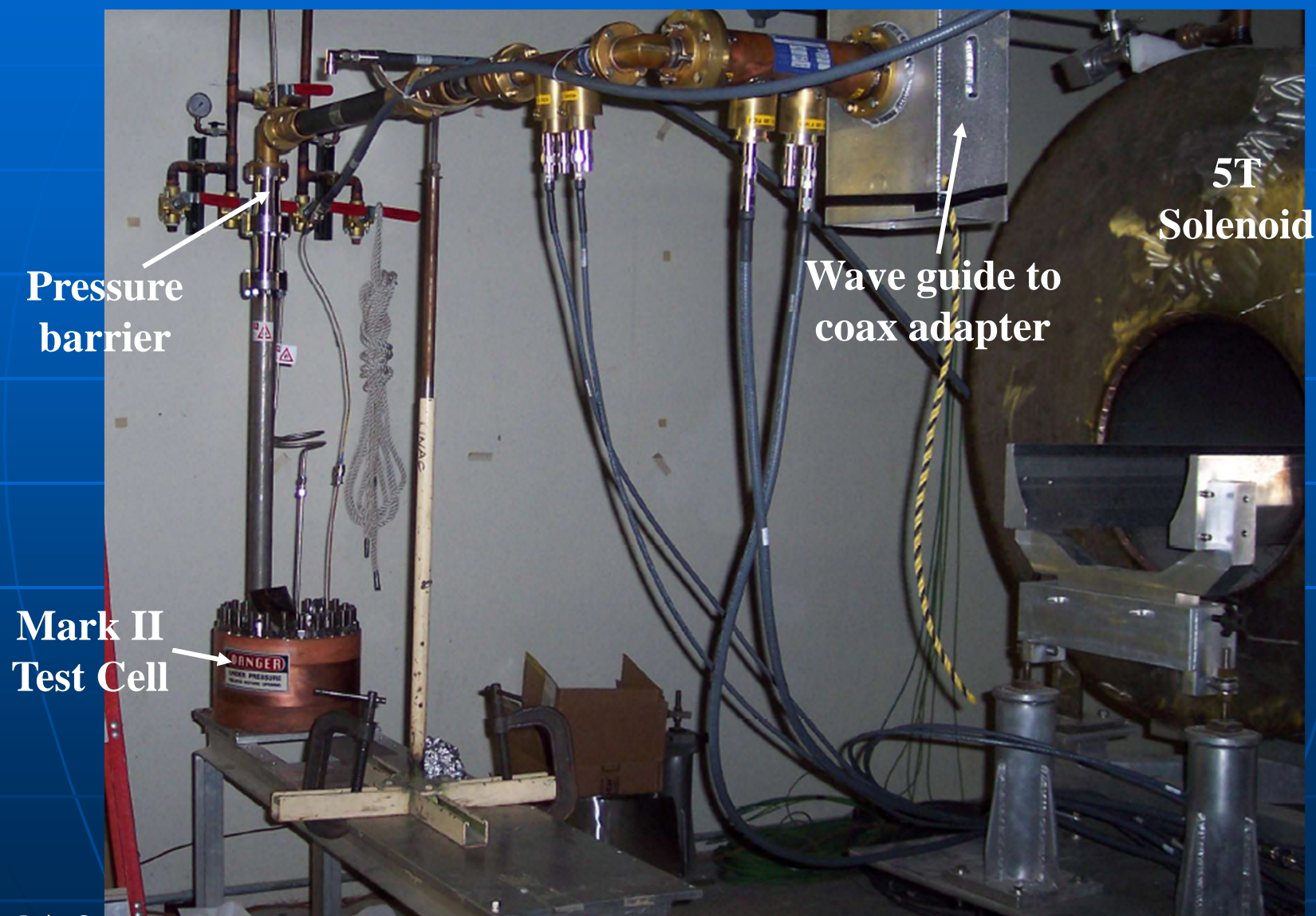
Pill box  
Cavity

Coax  
Feed per  
cavity



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## MuCool Test Area (MTA)



Rol - Sept 13, 2011

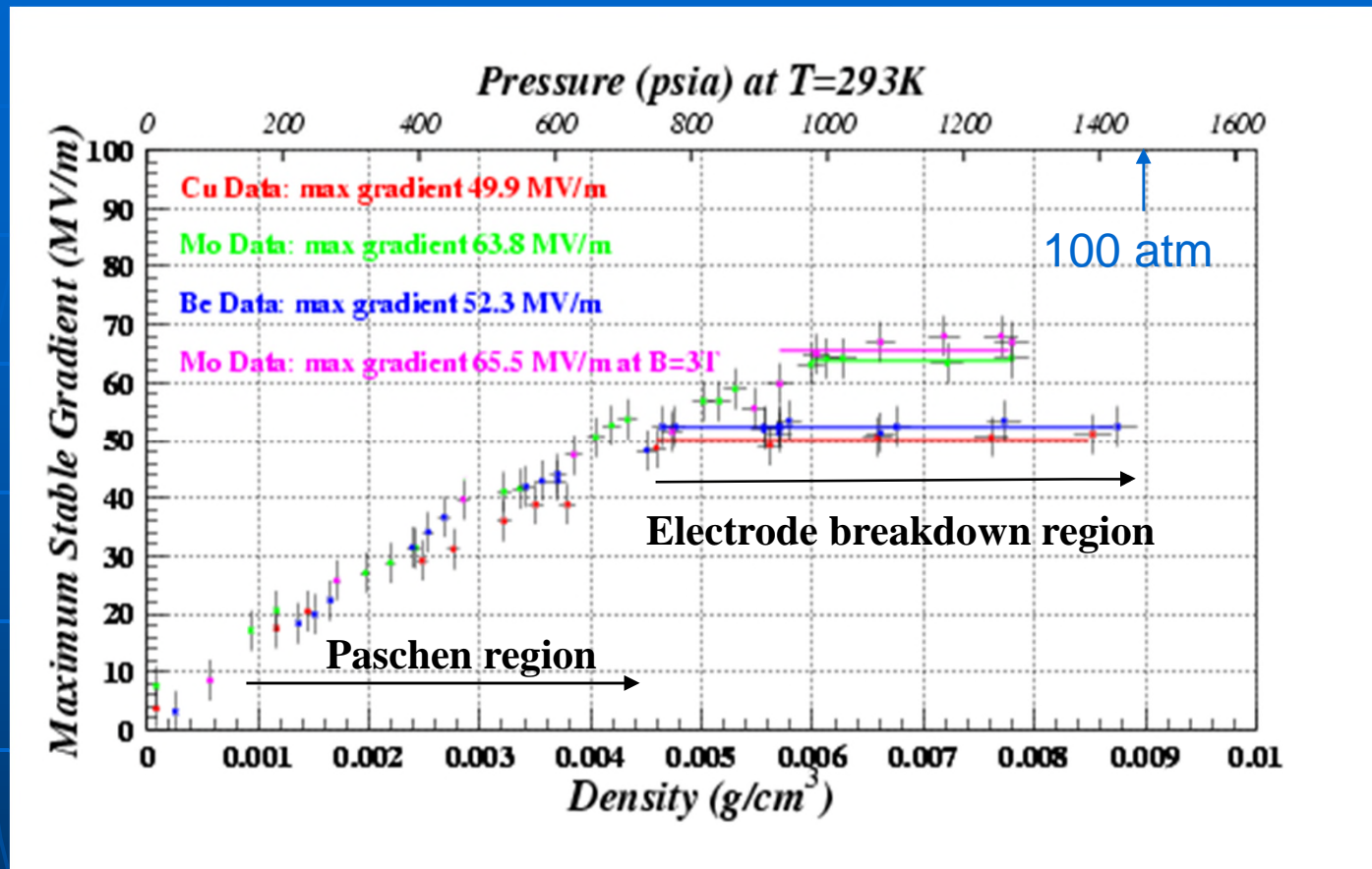
COOL11

22



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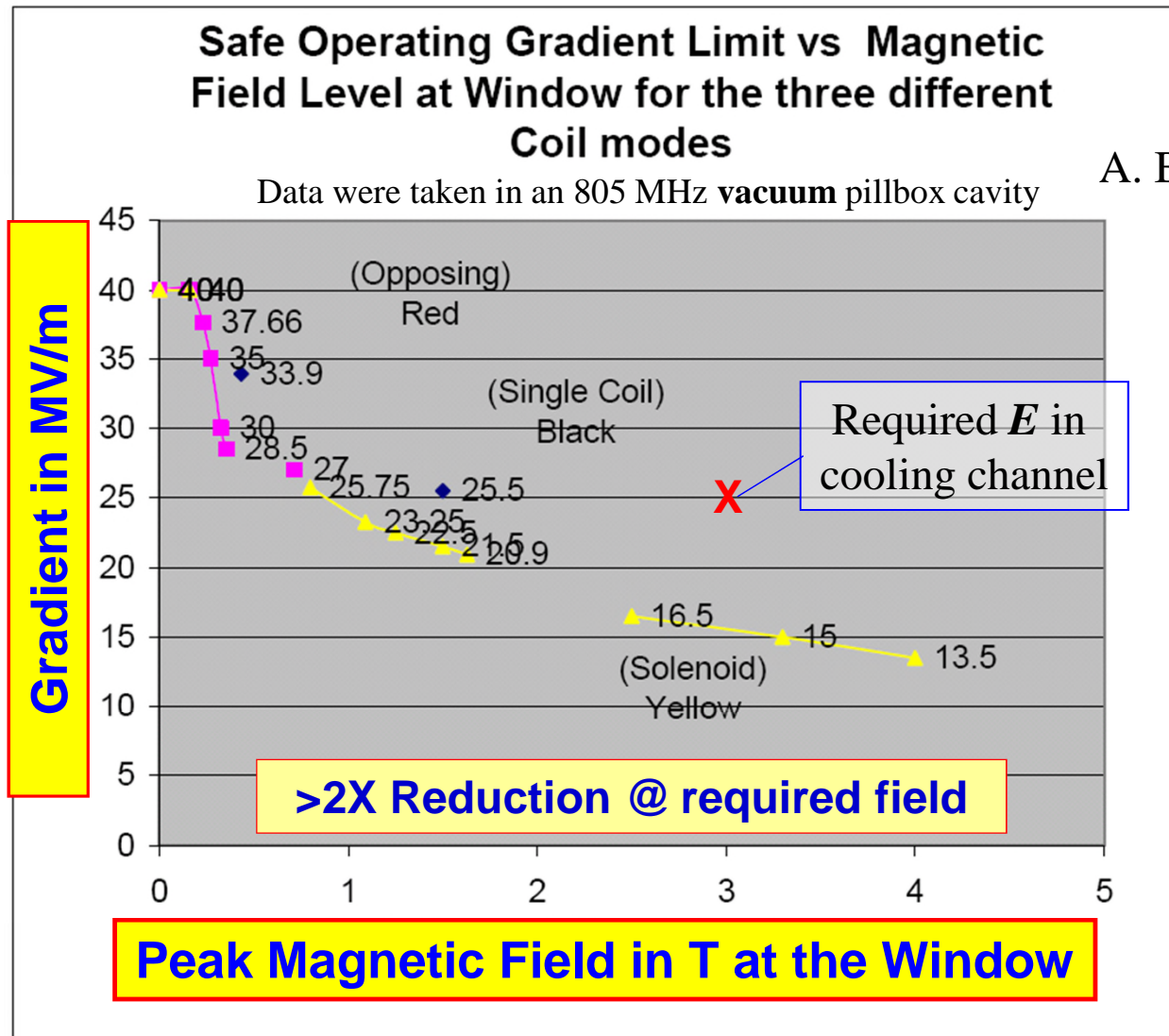
# HPRF Test Cell Measurements in MTA



- Paschen curve verified
- Maximum gradient limited by breakdown of metal.
- Cu and Be have same breakdown limits ( $\sim 50$  MV/m), Mo( $\sim 63$  MV/m), W( $\sim 75$  MV/m).
- Results show no B dependence, much different metallic breakdown than for vacuum cavities.
- **Need beam tests to prove HPRF works.**



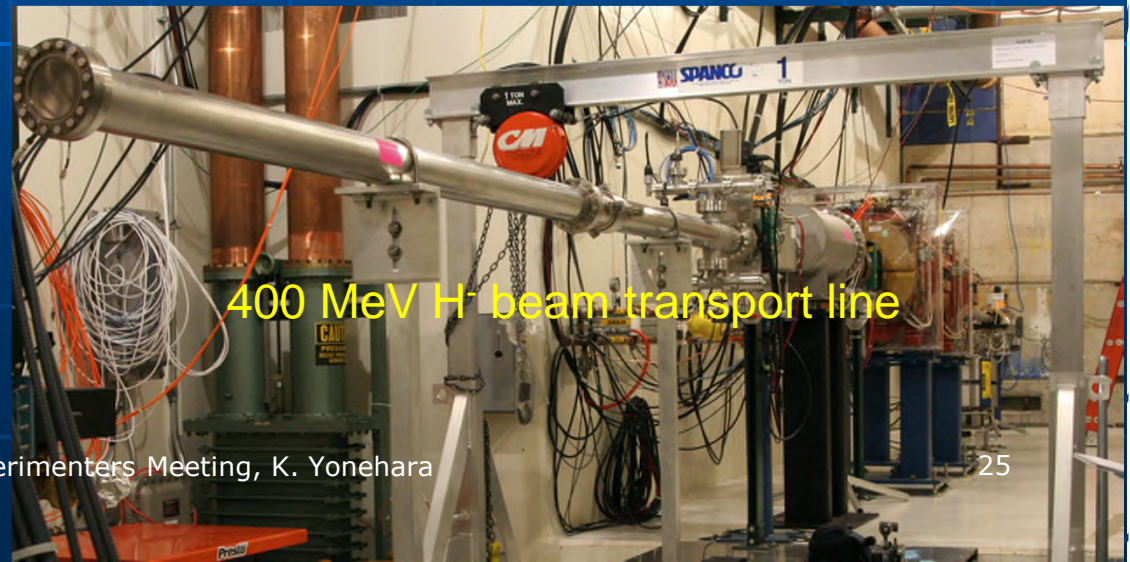
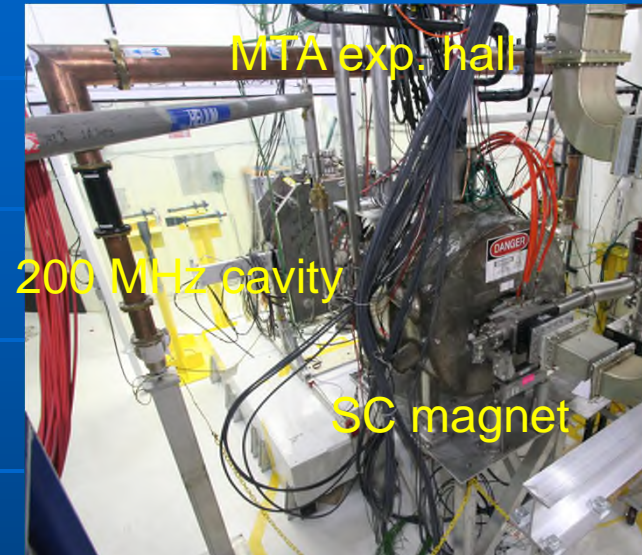
# Problem: B field effect on vacuum RF cavity





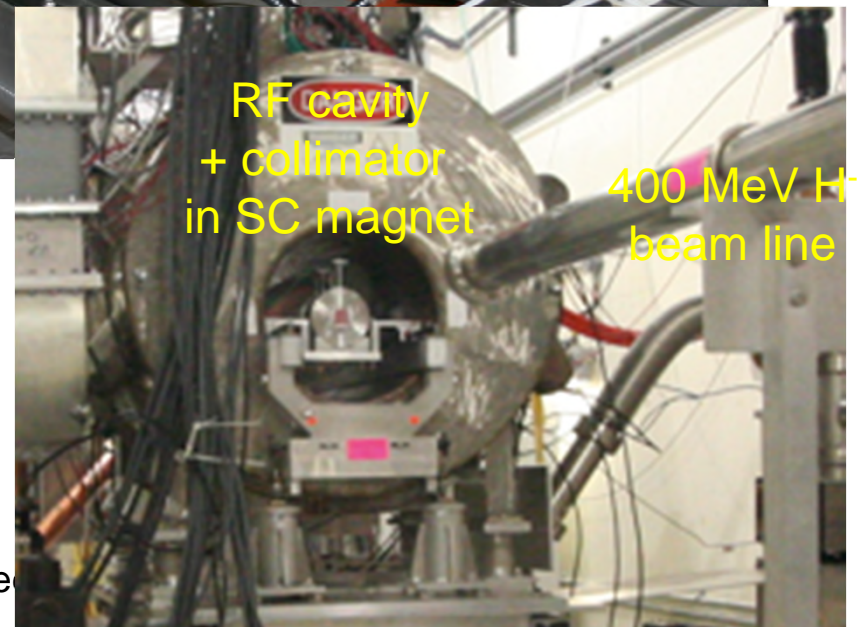
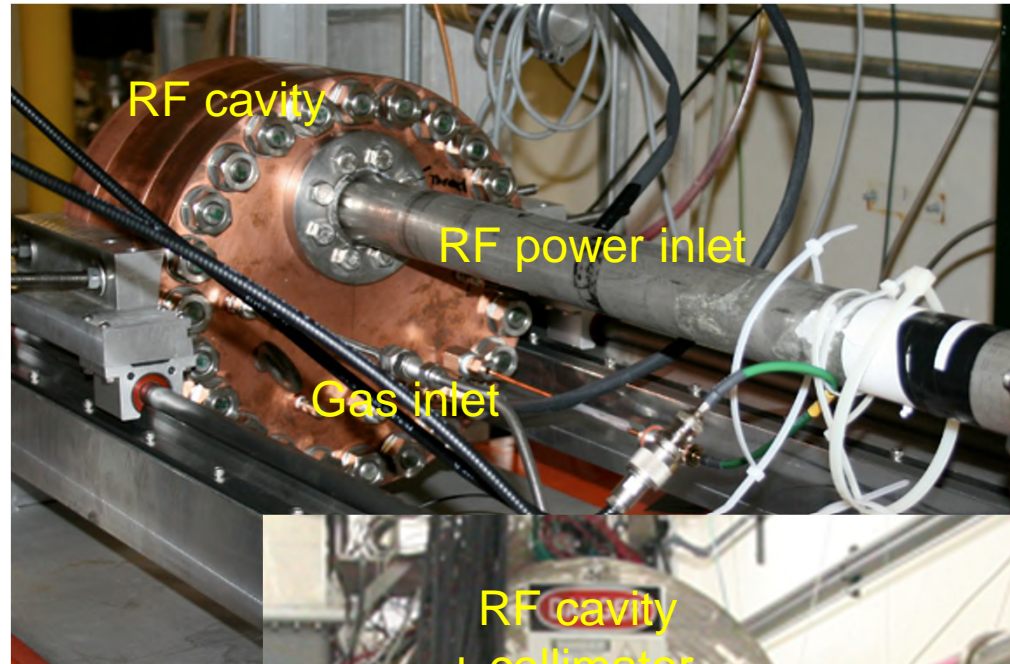
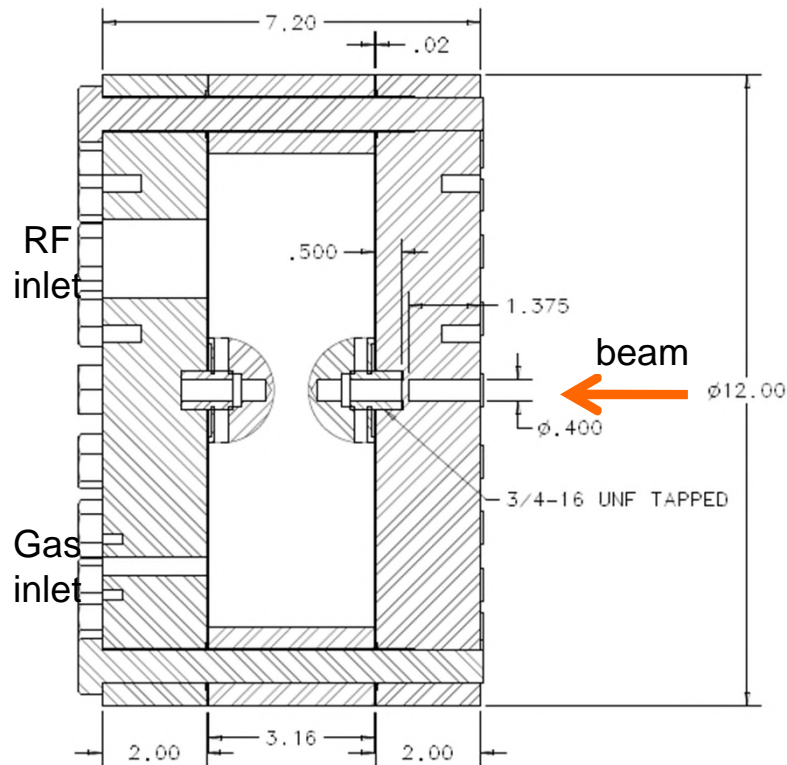
# Mucool Test Area (MTA) & work space

Multitask work space to study RF cavity under strong magnetic fields  
& by using intense  $H^-$  beams from Linac





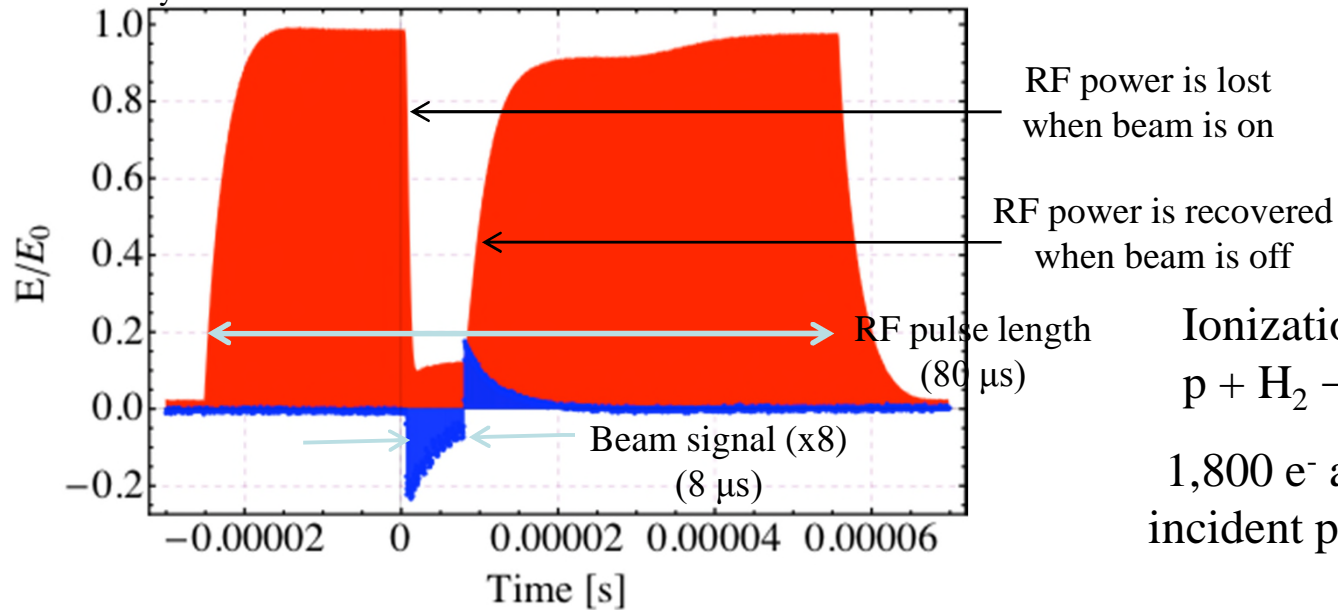
# First results HPRF cavity in beam



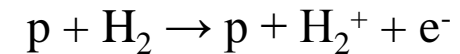
- 400 MeV H<sup>-</sup> beam
  - Beam pulse length 7.5  $\mu$ s
    - 5 ns bunch gap
    - $10^9$  H<sup>-</sup>/bunch
  - 18 % of transmission in collimator system
  - $1.8 \cdot 10^8$  protons/bunch reaches to the cavity
- 8/22/11 All Experimenters Meeting  
Yonehara

# Study interaction of intense beam with dense H2 in high gradient RF field

$\nu = 802 \text{ MHz}$   
Gas pressure = 950 psi  
Beam intensity =  $2 \cdot 10^8$  /bunch



Ionization process

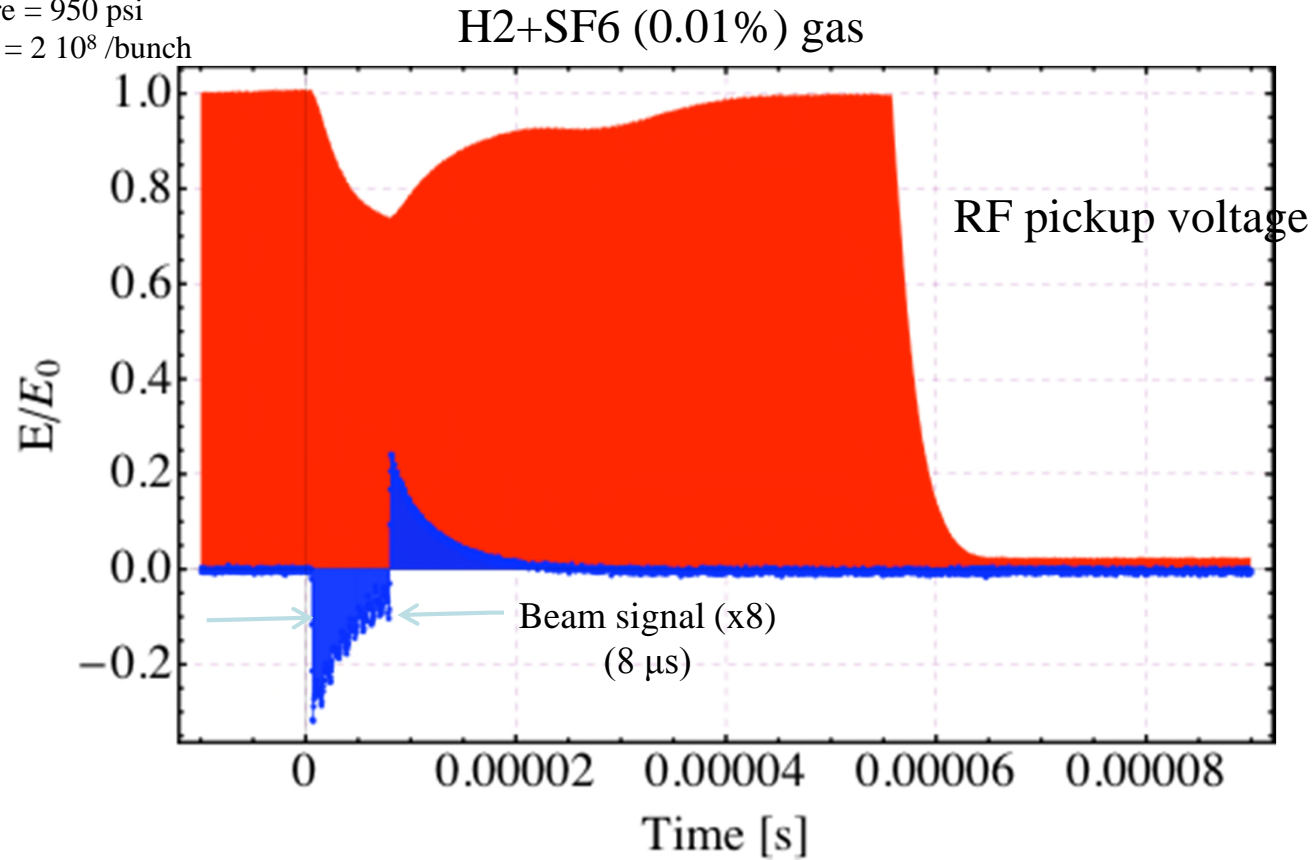


1,800  $e^-$  are generated by  
incident p @  $K = 400 \text{ MeV}$

Huge RF power lost due to electrons' power consumption  
**But, No Breakdown!!**

# Electronegative gas

$\nu = 802$  MHz  
H<sub>2</sub> + SF<sub>6</sub> (0.01 % condensation)  
Gas pressure = 950 psi  
Beam intensity =  $2 \cdot 10^8$  /bunch



SF<sub>6</sub> removes a residual electron  
Great improvement!





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# Conclusion/Suggestion

In the last 10 years,

theoretical and technological advances

in muon cooling and phase space manipulations,

supported by numerical simulations,

have improved the prospects

for a high-L high-E muon collider

The next COOL workshop

should have more on this subject.