HIGH TRANSFORMER RATIO PWFA FOR APPLICATION ON XFELS∗

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Abstract
The fourth generation of light sources (such as LCLS and the XFEL) require high energy electron drivers (15-20GeV) of very high quality. We are exploring the possibility of using a high transformer ratio PWFA to meet their challenging requirements. This may have the potential to reduce the size of the electron drivers by a factor of 5 or more, therefore making these light sources much smaller and more affordable. In our proposed design, a high charge (5-10nC) low energy driver (1-3GeV) with an elongated current profile is used to drive a plasma wake in the relativistic blowout regime[1] with a high transformer ratio (5 or more). A second ultra-short beam that has high quality and lower charge (1nC) can be loaded into the wake at a proper phase and be accelerated to high energy (5-15GeV) in very short distances (10s of cms). The parameters can be optimized, such that high quality (0.1 percent energy spread and 1mm mrad normalized emittance) and high efficiency (60-80 percent) can be simultaneously achieved. In this paper, we will present some preliminary theoretical analysis and PIC simulations of this concept.

HIGH TRANSFORMER RATIO PWFA CONCEPT

Recently, PWFA experiments at SLAC have made significant progress. In the E164 experiment, the tail of the 28GeV electron beam had obtained more than 4GeV energy gain in a 10 centimeter long self-ionized plasma[2]. Later, in the E167 experiment, the tail of the 42GeV electron beam had obtained more than 42GeV energy gain in less than one meter[3]. These experiments show the great potential of PWFA as an effective method of energy transfer between charged particle bunches at least in sub 100 GeV level. In the future development of PWFA, using shaped particle bunches to optimize the acceleration and efficiency are very essential. In the long term, concepts such as PWFA based linear collider (PWFA-LC) can be explored, which may have impact on high energy physics. In the near term, we feel that it is also very interesting to explore how to shape the bunches to achieve high transformer ratio and high efficiency PWFA such that the size and cost of 10GeV level LINAC can be reduced significantly. This kind of PWFA may find applications in X-ray free electron lasers. For example, the fourth generation of light sources (such as LCLS and the XFEL) require high energy electron drivers (15-20GeV) of very high quality (charge 1nC, peak current 5kA, emittance 1mm mrad and energy spread 0.1 percent), therefore a high transformer ratio PWFA that can meet these challenging requirements may have the potential to make the XFELs more compact and more affordable.

The concept of large transformer ratio PWFA has been around for more than 20 years. The basic idea is to use a low energy high charge driver to accelerate a low charge trailing bunch to much higher energy. For this to happen, the transformer ratio must be high (the ratio between the accelerating field and the decelerating field). Furthermore, in order to achieve high transfer efficiency and low energy spread at the same time, the plasma wake must have uniform accelerating/decelerating fields within the bunches. In the linear regime, the optimal parameters have been explored by using the linear plasma wakefield theory. It can be shown using 1D linear theory, that the transformer ratio of the wakefield from a symmetric longitudinal beam profile is always less than two[4]. To increase the transformer ratio, an asymmetric profile should be used[5]. A particularly interesting example to achieve large transformer ratio as well as uniform decelerating field is to use a linearly ramped beam profile with a sharp termination and a δ function precursor[5]. Later, it was also shown that larger transformer ratios can be obtained in the 1D nonlinear regime[7]. However, it is now very well understood that the 1D linear and nonlinear regime are not easily accessible experimentally, instead a 3D highly nonlinear regime where plasma electrons blowout occurs is normally the case[1]. So it is natural to ask the same kind of questions for this highly nonlinear 3D regime: what kind of bunch shapes can lead to constant accelerating/decelerating fields and a high transformer ratio in the relativistic blowout regime? In the second section, we will answer this question through a simple theory and PIC simulations. The results show that a linear ramped profile is also the choice in the blowout regime, although with very different physical mechanism comparing with the linear case. In the third section, we will show a sample QuickPIC simulation based on the simple theory. The parameters of this simulation are relevant to the XFEL application.

A SIMPLE THEORY OF TRANSFORMER RATIO IN THE BLOWOUT REGIME

We can start from the hint of the linear theory. For a linearly ramped drive beam in the blowout regime, the transition from the 1D linear regime to the 3D nonlinear blowout regime can proceed as follows by assuming a fixed longitudinal current profile and fixed total charge. Initially, one can choose a large spot size such that \( n_b/n_p << 1 \), there-

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fore the linear theory predicts a nearly uniform decelerating field within the beam and a large transformer ratio (assuming a δ function precursor is used in the linear regime). If we keep decreasing the spot size, eventually \( n_b/n_p > 1 \) will occur, therefore reaching the blowout regime. So will the special properties based on predictions from linear theory still hold when blowout is reached? will they still have the same transformer ratio?

We can get some insight from PIC simulations. In Fig.1, a linearly ramped electron beam driver is used to excite the plasma wake in the relativistic blowout regime. This beam has a normalized length \( k_pL_0 \approx 22 \) and a flat top transverse profile (\( n_{d0} = 100n_p \) for \( k_pr < 0.5 \) and it drops to zero linearly at \( k_pr = 0.6 \)). These parameters give a maximum charge per unit length \( \Lambda_0 \approx 15 \) (\( \Lambda = \frac{n_b}{n_p} \int k_pr \frac{n_b}{n_p} dk_pr \)). In this simulation, the maximum blowout radius is about \( 7.5k_p^{-1} \), which is very close to theoretical estimation from \( 2\sqrt{\Lambda_0} = 7.75 \)[1]. We can see that the decelerating field within the driver is still close to uniform as in the linear case and has an amplitude about \( 0.7mc\omega_p/e \). So it seems the answer to the first question is yes. However, the answer to the second question is no, as we can see the transformer ratio here is around 6, which is very different from linear predictions (between 11 and 22 depending on if a δ function precursor is used).

![Figure 1: Plasma wake driven by an electron beam with a linearly ramped current profile \( n_b/n_p = 100, k_p\alpha = 0.5, L_0 = 22 \) (a) plasma phase space x2x1 (b) lineout of E1](image)

It turns out that the observed results can be easily explained by using simple arguments from the nonlinear theoretical framework of the blowout regime[1]. In the relativistic blowout regime, by ignoring the contribution from the linear response region and the narrow electron sheath, the normalized wake potential \( \psi = \phi - A_z \) (normalized with \( mc^2/e \)) within the ion channel can be approximately written as,

\[
\psi(r, \xi) \approx \frac{r_b^2(\xi)}{4} - \frac{r^2}{4}
\]

for \( r_b > 1 \), where \( r_b(\xi) \) is the normalized blowout radius at each longitudinal slice. For a long electron beam with an adiabatically increasing current profile \( \Lambda(\xi) = \frac{\psi}{\psi_0} \Lambda_0 \), where \( \Lambda_0 \) is calculated at the maximum beam density, the blowout radius \( r_b(\xi) \) also increases adiabatically, and can be estimated from the theoretical formula

\[
k_p r_b(\xi) \approx 2\sqrt{\Lambda(\xi)}
\]

With this expression of \( r_b \), we get

\[
\psi(0, \xi) \approx \frac{\xi}{L_0} \Lambda_0
\]

and hence

\[
E_z(\xi) = \frac{d\psi}{d\xi} \approx \frac{\Lambda_0}{L_0}
\]

Therefore, the decelerating field is roughly a constant that is mainly determined by the peak beam current \( \Lambda_0 \) and the beam length \( L_0 \). To see the accuracy of this result we substitute in the simulation parameters, \( \Lambda_0 = 15 \) and \( L_0 = 22 \), giving \( E_z \approx 0.68 \), which is very close to the simulation result 0.7. We can also get an estimate of the transformer ratio if we assume that the beam current is terminated sharply at \( \xi = L_0 \). The wake field behind the beam is mainly determined by the maximum blowout radius \( k_p r_{m} \), so the maximum useful accelerating field \( E_{\text{max}} \approx \frac{1}{2} r_{m} \approx \sqrt{\Lambda_0} \). Therefore, the maximum useful transformer ratio for this drive beam is

\[
\frac{E_+}{E_-} \approx \sqrt{\frac{\Lambda_0}{\Lambda_0}} = \frac{L_0}{\sqrt{\Lambda_0}}
\]

which is \( 22/\sqrt{15} \approx 5.7 \) for this simulation. We also notice that a precursor is not needed in the nonlinear blowout regime.

**PARAMETER DESIGN AND A SAMPLE SIMULATION**

From the simple theory, one can see that to obtain a larger transformer ratio, a longer beam duration is needed. The practical beam duration is eventually limited by electron hosing instability due to its fast growth for long beam.
Based on estimation from the hosing theory[8], one can expect that a practical transformer ratio is in the range of $5 \sim 10$. Therefore, to reach the parameters relevant to XFELs, a high charge (5-10nC) low energy driver (1-3GeV) with an elongated current profile (with a duration around half ps) is needed. Assuming a transformer ratio of $5$, a second ultra-short beam with proper current profile and lower charge (1nC) can be loaded into the wake at a proper phase and be accelerated to high energy (5-15GeV) in very short distances (10s of cms). The efficiency can be quite high based on the estimation from the beam loading theory[9]. In Fig.2 we show the results from a sample QuickPIC simulation[10]. The driver is a 0.56ps long 1GeV beam with 5nC charge, the trailer is a 23fs long 1GeV beam with 0.35nC. Both beams have a normalized emittance around 1mm mrad and a spot size of 5 microns. To reduce the possible hosing growth, a 10.8cm long parabolic plasma channel with a radius of 68 microns is used. The plasma densities are $0.5 \times 10^{17} \text{cm}^{-3}$ near the axis and $1 \times 10^{17} \text{cm}^{-3}$ near the edge.

Figure 2: A sample QUICKPIC simulation: 5GeV energy gain in 10 cm (a) beam and plasma density (b) p1x1 phase space after 10cm propagation

After 10cm propagation, the trailing beam has obtained 5GeV energy gain with a energy spread better than one percent, and the emittance of the trailer is also conserved. This suggests an energy conversion efficiency around 35 percents. How the beam quality is affected by hosing due to initial beam alignment error will be explored in future.

**SUMMARY**

In this paper, we explored the idea of using high transformer ratio PWFA (with a transformer ratio around 5 ) in the relativistic blowout regime to meet the requirements of a XFEL light source. First, we presented a simple theory of transformer ratio for a linearly ramped drive beam in the blowout regime. Based on this theory and the theory of hosing instability, we mapped out a possible range of parameters. A sample simulation with 5GeV energy gain is also shown to illustrate the idea. Further research will be pursued to fully justify this idea.

**REFERENCES**