POSITRON ACCELERATION BY USING A PARTICLE BEAM-DRIVEN WAKE FIELD IN PLASMA

W. An†, C. Huang, W. Lu, W. B. Mori, UCLA, Los Angeles, CA 90095, USA
T. C. Katsouleas, Duke University, Durham, NC 27708, USA

Abstract

The Plasma Wake Field Accelerator (PWFA) concept is very attractive because the accelerating gradient can be three orders of magnitude higher than that of a traditional RF accelerator[1]. In this paper the acceleration of positron beams in a particle beam-driven wake field is investigated both in the linear and weakly nonlinear regimes under the quasi-static approximation. The results show that a beam-loading efficiency around 50% can be achieved both in the linear and weakly nonlinear regime when the spot sizes of both the drive beam and trailing beam are $k_p \sigma_r \approx 1$. In the linear regime the total charge of the accelerated positron beam and the average accelerating gradient are both smaller than those in the weakly nonlinear regime. Issues related to energy spread are also discussed.

INTRODUCTION

In the Plasma Wake Field Accelerator (PWFA) concept, a plasma is used as a medium instead of the conventional metallic accelerating cavity. In PWFA, the wake field excited in a plasma by a high energy particle beam (the drive beam) has both accelerating and focusing fields. A second particle beam (the trailing beam) follows the drive beam and is accelerated in the wake field. In recent years, PWFA has been widely studied, in the hope that a new generation of compact accelerators can be produced. Although most of the studies have focused on electron acceleration, which is more easily done in the so-called “Blow-out” regime[2], the acceleration of positrons in PWFA is essential for the important application of an electron-positron collider (PWFA Linear Collider). Unlike electron acceleration, positron acceleration in PWFA can not be realized in the “Blow-out” regime due to the tiny focusing phase of the wake field. A positron beam might therefore be accelerated in a linear plasma wake field, in which the maximum accelerating field is smaller than $0.1mc\omega_p/e$ (where $\omega_p$ is the plasma frequency), or in a weakly nonlinear plasma wake field, in which the maximum accelerating field approaches $\approx 1.0mc\omega_p/e$. The basic parameters we consider are the accelerating field, the total charge of the accelerated beam, and the energy spread of the accelerated beam and the beam-loading efficiency which is,

$$\eta = \frac{\int d^3x \rho_{tb}(\vec{x}) E_z(\vec{x})}{\int d^3x \rho_{db}(\vec{x}) E_z(\vec{x})},$$

where $\rho_{db}$ and $\rho_{tb}$ are the charge density of drive beam and the trailing beam, $E_z$ is the accelerating field.

In this paper we investigate the acceleration of a positron beam which has a Gaussian transverse beam profile and a properly chosen longitudinal beam profile. The acceleration is analyzed under the quasi-static approximation, which assumes the beam profile of the drive beam and the trailing beam do not change. The results in linear regime and weakly nonlinear regime are obtained by using 3-D linear theory of PWFA[3] and 2-D cylindrical PIC simulations using the code OSIRIS[4].

POSITRON ACCELERATION IN A LINEAR REGIME

Under the linear limit, a positron drive beam can generate the same plasma wake field as an electron drive beam. So in this section, we consider an electron beam as the drive beam. The same results will be obtained when using a positron drive beam except for the phase of the wake field.

1-D case

We begin by considering a 1-D regime which would be valid if $k_p \sigma_r \gg 1$ for both the drive and trailing beams. From 1-D linear theory, the trailing beam with a linearly ramped down beam density can effectively reduce its own energy spread by flattening the accelerating field[3], which is showed in Figure 1. The drive beam has a Gaussian profile, $n_{db} = n_{db0}e^{-\xi^2/2\sigma_r^2}$, where $\xi = z - ct$. The trailing beam has a triangle beam profile, $n_{tb} = n_{tb0}[(\xi_0 - \xi)/\lambda_0 + 1]$. For the example in Figure 1, $n_{db0} = 0.09n_0$, $\sigma_r = 0.5k_p^{-1}$, $n_{tb0} = 0.089n_0$, $\lambda_0 = 1.968k_p^{-1}$, $\xi_0 = 5.18k_p^{-1}$, where $n_0$ is the initial plasma electron density and $k_p^{-1}$ is the plasma skin depth. The beam-loading efficiency is $\eta = 1 - E_a^2/E_0^2$, where $E_a$ is the accelerating field felt by the trailing beam and $E_0$ is the maximum accelerating field of the original wake field exited by the drive beam. In this case, $E_a = 0.45E_0$, therefore the beam-loading efficiency is $\eta = 79.8\%$. The trailing beam will have no energy spread because it is accelerated in a locally uniform accelerating field. In the 1-D case, there are no transverse forces, and the radial dependence of the accelerating field is not considered.

3-D case

In the 3-D case, we let both of the drive beam and the trailing beam have Gaussian transverse profiles. The beam
density of the drive beam is $n_{db}(r, \xi) = n_{db0}e^{-\frac{r^2}{2\sigma_{rd}^2}}e^{-\frac{\xi^2}{2\sigma_{z}^2}}$, where $n_{db0}$ is the maximum beam density of the drive beam, $\sigma_{z}$ and $\sigma_{rd}$ are the pulse length and spot size of the drive beam. The beam density of the trailing beam is $n_{tb}(r, \xi) = n_{tb0}[(\xi_0 - \xi)/l_0 + 1]e^{-\frac{r^2}{2\sigma_{td}^2}}$, where $n_{tb0}$ is the maximum beam density of the trailing beam, $\sigma_{rt}$ is spot size of the trailing beam and the longitudinal profile is the same as the 1-D case.

Table 1: Numerical results of different trailing beam accelerated in the same linear plasma wake field

<table>
<thead>
<tr>
<th>$k_p\sigma_{rt}$</th>
<th>$n_{tb0}/n_0$</th>
<th>$eE_z/mc\omega_p$</th>
<th>$Q_{tb}/Q_{db}$</th>
<th>$\eta/%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.19</td>
<td>0.029</td>
<td>0.77</td>
<td>79.8</td>
</tr>
<tr>
<td>0.9</td>
<td>0.21</td>
<td>0.032</td>
<td>0.69</td>
<td>78.2</td>
</tr>
<tr>
<td>0.7</td>
<td>0.27</td>
<td>0.039</td>
<td>0.53</td>
<td>72.0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.39</td>
<td>0.045</td>
<td>0.39</td>
<td>61.2</td>
</tr>
<tr>
<td>0.3</td>
<td>0.74</td>
<td>0.049</td>
<td>0.27</td>
<td>45.8</td>
</tr>
<tr>
<td>0.1</td>
<td>3.76</td>
<td>0.050</td>
<td>0.15</td>
<td>26.4</td>
</tr>
</tbody>
</table>

Table 1 shows the results for different trailing beams being accelerated after the same driven beam. The drive beam has $n_{db0} = 0.195n_0$, $\sigma_{z} = 0.5k_p^{-1}$, $\sigma_{rd} = 1.0k_p^{-1}$, the total charge $Q_{db} = 1.54n_0k_p^{-5}$ = $0.37nC/\sqrt{n_0/10^{-16}cm^{-3}}$ and the maximum accelerating field $E_0 = 0.1mc\omega_p/e$. The positron beam is loaded at the place where $E_a(r = 0, \xi = \xi_0) = 0.45E_0$, $Q_{tb}$ is the total charge of the trailing beam, and the average accelerating gradient is $E_z = \int d\xi d^2\rho_{tb}(\xi)E_z(\xi)/Q_{tb}$. Figure 2 shows the accelerating field $E_z$ felt by the positron trailing beam and the positron beam’s energy spectrum.

In these results, when the spot size of the positron beam is equal to the drive beam spot size, which is $1.0k_p^{-1}$. The beam-loading efficiency is equal to that of the 1-D case. Actually when spot sizes of both beams are the same, the beam-loading efficiencies are also the same in 1-D and 3-D cases. But the accelerating field felt by the positron beam has a totally nonuniform transverse profile which leads to a large energy spread in the 3-D case. When the spot size becomes smaller, both the efficiency and the total charge of the positron beam will become smaller, but the average accelerating field will increase. The energy spread decreases as well. But when the spot size of the trailing beam is too small, the maximum beam density will greatly exceed the plasma density and therefore be outside the limit.

Thus, in linear regime we may accelerate a positron beam with a spot size comparable to the spot size of drive beam ($\sigma_{rt}/\sigma_{rd} \sim 0.5$) and obtain a high beam-loading efficiency, which is $\sim 50\%$. The accelerated positron charge is around $0.13nC/\sqrt{n_0/10^{-16}cm^{-3}}$

**Figure 1:** The accelerating field $E_z$ in a 1-D linear plasma wake field.

**Figure 2:** (a) The accelerating field felt by the positron beam ($-5 < r/\sigma_{rt} < 5, \xi_0 < \xi < \xi_0 + l_0$) and (b) the energy spectrum of the positron beam when the spot size of the positron beam is (1) $k_p\sigma_{rt} = 1.0$; (2) $k_p\sigma_{rt} = 0.7$; (3) $k_p\sigma_{rt} = 0.3$.

**Figure 3:** Shows the simulation results of loading a positron beam in a weakly nonlinear plasma wake field driven by an electron beam. Both the drive beam and the trailing beam have the same spot size, which is equal to $1.0k_p^{-1}$. The drive beam has a Gaussian longitudinal profile with $\sigma_z = 1.4k_p^{-1}$. The maximum beam density of the drive beam is $n_{db0} = 0.5n_0$. It can excite an accelerating field with $E_0 = 0.46mc\omega_p/e$. The trailing beam is loaded at $E_a = 0.32mc\omega_p/e$ and has a properly chosen intensity.

**Table 1:** Numerical results of different trailing beam accelerated in the same linear plasma wake field.

**Figure 3:** Shows the simulation results of loading a positron beam in a weakly nonlinear plasma wake field driven by an electron beam. Both the drive beam and the trailing beam have the same spot size, which is equal to $1.0k_p^{-1}$. The drive beam has a Gaussian longitudinal profile with $\sigma_z = 1.4k_p^{-1}$. The maximum beam density of the drive beam is $n_{db0} = 0.5n_0$. It can excite an accelerating field with $E_0 = 0.46mc\omega_p/e$. The trailing beam is loaded at $E_a = 0.32mc\omega_p/e$ and has a properly chosen intensity.
trapezoidal profile in the longitudinal direction, which can flatten the accelerating field. But the pulse length of the trailing beam is only $0.5k_p^{-1}$, which is much smaller than that in the linear regime.

Table 2: Simulation results of different trailing beam accelerated in the same weakly nonlinear plasma wake field

<table>
<thead>
<tr>
<th>$k_p\sigma_{rt}$</th>
<th>$Q_{tb}/n_0k_p^{-3}$</th>
<th>$Q_{tb}/Q_{db}$</th>
<th>$e\bar{E}_z/mc\omega_p$</th>
<th>$\eta/%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>2.20</td>
<td>0.20</td>
<td>0.17</td>
<td>53.77</td>
</tr>
<tr>
<td>0.75</td>
<td>2.20</td>
<td>0.20</td>
<td>0.19</td>
<td>61.04</td>
</tr>
<tr>
<td>0.5</td>
<td>2.20</td>
<td>0.20</td>
<td>0.20</td>
<td>62.8</td>
</tr>
<tr>
<td>0.25</td>
<td>1.38</td>
<td>0.12</td>
<td>0.21</td>
<td>41.93</td>
</tr>
</tbody>
</table>

Next, we varied the spot size of the trailing beam and kept its longitudinal profile as well as its total charge unchanged. The simulation results are shown in Table 2. Figure 4 shows the accelerating field $E_z$ felt by the positron trailing beam and the positron beam’s energy spectrum, which are obtained from OSIRIS simulations.

In the weakly nonlinear regime, decreasing the trailing beam’s spot size enhances the average accelerating gradient and the beam-loading efficiency. But the energy spread does not decrease significantly. When the spot size is too small, the total charge of the trailing beam must be decreased or it will destroy the accelerating field. Therefore, in weakly nonlinear regime we can also accelerate a positron beam with $\sigma_{rt}/\sigma_{dt} \sim 0.5$ and obtain a high accelerating field and high beam-loading efficiency, which is $\sim 50\%$. The accelerated positron charge is around $0.53nC/\sqrt{n_0/10^{-16}cm^{-3}}$, which is larger than that in the linear regime. But the energy spread is not as good as the linear regime results.

**CONCLUSION**

In this paper, we have presented a preliminary investigation of positron acceleration in a plasma wake field driven by a beam with a spot size equals to $1.0k_p^{-1}$ for both the linear and weakly nonlinear regimes. A beam-loading efficiency $\sim 50\%$ can be achieved in both of these two regimes when the trailing beam has a spot size $\sigma_{rt}/\sigma_{dt} \sim 0.5$ and a properly shape current profile. The energy spread of the trailing beam is smaller in the linear regime than that in the weakly nonlinear regime. But in the linear regime the total charge of the accelerated positron beam and the average accelerating gradient are both smaller than those in weakly nonlinear regime. As a result, the $Q_{tb} \cdot \bar{E}_z$ is much larger in the weakly nonlinear regime. We next plan on investigating in more detail the effects transverse shaping both the drive and trailing bunches, increasing the length of the trailing bunch in the weakly nonlinear regime, hollow channels, and the self-consistent evolution of each.

**REFERENCES**