

# TRACKING ROUTINES FOR ARBITRARY INSERTION DEVICES

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## Abstract

A package of FORTRAN tracking routines for arbitrary insertion devices has been developed at BESSY and incorporated in the tracking code BETA [1]. A comparison of computing time and tracking results for phase space and dynamical aperture calculations is presented.

## 1 INTRODUCTION

The 1.7 GeV synchrotron light source BESSY II<sup>1</sup> (under construction) will be a low emittance machine with 16 straight sections. The variety of planned IDs requires flexible and adapted tracking tools to study their effects on the performance of the beam optics. Based on both analytical and numerical methods we have implemented several routines into the tracking code BETA [1].

## 2 METHODS AND IMPLEMENTATION

The program BETA as well as our routines are written in FORTRAN. We have provided three user interface routines. Two routines pass control to the user at the beginning and the end of the main program. The third one is the insertion device tracking routine IDTRCK\_MODI which replaces the original routine IDTRCK of BETA. We also have provided a user common block to transfer variables within the user routines. When the program starts it calls our initialisation routine, which reads from a file the information needed for the insertion device tracking, i. e. the name, type and tracking mode, and the parameters of the ID.

For the time being we have two main sets of insertion device routines:

- symplectic mapping routines, based on Generating Functions
  - analytical Taylor expansion of Generating Function for devices with large bending radii
  - tracking with the linear transfer matrix of the ID
  - numerically fitted polynomial of the Generating Function for arbitrary magnetic field configurations
- symplectic integration routine
  - analytically given arbitrary magnetic field configuration
  - arbitrary equally spaced 3D magnetic field map

For the user it is easy to implement his own routines into the interface routines. This allows a very flexible adaptation to special problems. The integration routines e.g. calls a routine which provides the magnetic field. This routine can be replaced by the user to provide his own insertion device description.

In the following subsections the routines of our package are described in more detail.

### 2.1 Analytical Generating Function

Several of our routines are based on Generating Functions. The advantage of this method is clearly that it is symplectic and it is well suited for mapping over final step length. That is different to integration methods which are valid only for infinitesimal step length.

The GF is approximately solved by a Taylor series expansion [2]. The Hamilton-Jacobi differential equation

$$\partial F / \partial z + H = 0,$$

with a Hamiltonian given by

$$H = (p_x - A_x/B\rho)/2 + (p_y - A_y/B\rho)^2/2 - A_z/B\rho$$

is solved for arbitrary magnetic fields described by the vector potential  $(A_x, A_y, A_z)$ . The coefficients of the Taylor series are analytical expressions of the magnetic potential. This method shows good convergence for most of the ID fields. Step length over several periods of an ID can easily be done with good accuracy. Compared to the generating function  $F = F(x, y, p_{xi}, p_{yi})$  given in [2] a better type is  $F = F(x_i, y_i, p_x, p_y)$ , where  $x_i, y_i$  are the initial positions and  $p_x, p_y$  are the final momenta of the particle. This has an advantage in solving the implicit coordinate relation for the GF and makes the tracking routines much faster.

For the formulation of the GF an analytical representation of the ID field is required. This analytical formulas are manipulated with the algebraic computer code REDUCE [4]. Three FORTRAN modules are generated and inserted in a FORTRAN tracking code. Parameters like field strength, particle energy, period length and so on are kept as variables and will be defined in the FORTRAN code.

These modules have to be created for different ID field expressions. Presently we have a set of modules for planar IDs and two different helical IDs. One routine is for a Sasaki type of helical IDs [3]. The other module can be used for IDs described by a potential function of the type  $V = a_1 \cos k_z z + a_n \cos nk_z z + b_1 \sin k_z z + b_n \sin nk_z z$ . This is a very general field description [5]. The  $a, b$  coefficients are polynomials in  $x, y$  up to 7<sup>th</sup> order. In the longitudinal direction the sin- and cos-like terms can be

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given by two Fourier components. The field potential is only approximately Maxwellian, but in combination with the Generating Function the mapping routine becomes symplectic.

This method based on analytical approximated GF tracking routines is also available as one of the option for IDs tracking simulation (planar and helical ID) in RACE-TRACK [6].

### 2.2 Numerically fitted Generating Function

The Generating Function  $F(x_i, p_x, y_i, p_y)$  describes the mapping of a particle through an insertion device approximated by a four-dimensional polynomial of the 4th or 6th order [8], [9]. The coefficients of the polynomial are fitted from the start and end points for a set of particle trajectories passing through the device. The corresponding tracking and fitting is done by the program WAVE [7]. The coefficients of the fit are passed to the tracking routine via a data file. The method is described in a technical note and reference [9]. This method allows to track through an insertion device in a single step. The advantage of this numerical method is that it is not restricted to large bending radii and can e.g. be used for superconducting wavelength shifter. A disadvantage is that coefficients have to be derived for off momentum particles separately. Another disadvantage is that besides BETA other programmes are needed.

### 2.3 Integration tracking routine

The integration routine tracks the particle step by step through a given magnetic field. Within each step it calls the selected or user provided magnetic field routine passing the coordinates of the particle and getting back the magnetic field at this position. Then it transports the particle over one step assuming a constant field within this step. We usually track with 1 mm steps. The advantage of this method is its flexibility and reliability. It also offers the option to write out the particle trajectory inside the device. The main disadvantage is its time consumption.

If the field is given by a 3D field map, a large computer memory is required, but it offers the option to use measured or computer generated field maps.

## 3 PERFORMANCE STUDIES

To study the overall performance of the various insertion device tracking routines we have considered the planned undulator U42 for BESSY II with a period length of  $\lambda_0 = 42mm$ . The on-axis peak field is  $B_0 = 0.69T$  at a full gap of  $15mm$ . The number of periods is 78. This type of device can be tracked with the original beta routine as well as with our routines. The analytical representation of the field is given by

$$\begin{aligned} B_x &= -k_x/k_y & B_0 \sin(k_x x) \sinh(k_y y) \cos(k_z z) \\ B_y &= & B_0 \cos(k_x x) \cosh(k_y y) \cos(k_z z) \\ B_z &= -k_z/k_y & B_0 \cos(k_x x) \sinh(k_y y) \sin(k_z z) \end{aligned}$$

$$\text{with } k_y^2 = k_x^2 + k_z^2.$$

The ration  $k_x/k_z$  has been set to  $k_x/k_z = 0.2$ .

### 3.1 Computing time

We started with an investigation of the computing time considering the standard BESSY II optics with the U42 as the only insertion device. The sextupoles were turned off for these runs. We used a DEC AlphaServer 2100 4/200 with 256 MByte memory running OpenVMS V6.2-1H3. During the test the machine was almost exclusively processing the tracking job. In case of the analytical GF routine one period was tracked in a single step. For the numerical GF routine the whole device was taken in a one step and the step length for numerical integration routine was set to one millimeter. Each tracking method was done in a first run with 1000 turns and in a second run with 11000 turns. Then the differences in CPU time and elapsed time of both jobs then correspond to 10000 turns. Finally we subtracted the times used for tracking without any insertion device. This way we obtained the tracking time for the insertion device routines. The differences in the CPU and elapsed time were negligible, thus only the CPU time is shown in fig. 1.

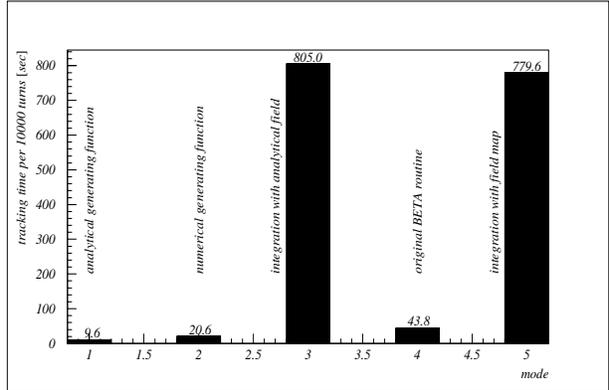


Figure 1: Tracking time for the various routines for 10000 turns.

### 3.2 Tracking precision

The tracking results of the routines were investigated by comparing phase space plots (linear optics with the U42 as the only ID). Fig. 2 shows in the upper windows two horizontal phase space plots. The first plot is done for different horizontal and vertical tunes. There is no significant difference between tracking results. For the second plot we set both tunes to the same value such that the resulting coupling resonance increased the differences between the various tracking schemes. Although the phase is slightly divergent our routines show a good agreement, i.e. the particle stays within the same phase space region. The behavior of the BETA tracking routine is different, the covered phase space is larger.

For an identical set of about 100 phase space coordinates at the entrance of the device we plotted in the lower

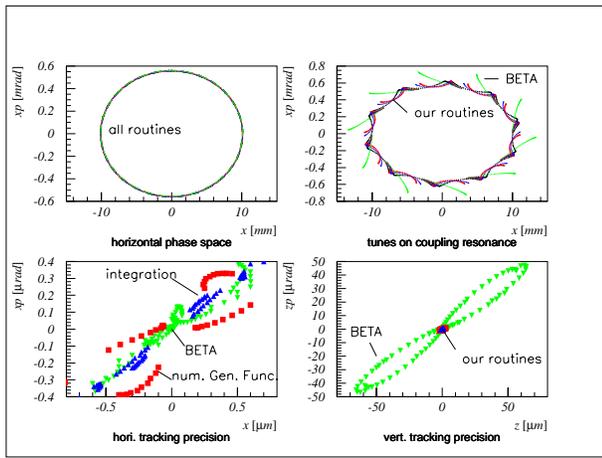


Figure 2: Comparison of tracking results for the original BETA routine and our Generating function and integration routines.

windows of fig.2 the deviations with respect to the results for the analytical Generating Function at the exit of the ID (same phase space region as before). The differences in the horizontal coordinates  $x, xp$  are similar for all routines. But for the vertical coordinate  $z, zp$  the BETA routine shows a much larger discrepancy than the integration and numerical Generating Function routines. Their deviation is of the same size as for the horizontal coordinates.

### 3.3 Dynamical aperture

As a last comparison we calculated dynamical apertures for the BESSY II optics (including sextupoles) with the U42 as the only ID. We asked for 1000 stable turns. Fig.3 shows the results. Although our routines are based on different methods and algorithms they show a good agreement. The original BETA routine shows a larger aperture. The largest aperture is obtained when the ID is tracked using the corresponding linear transfer matrix of the ID.

However, it should be noted that for vertical apertures larger than 1 cm, the simple formulas for the magnetic field of the device as given above yield unrealistic values. We obtain e.g.  $B_y(0, 1cm, 0) = 1.6T$  and  $B_y(0, 2cm, 0) = 6.7T$ . For these extreme values the analytical field expansion is no longer valid. All tracking routines using this simple description of IDs deal with this problem. Hence the resulting vertical apertures are not reliable. On the other hand, this in general will cause no problems, since the hardware aperture is much smaller in most cases.

## 4 CONCLUSION

The package of our routines shows consistent results for the different methods and algorithms. Some significant differences were found with respect to the BETA insertion device tracking routine. The provided routines allow to study the influence of arbitrary insertion devices on the beam dynamics.

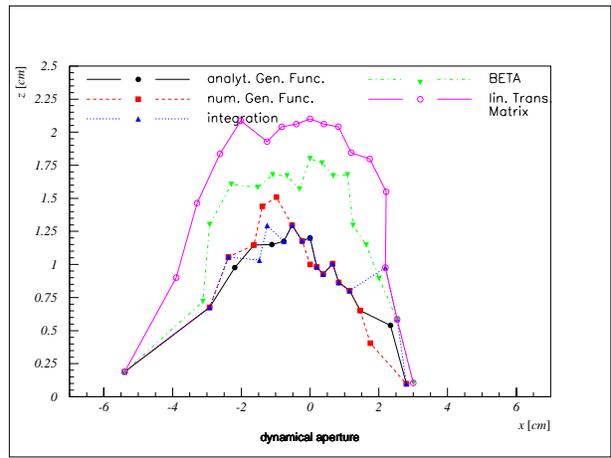


Figure 3: Comparison of dynamical apertures

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