Transverse to Longitudinal Emittance Exchange Results

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If I forgot your name - please accept my apologies!
EEX Papers at this conference

- **Fermilab**
  - FR5PFP020 - Emittance Exchange at the A0 Photoinjector
  - TH5RFP042 - Bunch Length Monitoring at the A0 Photoinjector Using a Quasi-Optical Schottky Detector

- **ANL**
  - TH5RFP005 - Measurement of the 4D Transverse Phase Space Distribution from an RF Photoinjector at the AWA

- **NIU**
  - FR5PFP039 - Verification of the AWA Photoinjector Beam Parameters Required for a Transverse-to-Longitudinal Emittance Exchange Experiment

- **Others**
  - TH5RFP040 - Resonant-Cavity Diagnostics for an Emittance Exchange Experiment
Transverse to Longitudinal Emittance Exchange - What and Why?

- The idea of Transverse to Longitudinal Emittance Exchange (EEX) is simple. Take a beam with emittances \((\varepsilon_x, \varepsilon_y, \varepsilon_z)\) and make a beam with emittances \((\varepsilon_z, \varepsilon_y, \varepsilon_x)\).

- Why?
  - Basic and unique beam dynamics manipulation
  - FEL’s
    - Possibility of a smaller transverse emittances gives a shorter gain length.
    - Larger longitudinal emittance might stabilize against instabilities
  - This phase space manipulation could have application in a linear collider
    - Possibly can combine EEX with a flat beam transform to produce the proper beam at the main linac entrance without an electron damping ring.
Transverse to Longitudinal Emittance Exchange - How?

- There have been two proposals for EEX in a linac
  1. Use a deflecting cavity in the middle of a chicane (Cornacchia and Emma, 2002)
  2. Use a deflecting cavity in the middle of two doglegs (Kim and Sessler, 2006)
     1. Emma, et.al. in 2006 combined this scheme with a round to flat beam transformer as well.
- Both FNAL and ANL use the Kim and Sessler scheme.
- Incoming beam is manipulated to have the appropriate transverse and longitudinal phase ellipses
- First dogleg provides dispersion at DMC.
- The deflecting cavity gives a longitudinal position dependant transverse kick and a transverse position dependant momentum kick.
- The second dogleg couples the remaining correlations to finish the exchange.
How does the exchange work??

- The transverse-longitudinal transport matrix $R$, and beam matrix $\sigma$ look like (in 2x2 block mode)

\[
R = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_z \end{pmatrix}
\]

- The beam matrix after the transport is given by

\[
\sigma_2 = R \sigma_1 R^T
\]

- If the $R$ matrix can be made to look like

\[
R = \begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix}
\]

- Then the beam matrix looks like

\[
\sigma_2 = \begin{pmatrix} B \sigma_z B^T & 0 \\ 0 & C \sigma_x C^T \end{pmatrix}
\]

New Horizontal Emittance is the old longitudinal emittance

New Longitudinal Emittance is the old Horizontal emittance
How does the exchange work?

- Assume that the beamline consists of a before cavity section, a DMC, and an after cavity section.

\[ R = M^{ac} M^{cav} M^{bc} \]

- Assume that the before cavity section produces some dispersion, \( \eta \), with a slope \( \eta' \).
- Assume that the cavity is a zero length element
  - What does the cavity strength need to be?

\[ k = \frac{eV_0 \omega}{E_c} = -\frac{1}{\eta} \]

- What are the needed properties for the after cavity section?

\[ \begin{pmatrix}
M^{ac}_{16} \\
M^{ac}_{26}
\end{pmatrix} = \begin{pmatrix}
M^{ac}_{11} & M^{ac}_{12} \\
M^{ac}_{21} & M^{ac}_{22}
\end{pmatrix} \begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} \]

- These equations come out of nothing more than the symplectic condition and the condition that the A and D blocks of the R matrix are all zeros.
- Note: The vertical emittance is unaffected by the transformation.
Fly's in the Ointment

- There are many effects that may leave residual coupling, dilute, or obscure the emittance exchange.
  - Linear Flies - can lead to residual coupling of the emittances, leading to an emittance increase
    - I've assumed an infinitely thin cavity, a finite length cavity will leave residual coupling
    - Building an imperfect beamline such as using a chicane vs. a double dogleg as Cornacchia and Emma pointed out.
    - Incorrect cavity strength - too strong is as bad as too weak.
  These can be minimized or eliminated by manipulating the incoming beam phase spaces
  - Ugly Flies - these can blow up the emittances, possibly washing out the effect of the exchange
    - Space charge
    - Coherent Synchrotron Radiation
  These can be minimized by lowering the beam charge.
Watching the Exchange - The Fermilab experiment

Input to the EEX line

Horizontal Phase Space

Longitudinal Phase Space
Watching the Exchange - The Fermilab experiment

Input to the EEX line
Before Dipole 2

Horizontal Phase Space

Longitudinal Phase Space
Watching the Exchange - The Fermilab experiment

Input to the EEX line
Before Dipole 2
Before DMC
Watching the Exchange - The Fermilab experiment

Input to the EEX line
Before Dipole 2
Before DMC
After DMC
Watching the Exchange - The Fermilab experiment

Input to the EEX line
Before Dipole 2
Before DMC
After DMC
Before Dipole 4
Watching the Exchange - The Fermilab experiment

Input to the EEX line
Before Dipole 2
Before DMC
After DMC
Before Dipole 4
Exchange Complete
The Experiments

- Fermilab
  - Fermilab’s A0 Photoinjector is exchanging a large longitudinal emittance with a small transverse emittance.

- ANL
  - Argonne’s AWA is exchanging a small longitudinal emittance with a large transverse emittance.
Fermilab's AO Photoinjector
- L band 1.5 cell NC RF gun with Cs$_2$Te photocathode
  - 35 MV/m maximum cathode gradient
- TESLA technology accelerating cavity
  - 12 MV/m accelerating gradient
- Round to Flat beam transformer
- Transverse to Longitudinal Emittance Exchange Beamline
- Quadrupole transport channel
- User experimental area
Beam Parameters

- 16 MeV total energy
- $\Delta p/p \approx 0.1\% @ 16\text{MeV}$ (250 pC)
- Bunch length $\approx 0.75\text{ mm}$ (250 pC)
- $\gamma c_z \approx 20\text{ mm-mrad}$ (RMS @ 250 pC)
- $\gamma c_x, \gamma c_y \approx 5\text{ mm-mrad}$ (RMS @ 250 pC)
Preliminary investigations showed encouraging results. For instance, as we increased the TM$_{110}$ cavity strength we saw a reduction in momentum spread...
Early EEX Signature from Spectrometer

Spectrometer Screen

Cavity: OFF

~ 550keV
Early EEX Signature from Spectrometer

\[ \sim 550\text{keV} \]
Early EEX Signature from Spectrometer

\[ \sim 550\text{keV} \]

Spectrometer Screen

Cavity 20%
Early EEX Signature from Spectrometer

\[ \sim 550\text{keV} \]
Early EEX Signature from Spectrometer

~ 550keV

Spectrometer Screen

Cavity 40%
Early EEX Signature from Spectrometer

~ 550keV

Spectrometer Screen

Cavity 50%
Early EEX Signature from Spectrometer

~ 550keV

Spectrometer Screen

Cavity 60%
Early EEX Signature from Spectrometer

Cavity 70%

~ 550keV
Early EEX Signature from Spectrometer

~ 550keV

Cavity 80%
Early EEX Signature from Spectrometer

~ 550 keV

Spectrometer Screen

Cavity 100%
Measuring the $R_{14}$ and $R_{34}$ through the EEX line

Evolution of the beam trajectory as the cavity strength is increased, and energy is changed

$k = 0 \% k_{\text{ideal}}$

Lines: Model
Dots: Horizontal BPM measured difference data

$\delta P = \pm 1.05 \%$ in 0.35 $\%$ increments
Measuring the $R_{14}$ and $R_{34}$ through the EEX line

Evolution of the beam trajectory as the cavity strength is increased, and energy is changed.

$k = 25\% \left( k_{\text{ideal}} \right)$

Horizontal BPM Difference (mm)

Vertical BPM Difference (mm)

Lines: Model
Dots: Horizontal BPM measured difference data

$\delta P = \pm 1.05\%$ in 0.35\% increments
Measuring the $R_{14}$ and $R_{34}$ through the EEX line

Evolution of the beam trajectory as the cavity strength is increased, and energy is changed

$k = 50 \% k_{\text{ideal}}$

$\delta P = \pm 1.05 \% \text{ in 0.35 \% increments}$
Measuring the $R_{14}$ and $R_{34}$ through the EEX line

Evolution of the beam trajectory as the cavity strength is increased, and energy is changed

$k = 75\% k_{\text{ideal}}$

Horizontal BPM Difference (mm)

Vertical BPM Difference (mm)

Lines: Model
Dots: Horizontal BPM measured difference data

$\delta P = \pm 1.05\%$ in 0.35\% increments
Measuring the $R_{14}$ and $R_{34}$ through the EEX line

Evolution of the beam trajectory as the cavity strength is increased, and energy is changed.

$k=90\% k_{\text{ideal}}$

Lines: Model
Dots: Horizontal BPM measured difference data

$\delta P = \pm 1.05\%$ in 0.35\% increments
Measuring the $R_{14}$ and $R_{34}$ through the EEX line

Evolution of the beam trajectory as the cavity strength is increased, and energy is changed.

$k=90\% k_{\text{ideal}}$

Momentum deviation removed, converted into positions and angles.

$\delta P = \pm 1.05\%$ in 0.35\% increments.
Measured EEX Transport Matrix FR5PFP020

EEX transport matrix as a function of deflecting cavity strength

Circles are measurements, green lines are a weighted linear fit
Red lines are calculated expected values

Measured full 6 x 6; the vertical plane is unaffected by the cavity status...
Measured EEX Transport Matrix

EEX transport matrix as a function of deflecting cavity strength

Circles are measurements, green lines are a weighted linear fit
Red lines are calculated expected values

Measured full 6 x 6; the vertical plane is unaffected by the cavity status...
Measured EEX Transport Matrix FR5PFP020

EEX transport matrix as a function of deflecting cavity strength

Circles are measurements, green lines are a weighted linear fit
Red lines are calculated expected values

Measured full 6 x 6; the vertical plane is unaffected by the cavity status...
Measured EEX Transport Matrix FR5PFP020

EEX transport matrix as a function of deflecting cavity strength

\[
\begin{array}{c}
\text{OUT} \\
X \\
X' \\
Z \\
\delta
\end{array}
= 
\begin{array}{c}
\text{IN} \\
X \\
X' \\
Z \\
\delta
\end{array}
\]

Circles are measurements, green lines are a weighted linear fit
Red lines are calculated expected values

Measured full 6 x 6; the vertical plane is unaffected by the cavity status...
<table>
<thead>
<tr>
<th>Plane</th>
<th>$\varepsilon$[mm-mrad] input</th>
<th>$\varepsilon$[mm-mrad] output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>4.7</td>
<td>20</td>
</tr>
<tr>
<td>Vertical</td>
<td>5.1</td>
<td>6.0</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>21</td>
<td>7.0</td>
</tr>
</tbody>
</table>

Successful exchange of horizontal and longitudinal emittances!!!
Future of AOPI EEX Program
Future of AOPI EEX Program

- Re-measure $R_{23}$ and $R_{43}$ element
Future of AOPI EEX Program

- Re-measure $R_{23}$ and $R_{43}$ element
- Space Charge Studies
Future of AOPI EEX Program

- Re-measure $R_{23}$ and $R_{43}$ element
- Space Charge Studies
- transverse-modulation $\rightarrow$ temporal Modulation

(pictures from Piot and Sun)
ANL’s Argonne Wakefield Accelerator
EX@AWA: overviewgoals

- Exchange incoming emittance from $(\varepsilon_x, \varepsilon_z) = (22, 3) \, \mu m$ to $(3, 22) \, \mu m$
- Exchanger beamline includes a $\frac{1}{2} + 1 + \frac{1}{2}$ cells deflecting cavity
- Possibly perform parametric study with varying incoming emittance partition, e.g., using a flat beam transform

<table>
<thead>
<tr>
<th>parameters</th>
<th>Value</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge</td>
<td>100</td>
<td>pC</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>22.3</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$\varepsilon_z$</td>
<td>2.90</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>Energy</td>
<td>12.1</td>
<td>MeV</td>
</tr>
</tbody>
</table>

Incoming beam parameters after numerical optimization
**EX@AWA: modeling**

- Explored and optimized the emittance exchange scheme at low energy [taking into account collective effects (space charge)]
- Emittance exchange very sensitive to incoming transverse Courant-Snyder parameters
- Best exchange numerically achieved so-far:

<table>
<thead>
<tr>
<th>Space Charge</th>
<th>$\epsilon_x$ (m)</th>
<th>$\epsilon_z$ (m)</th>
<th>$\epsilon_x$ (m)</th>
<th>$\epsilon_z$ (m)</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFF</td>
<td>22.3</td>
<td>2.90</td>
<td>4.4</td>
<td>22.67</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>ON</td>
<td>21.58</td>
<td>2.54</td>
<td>4.7</td>
<td>20.90</td>
<td>$\mu$m</td>
</tr>
</tbody>
</table>

Emittances (left) and beam size (right) evolution along the EX-beamline (PIC simulations performed with Impact-T from LBNL)
EX@AWA: experimental progress

- To date measured the achievable emittance partition downstream of the linac (before EX)
- The emittance-exchanging beamline will be installed this summer

Transverse emittance measured with quadrupole scan

\[ \varepsilon_x = 18.4 \pm 2 \, \mu m \]
\[ \varepsilon_y = 21.2 \pm 2 \, \mu m \]

Longitudinal emittance measured with Linac Phase Scan

Longitudinal emittance from ImpactT simulation

\[ \varepsilon_z = 6.5 \, \mu m \]
Conclusion

• The A0 Photoinjector has constructed a transverse to longitudinal emittance exchange beamline to swap a small transverse emittance with a large longitudinal emittance.
  • A0 Photoinjector has successfully shown an emittance exchange!

• AWA is also pursuing an emittance exchange experiment. They will swap a large transverse emittance with a small longitudinal emittance.
  • Hardware is in hand
  • Installation will begin this summer
  • Work continues on simulations and understanding the input beam parameters

• Other ideas of how to use these manipulations are also around.
  • Couple with a round to flat beam transformer
  • Making a microbunch train
Thanks for your attention
**TM_{110} Deflecting Mode Cavity (DMC)**

![Diagram of the TM_{110} deflection cavity with electric field Ez and magnetic field By.](image)

*Derived from Figure 1 of C&E.*

*Electric field at synchronous phase.*

*Magnetic field a quarter period later.*

- No longitudinal electric field on axis.
- Electric field imparts an energy kick proportional to distance off axis.
- Electro-magnetic field provides deflection as a function of arrival time.
- This type of cavity can be used as a crab cavity or for bunch length measurement.

\[
M_{Thin-Cav} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & k & 0 \\
0 & 0 & 1 & 0 \\
k & 0 & 0 & 1 \\
\end{pmatrix}
\]

\[
k = \frac{eV_0@}{E_c}
\]

\(k\) is the integrated transverse kick normalized to the beam energy \(E\).
Making an Emittance Exchange - Part I

- The 4x4 emittance matrix at two points in an accelerator are related by:
  \[
  \sigma_1 = \begin{pmatrix}
  \sigma_x^2 & \sigma_{xy} & 0 & 0 \\
  \sigma_{xy} & \sigma_y^2 & 0 & 0 \\
  0 & 0 & \sigma_z^2 & \sigma_{z\delta} \\
  0 & 0 & \sigma_{z\delta} & \sigma_{\delta}^2
  \end{pmatrix}
  \sigma_2 = R \sigma_1 R^T
  \]
  
- \( R \) is the 4x4 transport matrix between these points
  
  \[
  R = \begin{pmatrix}
  A & B \\
  C & D
  \end{pmatrix}
  \]

- \( B \) and \( C \) typically have zero determinant and couple transverse and longitudinal emittances through dispersion.

- The emittances after the transport line are given by:
  \[
  \varepsilon_{x2} = |A|^2 \varepsilon_{x1} + |B|^2 \varepsilon_{z1} + \lambda^2 \varepsilon_{x1} \varepsilon_{z1}
  \]
  \[
  \varepsilon_{z2} = |C|^2 \varepsilon_{x1} + |D|^2 \varepsilon_{z1} + \lambda^2 \varepsilon_{x1} \varepsilon_{z1}
  \]
  \[
  \lambda^2 \varepsilon_{x1} \varepsilon_{z1} = tr \left[ \begin{pmatrix}
  A \sigma_{x1} A^T \\
  B \sigma_{z1} B^T
  \end{pmatrix} \right] = tr \left[ \begin{pmatrix}
  C \sigma_{x1} C^T \\
  D \sigma_{z1} D^T
  \end{pmatrix} \right]
  \]

Derivation follows C&E
Making an Emittance Exchange - Part II

- These equations show that for perfect exchange we need:
  \[ |A| = |D| = 0 \]
  \[ |B| = |C| = 1 \]
  \[ \lambda^2 = 0 \]

- How to get \( \lambda^2 = 0 \)?
  \[ A_{ij} = D_{ij} = 0 \]

- If \( \lambda^2 \neq 0 \) the emittances are coupled.
  - Proper adjustment of the \( \sigma \) matrix can reduce or remove the coupling.

Derivation follows C&E