

HELICAL COOLING CHANNEL DEVELOPMENTS*

R. P. Johnson[#], C. Yoshikawa, Muons, Inc., Batavia, IL, U.S.A.
 Y. S. Derbenev, V. S. Morozov, JLab, Newport News, VA, USA

Abstract

Several beam phase space manipulation and cooling stages are required to provide the extraordinary reduction of emittances required for an energy-frontier muon collider. From the pion production target, the pions and their decay muons must be collected into RF bunches, rotated in phase space to reduce momentum spread, cooled in 6 dimensions by 6 orders of magnitude, cooled in each transverse plane by another order of magnitude, and accelerated and matched to the RF system used to accelerate the muons to the final collider energy. Many of these stages have Helical Cooling Channel (HCC) [1] solutions based on superimposed solenoid, helical dipole, and helical quadrupole magnetic fields. The HCC was invented to achieve efficient ionization cooling with continuous emittance exchange. We first describe the essential HCC equations and describe how they can be applied for longitudinal and transverse emittance matching. We then describe simulations of HCC segments with a continuous gaseous hydrogen energy absorber suitable for basic 6d cooling as well as new results of related pressurized RF cavity beam tests. We then describe a new and creative application of the theory and use of the HCC that has been developed for Parametric-resonance Ionization Cooling (PIC), and the phase space matching needed for transitions between various cooling channel subsystems

INTRODUCTION

Considerable progress has been made in developing promising subsystems for muon beam cooling channels to provide the extraordinary reduction of emittances required for an energy-frontier muon collider. A high-performance front end from the target to the cooling systems has been designed and simulated [2], and many advances in theory, simulation codes, and hardware development have been achieved, especially regarding the 6d HCC described below. However, the HCC theory is not necessarily restricted to channels having solenoid fields. For example, the Twin Helix [3], which is also described below, does not possess a solenoid field component. The HCC theory and its extensions can describe a wide variety of beam dynamics and is thus well suited to provide the platform from which matching sections can be designed. We now review the theory of the HCC and examine how it can be used for emittance matching between cooling segments that have been independently developed.

Basic Helical Cooling Channel

In the HCC, a solenoid field is augmented with a transverse helical field that provides a constant dispersion

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[#]rol@muonsinc.com

along the channel as necessary for the emittance exchange that allows longitudinal cooling. The Hamiltonian that describes motion in this magnetic configuration is easily solved by a transform into the frame of the rotating helical magnet, where it is seen that the addition of a helical quadrupole field provides beam stability over a very large acceptance.

The helical dipole magnet creates an outward radial force due to the longitudinal momentum of the particle while the solenoid magnet creates an inward radial force due to the transverse momentum of the particle:

$$F_{h-dipole} \approx p_z \times b; \quad b \equiv B_{\perp}; \quad F_{solenoid} \approx -p_{\perp} \times B; \quad B \equiv B_z,$$

where B is the field of the solenoid, the axis of which defines the z axis and b is the field of the transverse helical dipole. By moving to the rotating frame of the helical fields, a time- and z -independent Hamiltonian can be formed to derive the beam stability and cooling behaviour [1]. The motion of particles around the equilibrium orbit is shown schematically in Figure 1.

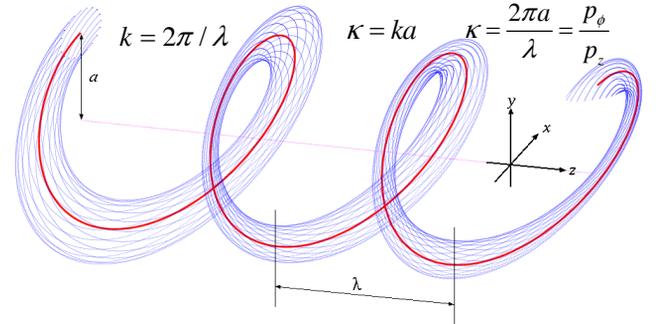


Figure 1: Schematic of beam motion in a HCC.

The equilibrium orbit shown in red follows the equation that is the Hamiltonian solution:

$$p(a) = \frac{\sqrt{1+\kappa^2}}{k} \left[B - \frac{1+\kappa^2}{\kappa} b \right] \quad (1)$$

The dispersion factor \hat{D} can be expressed in terms of the field components B , b , and the transverse magnetic field radial gradient $\partial b / \partial a$ on the particle's orbit:

$$\hat{D} = \frac{p}{a} \frac{da}{dp} = \left(\frac{a}{p} \frac{dp}{da} \right)^{-1}; \quad \hat{D}^{-1} = \frac{\kappa^2 + (1-\kappa^2)q}{1+\kappa^2} + g; \quad g \equiv \frac{-(1+\kappa^2)^{3/2}}{pk^2} \frac{\partial b}{\partial a},$$

where g is the effective field index at the periodic orbit.

The magnetic field ratio on the equilibrium trajectory satisfies the condition

$$\frac{b}{B} = \frac{\kappa}{1+\kappa^2} \left(1 - \frac{k}{k_c} \right) = \frac{\kappa}{1+\kappa^2} \left(\frac{q}{q+1} \right), \quad \text{where } q \equiv \frac{k_c}{k} - 1.$$

For stability, the following condition has to be satisfied

$$0 < G \equiv (q-g)\hat{D}^{-1} < R^2 \equiv \frac{1}{4} \left(1 + \frac{q^2}{1+\kappa^2} \right)^2. \quad (2)$$

Use of a continuous homogeneous absorber takes advantage of a positive dispersion along the entire cooling

path, a condition that has been shown to exist for an appropriately designed helical dipole channel. We have also shown that this condition is compatible with stable periodic orbits.

HCC LONGITUDINAL TRANSITIONS

Longitudinal emittance matching in transition sections can be facilitated, subject to simultaneously satisfying stability criterion (2), by continuously varying the RF bucket area to match RF parameters from one cooling section to the next. The RF bucket area is given by:

$$A_{bucket} \cong \frac{16}{w_{rf}} \sqrt{\frac{eV'_{max} \lambda_{RF} m_{\mu} c^2}{2\pi |\eta_H|}} \left[\frac{1 - \sin(\phi_s)}{1 + \sin(\phi_s)} \right] \quad (3)$$

Where the term in brackets is an approximation for the moving-bucket factor, w_{rf} is the RF frequency in radians/second, V'_{max} is the maximum E-field voltage gradient, λ_{RF} is the RF wavelength, m_{μ} is the mass of the muon, ϕ_s is the synchronous particle RF phase, and η_H is the translational mobility or slip factor, derived in [1] for the HCC as:

$$\eta_H = \frac{\sqrt{1 + \kappa^2}}{\gamma \beta^3} \left(\frac{\kappa^2}{1 + \kappa^2} \hat{D} - \frac{1}{\gamma^2} \right) \quad (4)$$

where $1/\gamma_T^2 = (\kappa^2 / (1 + \kappa^2)) \hat{D}$ and the dispersion factor \hat{D} relates to apparatus quantities and design momentum via:

$$\hat{D}^{-1} = \frac{a}{p} \frac{dp}{da} = \frac{\kappa^2 + (1 - \kappa^2) [B \sqrt{1 + \kappa^2} / pk - 1]}{1 + \kappa^2} - \frac{(1 + \kappa^2)^{3/2}}{pk^2} \frac{\partial b}{\partial a} \quad (5)$$

where p is the reference momentum; a reference radius, $\kappa = p/p_z = \text{helix pitch}$, B the solenoid B_z , $k = 2\pi/\lambda$; λ is the helix period, and $\partial b/\partial a$ the quadrupole component.

Thus, in matching between sections with different longitudinal dynamics, the RF bucket area can be continually manipulated by varying any of the following: the gradient of the dipole field ($\partial b/\partial a$), the reference momentum (p), the accelerating phase (ϕ_s), the transition energy γ , or the maximum gradient (V'_{max}).

HCC TRANSVERSE TRANSITIONS

In the case of transverse matching, equation (1) would be used to compute the evolution of the solenoid B and helical dipole b fields between cooling segments, where $\partial b/\partial a$ is subject to constraint (2).

G4beamline HCC 6d Cooling Simulations

The analytic relationships above have been used to guide GEANT [4] simulations using G4beamline [5] and ICOOL [6]. Simulation results [7] show a 190,000-fold 6d emittance reduction for a series of eight 250 MeV/c HCC segments, where the reference orbit radii are decreased and fields are increased as the beam cools. Longitudinal and transverse emittances at the end of each HCC segment are shown in Table 1 and are also plotted as red dots in Figure 2. The peak RF field is 27 MV/m and 60 μm Be windows make the cavities true pillboxes. The

hydrogen gas pressure is 160 atm at 300 K. Forty per cent of the beam is lost in the 303 m long channel. About 22% of the beam is lost due to muon decay while the rest of the loss is due to emittance mismatches, which can be improved. A new Helical Solenoid (HS) magnet design [8] that uses simple offset coils to generate the required solenoid, dipole and quadrupole field components, was invented and superconducting prototypes are being tested.

Table 1: The parameters of the 8 HCC cooling channel segments used by Yonehara [7].

| # | Z | b | b' | b _z | λ | ν | ϵ_T | ϵ_L | ϵ_{6D} | ϵ |
|---|-----|-----|------|----------------|-----------|-------|---------------|--------------|-----------------|------------|
| | m | T | T/m | T | m | GHz | μm | mm | mm^3 | |
| 1 | 0 | 1.3 | -0.5 | -4.2 | 1.0 | 0.325 | 20.4 | 42.8 | 12900 | 1.0 |
| 2 | 40 | 1.3 | -0.5 | -4.2 | 1.0 | 0.325 | 5.97 | 19.7 | 415.9 | 0.92 |
| 3 | 49 | 1.4 | -0.6 | -4.8 | 0.9 | 0.325 | 4.01 | 15.0 | 10.8 | 0.86 |
| 4 | 129 | 1.7 | -0.8 | -5.2 | 0.8 | 0.325 | 1.02 | 4.8 | 2.0 | 0.73 |
| 5 | 219 | 2.6 | -2.0 | -8.5 | 0.5 | 0.65 | 0.58 | 2.1 | 3.2 | 0.66 |
| 6 | 243 | 3.2 | -3.1 | -9.8 | 0.4 | 0.65 | 0.42 | 1.3 | 0.14 | 0.64 |
| 7 | 273 | 4.3 | -5.6 | -14.1 | 0.3 | 0.65 | 0.32 | 1.0 | 0.08 | 0.62 |
| 8 | 303 | 4.3 | -5.6 | -14.1 | 0.3 | 1.3 | 0.32 | 1.0 | 0.07 | 0.60 |

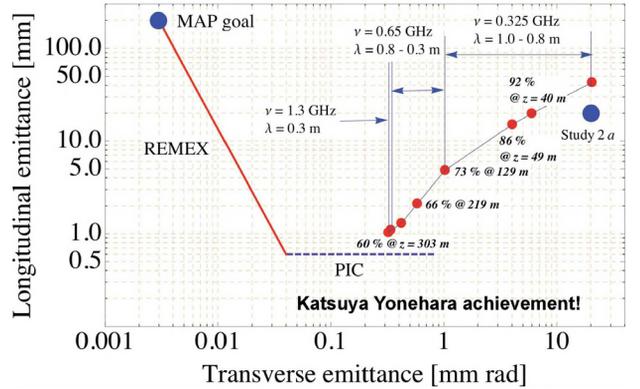


Figure 2: Fernow-Neuffer plot of the G4beamline simulated emittance evolution shown in Table 1.

RF BEAM TESTS

The simple idea that emittance exchange can occur in a practical homogeneous absorber without shaped edges followed from the observation that RF cavities pressurized with a low Z gas are possible [9]. Recent experiments with the proton beam at the Fermilab MuCool Test Area have verified that such cavities will not suffer RF breakdown in a beam. Other parameters have been measured that verify many features of models of the cavities as well. The recombination rate of the ionized electrons produced by the beam is fast enough that the RF amplitude is likely to be adequate for the muon beam that will be less than 100 ns long. Figure 3 shows the first tests of a pressurized RF cavity being hit by a charged particle beam. The voltage drops due to the absorption of energy by the ionized electrons and to the change in impedance of the cavity due to the plasma causing the power from the klystron to be reflected.

The use of a 1 part in 10,000 SF₆ dopant has been shown to largely mitigate the drop in RF voltage caused by the motion of ionized electrons in the RF field that

heat the gas. Namely, the electrons attach to the SF₆ and the large mass of the resulting ion inhibits its motion in the RF field and its energy transfer to the hydrogen gas. Figure 3 shows the influence of a proton beam in a high pressure gas filled cavity without the aid of SF₆ to inhibit the effect of electrons, while Figure 4 illustrates the improvement by addition of the SF₆ dopant. Preliminary models indicate that for short bunch trains (< 100 ns), the doped RF cavity stability allows HCC use as in Figure 2.

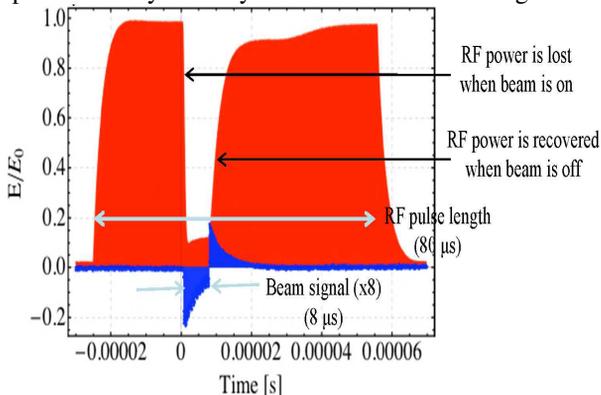


Figure 3: Beam influence on 950 psi hydrogen-filled RF test cell. Red is the 802 MHz RF envelope which rises to $E_0 \sim 20$ MV/m. Blue is the toroid signal from the beam from the 400 MeV H- Linac to the MTA.

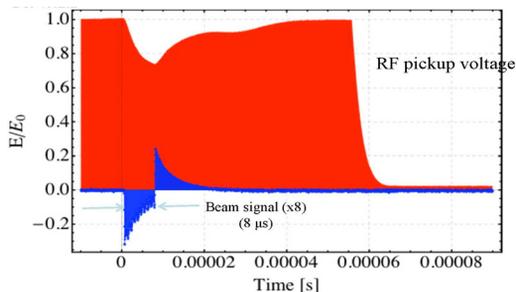


Figure 4: Beam influence on 950 psi hydrogen-filled RF test cell with a 0.01% dopant addition of SF₆. The gradient drops by about 30% during the 8 μs beam pulse, where some of that is due to the impedance mismatch caused by the plasma that in turn causes most of the klystron power to be reflected.

OTHER HCC SEGMENT EXAMPLES

In all cases we can imagine, the pions are produced in a strong solenoid that becomes weaker along the channel so that transverse momenta are folded forward. The decay muon phase space must be matched to the acceptance of the HCC for 6d cooling, a matching that goes from solenoid with no helical dipole to an HCC.

At the end of a 6d HCC segment as shown in Figure 2, you could either match into a strong field solenoid channel for extreme cooling or to a PIC channel. While the former is an example of HCC to solenoid transition, the PIC channel being discussed is even more of a challenge in that it is made up of two helical magnets with no solenoid field component.

PIC and the Twin-Helix Example

Parametric-resonance Ionization Cooling (PIC) requires a half integer resonance to be induced in a ring or beam-line such that the normal elliptical motion of particles in $x-x'$ phase space becomes hyperbolic, with particles moving to smaller x and larger x' as they pass down the beamline. (This is almost identical to the technique used for half integer extraction from a synchrotron where the hyperbolic trajectories go to small x' and larger x to pass the wires of an extraction septum.) Thin absorbers placed at the focal points of the channel then cool the angular divergence of the beam by the usual ionization cooling mechanism, where each absorber is followed by RF cavities. Thus, in PIC the phase space area is reduced in x due to the dynamics of the parametric resonance and x' is reduced or constrained by ionization cooling.

The main constraint in parametric-resonance ionization cooling channel design is the requirement to combine low dispersion at the wedge absorber plates (for emittance exchange to compensate energy straggling) with large dispersion in the space between plates (where sextupoles can be placed to compensate for chromatic aberration). The desired large angular divergence of + 200 mr at the absorber plates also implies significant corrections for spherical aberrations and the next step in the development of this channel is to compensate for aberrations. The horizontal and vertical optics also have to be correlated such that there must be places where each plane has a focus at the absorber plates.

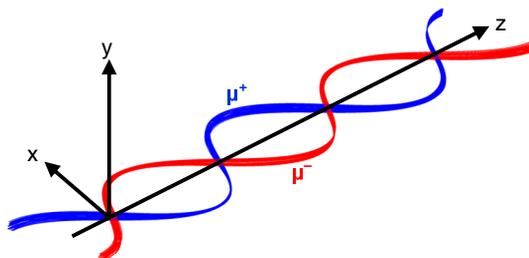


Figure 5: G4beamline display of planar trajectories of muons of both signs in the Twin Helix magnet system.

Perhaps one of the more interesting recent cooling channel developments follows from the realization that two helical dipole magnetic fields of opposite helicity can be superimposed to create a purely vertical magnetic field that varies sinusoidally in amplitude along the magnet axis. This magnet system, called the Twin-Helix [10], produces muon orbits as shown in Figure 5, with a possible coil configuration shown in Figure 6. Here the horizontal and vertical betatron wavelengths differ by a factor of two so energy loss wedge absorbers are placed at every focal point in one plane where the same position corresponds to every other focal point in the other plane. The dispersion is also correlated in that it is small but not zero at the focal points and RF cavities, but large between absorbers where aberration correction magnets can be located.

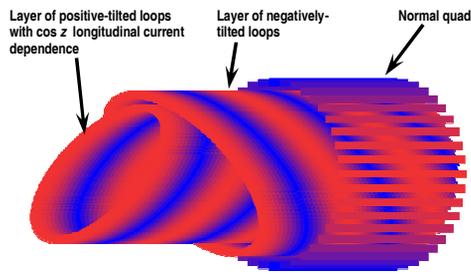


Figure 6: Possible coil configuration for the Twin Helix magnet system.

Space Charge

At some points in the muon cooling channel, the bunching could be extreme, where some estimates are as high as 10^{13} muons in an RF bucket, and the energy low enough to anticipate that space charge effects can be problematic. Through a project supported by the SBIR-STTR program, space charge calculation capability has been added to G4beamline. The transitions between cooling channel segments will incorporate the appropriate criteria to manage any space charge tune shifts.

Gas, Vacuum, and Liquid transitions

Transitions between cooling channel segments may also involve windows or pressure barriers to separate vacuum, pressurized gas, or liquid hydrogen regions. It is quite likely that the beam sizes or beta functions at the window locations will have to be included in the constraints for the matching regions in order to reduce emittance growth from multiple scattering or to help solve engineering problems.

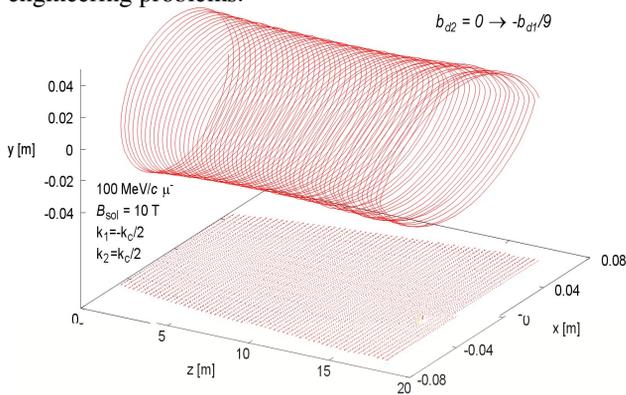


Figure 7: Adiabatic turn-on of the secondary helical dipole. The cyclotron wave number is $k_c = qB/cp_z$. The wave numbers k_1 and k_2 refer to the primary and secondary dipole fields, respectively.

As for creating the matching sections, one approach is an adiabatic turn-on of various components of an HCC. A demonstration of this is shown in Figure 7, where a particle is initially in an HCC that consists of a single helical dipole component. The desire here is to match into another HCC that consists of two helical dipoles, the second of which has a magnetic strength of $-1/9$ of the primary dipole, and what is shown is an adiabatic turn on

of that second helical dipole component. Note how the initially circular orbit transforms into an elliptical trajectory.

Since a solenoid may be thought of as a special case of an HCC with the dipole component turned off, this adiabatic approach seems promising to match between a solenoid and any type of helical channel. Furthermore, the HCC theory may be extended to segments without solenoid field components such as the Twin-Helix.

CONCLUSIONS

The HCC theory and its extensions can be used to solve a wide variety of emittance manipulation and beam cooling problems that are needed to create intense muon beams suitable for a collider. A new use under development is to provide the emittance matching sections between cooling section segments which have very different parameters. Helical solenoid engineering solutions for HCC fields and recent proof of principle hydrogen pressurized RF cavity experiments with intense beams give confidence that practical, complete muon cooling designs will enable an energy frontier collider.

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