ACCURATE ENERGY MEASUREMENT OF AN ELECTRON BEAM IN A
STORAGE RING USING COMPTON SCATTERING TECHNIQUE∗

C. Sun†, Y. K. Wu, J. Li, FEL Laboratory, Duke University, Durham, NC 27708-0319, USA
G. Rusev, A. P. Tonchev, TUNL, Duke University, Durham, NC 27708-0308, USA

Abstract

A gamma-ray beam produced by Compton scattering of
a laser beam and a relativistic electron beam has been used
to determine electron beam parameters. In order to accu-
rately measure the electron beam energy, a fitting model
based upon Compton scattering cross section is introduced
in this paper. With this model, we have successfully deter-
mined the energy of the electron beam in Duke storage ring
with a relative uncertainty of $3 \times 10^{-5}$ using a Compton
gamma beam from the High Intensity $\gamma$-ray Source (HI-$\gamma$S)
facility at Duke University.

INTRODUCTION

A gamma-ray beam produced by Compton scattering of
a laser beam and an electron beam carries the information
of the electron beam. The high energy edge of the gamma-
ray beam spectrum can be used to determine electron beam
parameters, such as the beam centroid energy and energy
spread. In several published works [1, 2, 3], the high en-
ergy edge of the gamma beam spectrum was simply ex-
pressed as a convolution between a modified step function
and a Gaussian function. The influences of the gamma
beam collimation as well as the electron beam emittance
on the gamma beam spectrum were not taken into account.
However, under many circumstances these influences could
have a significant impact on the accuracy of electron beam
energy measurements.

To overcome this problem, we have developed a new fit-
ing model which can describe the gamma beam spectrum in
detail while taking into account the collimation and emit-
tance effects. Using this model, we have accurately mea-
sured the energy of the electron beam in the Duke storage
ring with a relative uncertainty of $3 \times 10^{-5}$.

SPECTRUM DESCRIPTION

Based upon the Compton scattering cross section, the
angular and energy distribution of the gamma photons pro-
cuced by the collision of electron and laser bunches can be
expressed as [4]
\[
\frac{d^2N_{\gamma}}{d\Omega_L dE_{\gamma}} = N_e N_p \int \frac{d^2\sigma}{d\Omega dE_e} c(1 - \beta \cos \theta_i)
\]

\[
\times f_e(V, p_e, t) f_p(V, k, t) dV_e dkdV dt,
\]

where $dN_\gamma$ is the number of the gamma photons in an en-
ergy range of $E_{\gamma}$ to $E_{\gamma}+dE_{\gamma}$ and solid angle $d\Omega_L$ viewed
in a laboratory frame; $N_e$ and $N_p$ are the total numbers
of electrons and laser photons in their bunches; $d\sigma/d\Omega$ is
the angular differential cross section for Compton scatter-
ing; $c$ is the speed of light; $\beta = v/c$ is the velocity of
the electron scaled by the speed of the light; $\theta_i$ is the angle
between the momenta of the incident electron and photon.
$f_e(V, p_e, t)$ and $f_p(V, k, t)$ are the phase space distribu-
tion functions of the electron and photon beams. The integra-
tion is computed for the entire collision time and volume
as well as the momenta of electrons and laser photons via
\[
\int \cdots dV_e dkdV dt.
\]

Assuming Gaussian distributions of the electron and
laser beams, neglecting the vertical emittance of the elec-
tron beam and the energy spread of the laser beam, and
further assuming head-on collisions happening at the waist
of the laser beam, the energy spectrum of the collimated
gamma beam can be obtained by partially integrating Eq. (1) [4],
\[
dN_{\gamma} = \frac{\gamma^2 L^2 N_e N_p}{16 \pi^3 \hbar c \sqrt{\xi_x \sigma_x \gamma}} \int_{-x_o}^{x_o} \int_{-y_o}^{y_o} \int_{-\theta_{max}}^{\theta_{max}} \left( \frac{\gamma}{1+2\gamma a} \right)
\]

\[
\times \left[ \frac{4\gamma^2 E_p}{E_{\gamma}(1+\gamma^2 \theta_j^2)} + \frac{E_{\gamma}(1+\gamma^2 \theta_j^2)}{4\gamma^2 E_p} - \frac{\gamma^2 \theta_j^2}{(1+\gamma^2 \theta_j^2)^2} \right]
\]

\[
\times \exp \left[ \frac{(-\theta_x - x_e)^2}{2\sigma_{\theta_x}^2} - \frac{(-\gamma - \gamma_0)^2}{2\sigma_{\gamma}^2} \right] d\theta_x dx_e dy_e,
\]

where

\[
\gamma = \frac{2E_{\gamma}a}{4E_p - E_{\gamma} \theta_j^2} \left( 1 + \sqrt{1 + \frac{4E_{\gamma}^2 - E_{\gamma} \theta_j^2}{4a^2 \gamma^2}} \right);
\]

\[
a = \frac{E_p}{mc^2}; \quad \theta_j = \theta_x^2 + \left( \frac{y_c}{L} \right)^2; \quad \sigma_{\theta_x} = \sqrt{\frac{\varepsilon_{x \xi_x}}{\beta_x \xi_x}};
\]

\[
\xi_x = 1 + \left( \frac{\beta_x}{L} \right)^2; \quad \varepsilon_x = \left( \frac{2k\beta_x e_x}{\beta_0} \right); \quad \varepsilon_x = 1 + \left( \frac{2k\beta_x e_x}{\beta_0} \right);
\]

\[
\theta_{max} = \sqrt{\frac{4E_p}{E_{\gamma}} - \left( \frac{y_c}{L} \right)^2};
\]

$r_e$ is the classical electron radius; $\hbar$ is the reduced Planck
constant; $\varepsilon_x$ and $\beta_x$ are the emittance and the beta func-
tion of the electron beam in the horizontal direction, re-
spectively; $\gamma_0 = E_c/mc^2$ and $\sigma_\gamma = \sigma_{E_c}/mc^2$ represent


\*
Work supported by US Department of Defense Medical FEL Program
as administered by the AROS under contract number FA9550-04-01-0086 and US Department of Energy grant DE-FG05-91ER40665.

† suncc@fel.duke.edu
the electron beam energy and energy spread normalized by $mc^2$, respectively; $E_p$ is the laser photon energy; $\beta_0$ and $k = E_p/(hc)$ are the Rayleigh range and wavenumber of the laser beam, respectively; $L$ is the distance between the collision point and the collimator; $x_o$ and $y_o$ are half widths of horizontal and vertical apertures, and for a circular aperture, the radius of the aperture is given by $R = \sqrt{x_o^2 + y_o^2}$. Based upon the assumption of far field collimation ($L \gg R$), the solid angle $d\Omega_L$ in Eq. (1) has been replaced by $d\Omega = dx_c dy_c/L^2$, where $x_c$ and $y_c$ are integration variables ranging from $-x_o$ to $x_o$ and from $-y_o$ to $y_o$, respectively.

The collimator misalignment effect can be easily introduced in Eq. (2) by replacing the integration variables $x_c$ and $y_c$ with $x_c + x_{e}$ and $y_c + y_{e}$, respectively, where $x_e$ and $y_e$ are the misalignment offsets of the collimator in the horizontal and vertical directions with respect to the gamma beam center.

In order to use Eq. (2) to describe the energy spectrum $dN/\gamma E$ of a collimated Compton gamma-ray beam, the integrations with respect to $dx_c dy_c d\theta_x$ in Eq. (2) must be evaluated numerically. For this purpose, a numerical integration code has been developed [4]. The spectra calculated using this code for the different misalignment conditions of the collimator are shown in Fig. 1. Clearly, when the misalignment of the collimator is small compared to the radius of the collimation aperture, the misalignment mainly affects the low energy edge of the spectrum, while leaving the high energy edge unchanged.

Figure 1: The energy spectrum of gamma-ray beam produced by Compton scattering of a 460 MeV electron beam with a 790 nm laser beam. The gamma beam is collimated by a collimator with radius of 12.7 mm placed 60 m downstream of the collision point. The spectrum is calculated for different alignment of the collimator from 0.0 mm to 8.0 mm in the horizontal direction.

Figure 3: A typical HIγS beam spectrum measured by a large volume 123% efficiency HPGe detector. The radiation sources of $^{226}$Ra and $^{60}$Co as well as the nature background from $^{40}$K are used in the real time for the detector energy calibration.

**MEASUREMENT**

The schematic of the High Intensity $\gamma$-ray Source (HIγS) facility [6] at Duke university is shown in Fig. 2. The gamma-ray beam at HIγS is generated by colliding a Free-Electron Laser (FEL) [7] beam inside the laser resonator with an electron beam in the storage ring. The electron beam is first generated and accelerated to 180 MeV in a linear accelerator. The electron beam energy is then ramped up to a desired value in a booster synchrotron before injecting into the storage ring. The electron beam, consisting of two bunches separated by a half of the storage ring circumference, is used to drive the FEL. The FEL photons from the first (second) electron bunch collide with electrons in the second (first) bunch. The resultant high intensity gamma beam is transported in vacuum to the gamma beam target room. A lead collimator is placed 60 meters downstream of the collision point and before the target room.

The energy spectrum of the gamma beam is measured using a large volume 123% HPGe detector installed at the end of the target room 10 meters downstream from the collimator. The gamma-ray radiation sources of $^{226}$Ra and $^{60}$Co as well as the nature background from $^{40}$K are used for the detector energy calibration. The electron beam emittance and FEL wavelength are recorded by a synchrotron radiation beam profile monitor and a spectrometer, respectively.

A HIγS beam collimated by a lead aperture with radius of 12.7 mm is used to determine the electron beam energy and energy spread. A typical measured gamma beam spectrum with the simultaneously recorded gamma-ray calibration peaks are shown in Fig. 3.

Using Eq. (2), the nonliner least square fitting method

**Instrumentation**

**T03 - Beam Diagnostics and Instrumentation**
is applied to the high energy edge of the measured gamma beam spectrum using electron beam energy $E_e$ and energy spread $\sigma_{E_e}/E_e$ as parameters. To speed up the fitting process, a parallel computing technique which involves 32 central processing units (CPUs) of the Duke Shared Cluster Resource (DSCR) [5] is applied. The fitting result is illustrated in Fig. 4. The fit electron beam energy is 459.063 MeV.

The accuracy of the electron beam energy measurement is mainly affected by the uncertainties in the determination of the gamma beam spectrum edge as well as the FEL peak wavelength. These uncertainties can be further divided into two types: systematic errors and statistical errors. The systematic errors arise from the calibration of the HPGe detector and the spectrometer, while the statistical errors arise from the count fluctuations in the measured gamma beam spectrum and the measured FEL spectrum.

The overall uncertainty $\delta E_e$ (68% confidence level) of the electron beam energy measurement is given by the square root of the quadratic sum of the individual uncertainty contribution $\delta E^i_e$, i.e., $\delta E_e = \sqrt{\sum_i (\delta E^i_e)^2}$. In this measurement, the overall uncertainty of the energy measurement is estimated about 0.013 MeV. Thus, the relative uncertainty of $3 \times 10^{-5}$, including both systematic and statistical errors, is achieved for the electron beam energy measurement.

**SUMMARY**

Based upon the Compton scattering cross section, we have derived a formula to describe the energy spectrum of a collimated Compton gamma-ray beam. Using this formula as a fitting model and a gamma-ray beam from the High Intensity $\gamma$-ray source (HI$\gamma$S) facility, we have successfully measured the electron beam energy with a relative uncertainty of $3 \times 10^{-5}$ for 460 MeV beam in Duke Storage ring.

**REFERENCES**