Monte Carlo Mean Field Treatment of Microbunching Instability in the FERMI@Elettra First Bunch Compressor

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1. Self Consistent Vlasov-Maxwell Treatment
2. Field Calculation and Density Estimation
3. Microbunching Instability Studies
4. Discussion

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Basic Lab Frame Setup

\[ E_{\parallel} = 0 \]
\[ B_Y = 0 \]
Self Consistent Vlasov-Maxwell Treatment

3D Wave equation in lab frame with “2D” planar source:

\[
(\partial_Z^2 + \partial_X^2 + \partial_Y^2 - \partial_u^2)\mathcal{E} = H(Y)S(R, u), \quad \mathcal{E}(R, Y = \pm g, u) = 0.
\]

where \( u = ct \), \( \mathcal{E}(R, Y, u) = (E_Z, E_X, B_Y) \), \( R = (Z, X) \).

Vlasov equation in beam frame:

\[
f_s - \kappa(s)xf_z + F_zfp_z + p_xf_x + [\kappa(s)p_z + F_x]fp_x = 0
\]

where

\[
F_z = \frac{e}{v_tE_T}V \cdot E,
\]
\[
F_x = \frac{e}{E_T\beta^2} \left[ -X'_T(s)E_Z + Z'_T(s)E_X - v_TB_Y \right],
\]

and \( V = v_T(t(s) + pxn(s)) \), \( E = (E_Z, E_X) \) and \( B_Y \) are evaluated at \( R = R_T(s) + xn(s) \) and \( u = (s - z)/\beta_T \).
Field Calculation and Density Estimation

Field formula:

\[ E(R, u) := \int_{-g}^{g} H(Y) E(R, Y, u) dY \]

\[ = -\frac{1}{2\pi} \sum_{k=0}^{\infty} a_k \int_{-\infty}^{u-kh} dv \int_{-\pi}^{\pi} d\theta S(\hat{R}, v, k) \]

where \( \hat{R} = R + \sqrt{(u - v)^2 - (kh)^2} (\cos \theta, \sin \theta) \) and \( a_k = (-1)^k (1 - \delta_{k0}/2) \).

- localization in \( \theta \) for \( v \ll u - kh \implies \int d\theta \) with superconvergent trapezoidal rule
- non uniform behavior in \( v \implies \int dv \) with adaptive Gauss-Kronrod rule

Density estimation: from scattered beam frame points at \( s \rightarrow \) smooth/global lab frame charge/current density via a 2D Fourier method.

1D Example: 1D orthogonal series estimator of \( f(x) \), \( x \in [0, 1] \)

\[ f_J(x) := \sum_{j=0}^{J} \theta_j \phi_j(x), \quad \theta_j = \int_{0}^{1} \phi_j(x) f(x) dx, \quad \phi_0(x) = 1, \phi_j(x) = \sqrt{2} \cos(j\pi x), j = 1, 2, ... \]

Since \( f(x) \) is a probability density (\( X, X_n \) random variables distributed via \( f \))

\[ \theta_j = E\{\phi_j(X)\}, \quad \text{thus from Monte Carlo a natural estimate is} \quad \hat{\theta}_j := \frac{1}{N} \sum_{n=1}^{N} \phi_j(X_n) \]
Beam to Lab Charge/Current Density Transformation

- To solve Maxwell equations in lab frame must express lab frame charge/current density in terms of beam frame phase space density
- To a good approximation lab frame charge/current densities are

\[
\rho_L(R, Y, u) = H(Y) \rho_B(r, \beta_r u),
\]
\[
J_L(R, Y, u) = \beta_r c H(Y) [\rho_B(r, \beta_r u) t(\beta_r u + z) + \tau_B(r, \beta_r u) n(\beta_r u + z)],
\]
\[
\rho_B(r, s) = Q \int dp_z dp_x f(\zeta, s), \quad \tau_B(r, s) = Q \int dp_z dp_x p_x f(\zeta, s),
\]

where \( \zeta = (z, p_z, x, p_x) \)

**Remark:** subtlety in the change of independent variable \( u = ct \to s \)

Derivation to be published in a forthcoming paper
Microbunching can cause an instability which degrades beam quality.

This is a major concern for free electron lasers where very bright electron beams are required.

FERMI@Elettra first bunch compressor system proposed as a benchmark for testing codes at the first Workshop on Microbunching Instability held in Trieste in 2007.
### FERMI@Elettra First Bunch Compressor Parameters

Table 1: Chicane parameters and beam parameters at first dipole

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy reference particle</td>
<td>$E_r$</td>
<td>233</td>
<td>MeV</td>
</tr>
<tr>
<td>Peak current</td>
<td>$I$</td>
<td>120</td>
<td>A</td>
</tr>
<tr>
<td>Bunch charge</td>
<td>$Q$</td>
<td>1</td>
<td>nC</td>
</tr>
<tr>
<td>Norm. transverse emittance</td>
<td>$\gamma \epsilon_0$</td>
<td>1</td>
<td>$\mu$m</td>
</tr>
<tr>
<td>Alpha function</td>
<td>$\alpha_0$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Beta function</td>
<td>$\beta_0$</td>
<td>10</td>
<td>m</td>
</tr>
<tr>
<td>Linear energy chirp</td>
<td>$h$</td>
<td>-12.6</td>
<td>1/m</td>
</tr>
<tr>
<td>Uncorrelated energy spread</td>
<td>$\sigma_E$</td>
<td>2</td>
<td>KeV</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>$R_{56}$</td>
<td>0.057</td>
<td>m</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>$\rho_0$</td>
<td>5</td>
<td>m</td>
</tr>
<tr>
<td>Magnetic length</td>
<td>$L_b$</td>
<td>0.5</td>
<td>m</td>
</tr>
<tr>
<td>Distance 1st-2nd, 3rd-4th bend</td>
<td>$L_1$</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>Distance 2rd-3nd bend</td>
<td>$L_2$</td>
<td>1</td>
<td>m</td>
</tr>
</tbody>
</table>
Initial spatial density in grid coordinates for $A=0.05$, $\lambda_0 = 100 \mu$m.

Init. phase space density $= (1 + A \cos(2\pi z/\lambda_0))\mu(z)\rho_c(p_z - hz)g(x, p_x)$. 
Gain factor

Gain := $|\tilde{\rho}(k_f, s_f)/\tilde{\rho}(k_0, 0)|$, $\tilde{\rho}(k, s) = \int dz \exp(-ikz)\rho(z, s)$ and $k_f = C(s_f)k_0$

for $\lambda_0 = 2\pi/k_0$. Here $C(s_f) = 1/(1 + hR_{56}(s_f)) = 3.54$, $s_f = 8.029\text{m}$.

Spectra Longitudinal Density I

For $\lambda_0 = 300 \ \mu m$ ($k_0 = 20944 \ 1/m$), Gain = $0.0243/0.0240 = 1.0128$

- $s=0 \ m, A=0.05$
- $s=0 \ m, A=0$
- $(20944,0.0240)$

For $\lambda_0 = 600 \ \mu m$ ($k_0 = 10472 \ 1/m$), Gain = $0.0236/0.0176 = 1.3409$

- $s=0 \ m, A=0.05$
- $s=0 \ m, A=0$
- $(10472,0.0176)$

For $\lambda_0 = 300 \ \mu m$ ($k_0 = 20944 \ 1/m$), Gain = $0.0243/0.0240 = 1.0128$

- $s=8.028 \ m, A=0.05$
- $s=8.028 \ m, A=0$
- $(74247,0.0243)$

For $\lambda_0 = 600 \ \mu m$ ($k_0 = 10472 \ 1/m$), Gain = $0.0236/0.0176 = 1.3409$

- $s=8.028 \ m, A=0.05$
- $s=8.028 \ m, A=0$
- $(37123,0.0236)$
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\textbf{Spectra Longitudinal Density II}

\( \lambda_0 = 100 \text{ } \mu \text{m} \) \( (k_0 = 62832 \text{ } 1/\text{m}) \), Gain = 0.0649/0.0248 = 2.6169

\( \lambda_0 = 80 \text{ } \mu \text{m} \) \( (k_0 = 78534 \text{ } 1/\text{m}) \), Gain = 0.0620/0.0248 = 2.5000

\( \lambda_0 = 60 \text{ } \mu \text{m} \) \( (k_0 = 104720 \text{ } 1/\text{m}) \), Gain = 0.0593/0.0248 = 2.3870

\( \lambda_0 = 40 \text{ } \mu \text{m} \) \( (k_0 = 157070 \text{ } 1/\text{m}) \), Gain = 0.0291/0.0248 = 1.1734

\( s=8.028 \text{ } \text{m}, A=0.05 \)
\( s=0 \text{ } \text{m}, A=0.05 \)
Monte Carlo Mean Field Treatment of $\mu$BI in the FERMI@Elettra BC1 / Gabriele Bassi

Longitudinal Density

$\lambda_0 = 100\mu m$ at $s = 0$ (top left), $\lambda_0 = 60\mu m$ at $s = s_f$ (bottom left), $\lambda_0 = 100\mu m$ at $s = s_f$ (top right), $\lambda_0 = 40\mu m$ at $s = s_f$ (bottom right).

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\[ \lambda_0 = 200 \mu \text{m}. \] 

\( s = 0.25 s_f \) (top left), \( s = 0.5 s_f \) (top right), \( s = 0.75 s_f \) (bottom left), \( s = s_f \) (bottom right).
2D spatial density and longitudinal force at $s = s_f$

$\lambda_0 = 200\mu m$ (top left), $\lambda_0 = 100\mu m$ (top right), $\lambda_0 = 200\mu m$ (bottom left), $\lambda_0 = 100\mu m$ (bottom right)
• FERMI@Elettra microbunching studies at $\lambda_0 \geq 40\mu m$:
  - Very small effect of $\mu BI$ on mean power and transverse emittance
  - Gain factor at long wavelengths shows breakdown coasting beam assumption
  - Gain factor at short wavelengths indicates deviations from analytical gain formula
  - A paper has been submitted to PRSTAB

• Work in progress and future work:
  - Study wavelengths shorter than $\lambda_0 = 40\mu m$
  - Study dependence on the amplitude of the initial modulation and on the uncorrelated energy spread
  - Study initial perturbation with more than one frequency
  - Complete studies for benchmark microbunching instability including RF cavities
Computational Issues

- Intensive memory requirement and expensive computational cost:
  - Typical simulations done on the parallel clusters ENCANTO in New Mexico and NERSC at LBNL: N procs = 200-1000, N particles = $2 \times 10^7$-$5 \times 10^8$, few hours of CPU time
  - Memory requirement: for $\lambda_0 = 50 \mu m$ store 3D array of dimension $1500 \times 128 \times 200$ on master processor (to avoid massive communications between slave processors)

- To reduce storage/computational cost:
  - Analytical work + state of the art numerical techniques: integration, interpolation, density estimation
  - Parallel computing
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FERMI@Elettra First Bunch Compressor II

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2D spatial density in grid coordinates at $s = s_f$ for $\lambda_0 = 200 \mu m$
2D spatial density in grid coordinates at $s = s_f$ for $\lambda_0 = 100\mu m$
2D spatial density in grid coordinates at $s = s_f$ for $\lambda_0 = 80 \mu m$
Longitudinal force in grid coordinates at $s = s_f$ for $\lambda_0 = 200\mu m$
Longitudinal force in grid coordinates at $s = s_f$ for $\lambda_0 = 100\mu m$. 
Longitudinal force in grid coordinates at $s = s_f$ for $\lambda_0 = 80\mu m$