Helical Cooling Channels for Muon Colliders

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Helical Cooling Channels, based on the same helical dipole Siberian Snake magnets used for spin control in synchrotrons and storage rings, are now proposed for almost all stages of muon beam cooling that are required for high luminosity muon colliders. We review the status of the theory, simulations, and technology development for the capture, phase rotation, 6-D ionization cooling, parametric-resonance ionization cooling, and reverse emittance exchange sections of one of the candidate scenarios for a high-luminosity, high-energy muon collider.

Please visit "Papers and Reports" and "LEMC Workshop" at http://www.muonsinc.com/
Overview

Slava Derbenev’s theory of the **Helical Cooling Channel** (HCC) is being exploited to provide a basis for the ionization cooling and emittance manipulation techniques needed for intense muon beams for energy frontier colliders.

- Since the last COOL meetings, a Muon Collider has become the most attractive path for an energy-frontier machine in the USA and is now seen on official Fermilab and DOE documents. (i.e. E of ILC too low, CLIC too power hungry)

- I will show steps that we now see as needed to provide muon beams for a high-luminosity, high energy (>10^{34}, ~3-5 TeV) Muon Collider that cold fit on the Fermilab site.

- I will describe the HCC aspects of the steps in more detail, concentrating on the newest technology developments
5 Steps for Muon Production & Cooling

1) A 4 MW proton beam hits a target in a 20 T solenoid field which tapers to ~2T

2) Pions decay to muons and are phase rotated and captured into a string of bunches in a straight solenoid configuration.

3) Muons enter successive HCC segments each with smaller dimensions (allowing higher magnetic fields and higher RF frequency), as the beam is cooled in 6d using H$_2$-filled RF cavities.
   - Adjustment of slip factor affords larger RF buckets for better matching
   - ~6 orders of magnitude 6-d emittance reduction in 300 m long HCC
   - Recent beam tests of a H$_2$-pressurized RF cavity are encouraging
   - Helical Solenoid invention for HCC being tested in models with HTS

4) Parametric Resonance Ionization Cooling is used for more transverse emittance reduction (10x in each plane) Derbenev!
   - Twin-Helix HCC with correlated optics a new invention (” & Morozov)

5) Reverse Emittance exchange with wedge absorbers reduces transverse emittance while longitudinal grows (to hourglass limit)
   (HCC theory also used for transitions between steps)
Compact Muon Collider
Fit in Fermilab campus

Muon Collider: 20??
Cost: Unknown
Energy: 0.5 ~ 4 TeV
Components: 10,000

CLIC: 20??
Cost: Estimate due in 2010
Energy: 0.5 ~ 3 TeV
Components: 260,000

Muon Collider Conceptual Layout

Project X
Accelerate hydrogen ions to 8 GeV using SRF technology.

Compressor Ring
Reduce size of beam.

Target
Collisions lead to muons with energy of about 200 MeV.

Muon Cooling
Reduce the transverse motion of the muons and create a tight beam.

Initial Acceleration
In a dozen turns, accelerate muons to 20 GeV.

Recirculating Linear Accelerator
In a number of turns, accelerate muons up to 2 TeV using SRF technology.

Collider Ring
Located 100 meters underground. Muons live long enough to make about 1000 turns.

Comparison of Particle Colliders
To reach higher and higher collision energies, scientists have built and proposed larger and larger machines.

LHC: 2009-present
Cost: US $4.6 billion
Energy: 14 TeV
Components: 11,000

ILC: 20??
Cost: US $8 billion in 2007
Energy: 0.5 TeV
Components: 38,000

VLHC: 20??
Cost: Unknown
Energy: 40 ~ 200 TeV

Numbers are taken from Nature 462, 260-261 (2009)
Principle of Ionization Cooling

• Each particle loses momentum by ionizing a low-Z absorber
• Only the longitudinal momentum is restored by RF cavities
• The angular divergence is reduced until limited by multiple scattering
• Successive applications of this principle with clever variations leads to small emittances for many applications
• Early work: Budker, Ado & Balbekov, Skrinsky & Parkhomchuk, Neuffer
The equation describing the rate of cooling is a balance between cooling (first term) and heating (second term):

$$\frac{d\varepsilon_n}{ds} = -\frac{1}{\beta^2} \frac{dE_\mu}{ds} \varepsilon_n + \frac{1}{\beta_\perp (0.014)^2} \frac{\beta^3}{2E_\mu m_\mu X_0}$$

Here $\varepsilon_n$ is the normalized emittance, $E_\mu$ is the muon energy in GeV, $dE_\mu/ds$ and $X_0$ are the energy loss and radiation length of the absorber medium, $\beta_\perp$ is the transverse beta-function of the magnetic channel, and $\beta$ is the particle velocity.

Bethe-Bloch
Moliere (with low Z mods)
Ionization Cooling is only transverse. To get 6D cooling, emittance exchange between transverse and longitudinal coordinates is needed.
6-Dimensional Cooling in a Continuous Absorber

- Helical cooling channel (HCC)
  - Continuous absorber for emittance exchange
  - Solenoid, transverse helical dipole and quadrupole fields
  - Helical dipoles known from Siberian Snakes
  - z- and time-independent Hamiltonian
  - Derbenev & Johnson, *Theory of HCC*, April/05 PRST-AB
Particle Motion in a Helical Magnet

Combined function magnet (invisible in this picture)
Solenoid + Helical dipole + Helical Quadrupole

Opposing radial forces
\[ F_{\text{h-dipole}} \approx p_z \times B_\perp; \quad b \equiv B_\perp \]
\[ F_{\text{solenoid}} \approx -p_\perp \times B_z; \quad B \equiv B_z \]

Transforming to the frame of the rotating helical dipole leads to a time and z – independent Hamiltonian

\[ b' \text{ added for stability and acceptance} \]
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Some Important Relationships

Hamiltonian Solution

\[ p(a) = \frac{\sqrt{1 + \kappa^2}}{k} \left[ B - \frac{1 + \kappa^2}{\kappa} b \right] \]

\[ k = 2\pi/\lambda \quad \kappa = ka \]

Equal cooling decrements

\[ q \equiv \frac{k_c}{k} - 1 = \beta \frac{1 + \kappa^2}{\sqrt{3 - \beta^2}} \]

\[ k_c = B\sqrt{1 + \kappa^2}/p \]

Longitudinal cooling only

\[ \hat{D} \equiv \frac{p}{a} \frac{da}{dp} = 2 \frac{1 + \kappa^2}{\kappa^2} \]

\[ q = 0 \]

~Momentum slip factor

\[ \eta = \frac{d}{d\gamma} \frac{\sqrt{1 + \kappa^2}}{\beta} = \frac{\sqrt{1 + \kappa^2}}{\gamma^3 \beta} \left( \frac{\kappa^2}{1 + \kappa^2} \hat{D} - \frac{1}{\gamma^2} \right) \]

\[ \frac{\kappa^2}{1 + \kappa^2} \hat{D} \sim \frac{1}{\gamma_{\text{transition}}^2} \]
Matching Between Step 2 Capture and Step 3 HCC

\[ A_{\text{bucket}} \approx \frac{16}{w_{rf}} \sqrt{\frac{eV'_{\max} \lambda_{RF} m_\mu c^2}{2\pi |\eta_H|}} \left[ \frac{1 - \sin(\varphi_s)}{1 + \sin(\varphi_s)} \right] \]

(1)

where

- the term in brackets is an approximation for the moving-bucket factor
- \( w_{rf} \) is the RF frequency in radians/second
- \( V'_{\max} \) is the maximum E-field voltage gradient
- \( \lambda_{RF} \) is the RF wavelength
- \( m_\mu \) is the mass of the muon
- \( \varphi_s \) is the synchronous particle RF phase, and
- \( \eta_H \) is the slip factor, derived in [1] for an HCC as:
Step 3: 6d Ionization Cooling w pressurized RF Cavities

Simulation parameters & results

Simulation has been made with analytical EM field expression in G4beamline

<table>
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<th>$z$ (unit)</th>
<th>$b$ (m)</th>
<th>$b'$ (T/m)</th>
<th>$b_z$ (T)</th>
<th>$\lambda$ (m)</th>
<th>$N$ (GHz)</th>
<th>$\varepsilon_T$ (mm rad)</th>
<th>$\varepsilon_L$ (mm)</th>
<th>$\varepsilon_{6D}$ (mm$^3$)</th>
<th>$\varepsilon$ (Transmission)</th>
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Other important parameters:
- RF $E_{peak} = 27$ MV/m
- Peak RF power = 10 to less than 0.2 MW/m
- 60 $\mu$m Be window at RF entrance
- GH2 pressure = 160 atm at 300 K

(optimization is on going, not covered in this talk)
Emittance Evolution in Homogeneous GH2 Filled HCC

- $10^6$ of 6D cooling factor is needed for MC
- HCC demonstrated 6D cooling factor $> 10^5$ with 60% transmission efficiency
- Additional cooling is needed (Parametric Ionization Cooling etc.)

![Graph showing emittance evolution](image-url)
Step 4: Parametric-resonance Ionization Cooling

Excite ½ integer parametric resonance (in Linac or ring)
- Like vertical rigid pendulum or ½-integer extraction
- Elliptical phase space motion becomes hyperbolic
- Use $xx'=$const to reduce $x$, increase $x'$
- Use IC to reduce $x'$

Detuning issues being addressed (chromatic and spherical aberrations, space-charge tune spread). Simulations underway.

Smaller beams from 6D HCC cooling essential for this to work!
PIC using a Twin-Helix

- PIC uses a half-integer parametric resonance.
- Twin-Helix is superposition of 2 opposite helicity HCCs (similar to getting linear polarization from two circularly polarized light beams)
- correlated h and v betatron periods -> simultaneous focusing in both planes.
- Energy absorber plate & energy-restoring RF cavity at beam focal point
- IC limits the angular spread, parametric resonance reduces beam spot size.
- The achievable normalized equilibrium transverse emittance is given by [2]

\[ \varepsilon_{\perp}^n = \frac{\sqrt{3}}{4\beta} (Z+1) \frac{m_e}{m_\mu} w , \]  

(1)

- \( w \) is the average absorber thickness in the beam direction. Compared to conventional IC, the this emittance is reduced by at least an order of magnitude by:

\[ \frac{\pi w}{\sqrt{3} \lambda_x} = \frac{\pi \gamma'_\text{acc}}{2\sqrt{3} \gamma'_\text{abs}} , \]  

(2)

- where \( \lambda_x \) is the period of the horizontal betatron oscillations and \( \gamma'_\text{acc} \) and \( \gamma'_\text{abs} \) are the RF acceleration and intrinsic absorber energy loss rates, respectively.
PIC using Twin-Helix (cont.)

Figure 3: ±200 mrad-μ tracks distributed over ±200 mrad in the horizontal plane before (top) and after (bottom) compensation of the horizontal spherical aberration.

Figure 5: Beam smear due to spherical aberrations after 2 helix periods. G4beamline simulation is compared to various-order COSY Infinity calculations.
Reverse Emittance Exchange, Coalescing

- $p(\text{cooling})=100\text{MeV/c}, p(\text{colliding})=2.5 \text{ TeV/c} \Rightarrow \text{room in } \Delta p/p \text{ space}
- Shrink the transverse dimensions of a muon beam to increase the luminosity of a muon collider using wedge absorbers
- Allow bunch length to increase to size of low beta
- Low energy space charge, beam loading, wake fields problems avoided
- 20 GeV Bunch coalescing in a ring Neutrino factory and muon collider now have a common path
Hardware Development

• Helical Solenoid invention to get HCC fields
  – 4-coil NbTi HS model tested (1st 6d HCC segment)
  – 6-coil YBCO HS model tested (last 6d HCC segment)

• High Pressure H₂ Cavity development
  – Test cell shows no HV max dependence on external B
  – First beam tests show agreement with models
    • No RF breakdown
    • Ionization electrons move far enough to heat H₂ – reduce Q
    • Mitigated with 0.01% SF₆ dopant – will allow MC application
Helical Magnet
Helical solenoid magnet

- Coil center follows on the helical reference track
- It generates proper helical dipole + field gradient
- By adding a solenoid coil, it tunes all three field components (b, b', bz)
- By modulating the coil position, it can make a beam adapter to connect between straight and helical magnet sections.
- Helical solenoid magnet generates more uniform field than analytical field.
- It means that helical solenoid magnet has larger acceptance than analytical one.

AAC2010, K. Yonehara, 17
Incorporate RF Cavity into Helical Magnet

- Plastic model to demonstrate integrating RF into helical magnet
- Segment RF cavity and helical solenoid coil
- Red: RF cavity
- White: Helical magnet

- 4-coil helical solenoid magnet to study support structure, splice ground insulation, field quality test, etc...

- CAD drawing to show one helical period
This is a 24 cavity per period 400 MHz design

- Approximate Diameter at the coax flange is 40”
MuCool Test Area (MTA)

Pressure barrier

Wave guide to coax adapter

Mark II Test Cell

Solenoid

5T
HPRF Test Cell Measurements in MTA

- Paschen curve verified
- Maximum gradient limited by breakdown of metal.
- Cu and Be have same breakdown limits (~50 MV/m), Mo(~63MV/m), W(~75MV/m).
- Results show no B dependence, much different metallic breakdown than for vacuum cavities.
- **Need beam tests to prove HPRF works.**
Problem: B field effect on vacuum RF cavity

Data were taken in an 805 MHz vacuum pillbox cavity.
Mucool Test Area (MTA) & work space
Multitask work space to study RF cavity under strong magnetic fields & by using intense H⁻ beams from Linac
First results HPRF cavity in beam

- 400 MeV H$^+$ beam
- Beam pulse length 7.5 μs
- 5 ns bunch gap
- $10^9$ H$/\text{bunch}$
- 18% of transmission in collimator system
- $1.8 \times 10^8$ protons/bunch reaches to the cavity

8/22/11 All Experimenters Meeting, K. Yonehara

400 MeV H$^+$ beam line

RF cavity + collimator in SC magnet

RF power inlet

Gas inlet

RF cavity

Diagram of cavity and beam line.
Study interaction of intense beam with dense H\(_2\) in high gradient RF field

- Beam signal (x8) (8 μs)
- RF pulse length (80 μs)
- Beam intensity = 2 \times 10^8 / bunch
- ν = 802 MHz
- Gas pressure = 950 psi

**Ionization process**
\[ p + H\_2 \rightarrow p + H\_2^+ + e^- \]

1,800 e\(^-\) are generated by incident p @ K = 400 MeV

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**Huge RF power lost due to electrons’ power consumption**
**But, No Breakdown!!**
Electronegative gas

H₂ + SF₆ (0.01% condensation)
Gas pressure = 950 psi
Beam intensity = 2 \times 10^8 / bunch

ν = 802 MHz

H₂+SF₆ (0.01%) gas

RF pickup voltage

SF₆ removes a residual electron
Great improvement!
Conclusion/Suggestion

In the last 10 years,

theoretical and technological advances

in muon cooling and phase space manipulations,

supported by numerical simulations,

have improved the prospects

for a high-L high-E muon collider

The next COOL workshop

should have more on this subject.