# A STUDY ON BEAM-BEAM INTERACTIONS FOR BTCF 

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## Abstract

A study on beam-beam interactions in the proposed Beijing Tau-Charm Factory (BTCF) is carried out. Various parameters of the collider, such as tunes, beam-beam parameters, bunch length, beta functions at the interaction point (IP), crossing angle and vertical dispersion at IP, are examined. It is concluded that with the optimized parameter set, the luminosity goal of BTCF is feasible.

## 1 MOTIVATION

BTCF is a double-ring electron-positron collider working in the $\tau$ lepton and charm meson energy region of $3 \sim 5$ GeV [1]. The target peak luminosity is $1 \times 10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$. The parameters of BTCF closely related to the beam-beam effects are listed in Table 1.

Table 1: BTCF parameters related to beam-beam effects.

| Parameter | Crossing Angle | Monochromator |
| :---: | :---: | :---: |
| $E(\mathrm{GeV})$ | 2.0 | 1.55 |
| $C(\mathrm{~m})$ | 385.447 |  |
| $\beta_{x} / \beta_{y}(\mathrm{~m})$ | $0.65 / 0.01$ | $0.01 / 0.15$ |
| $D_{y}^{*}(\mathrm{~m})$ | 0 | $\pm 0.35$ |
| $Q_{x} / Q_{y} / Q_{s}$ | $11.8 / 12.6 / 0.068$ | $13.08 / 11.11 / 0.057$ |
| $\epsilon_{x} / \epsilon_{y}(\mathrm{~nm} \cdot \mathrm{rad})$ | $153 / 2.3$ | $48 / 4$ |
| $\sigma_{e}\left(10^{-4}\right)$ | 5.84 | 8.0 |
| $\sigma_{z}(\mathrm{~cm})$ | 0.76 | 1.0 |
| $\tau_{x} / \tau_{y} / \tau_{e}(\mathrm{~ms})$ | $30 / 30 / 15$ | $25 / 59 / 95$ |
| $\xi_{y}$ | 0.04 | 0.015 |
| $\phi_{c}(\mathrm{mrad})$ | 2.6 | 0 |
| $N_{b}$ | 86 | 29 |
| $I_{b}(\mathrm{~A})$ | 0.57 | 0.2 |
| $L_{0}\left(10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}\right)$ | 1 | 0.1 |

It can be found from the Table 1 that the beam-beam interactions in BTCF have following features in comparison with other machines like B-factories [2]: (1) with lower beam energy and larger damping ratio $\tau_{x, y, e} / T_{0}, T_{0}$ being the revolution period, the beam-beam effects get stronger for the same $\xi$; (2) for the longer bunch $\left(\sigma_{z} / \beta^{*} \sim 1\right)$ and higher synchrotron tune, the synchro-betatron coupling gets more important; (3) the variety of collision fashions with crossing angle and vertical dispersion at IP makes the beam-beam behavior in the BTCF varied and interesting.

## 2 ISSUES WITH BEAM-BEAM INTERACTIONS

The luminosity in a collider is expressed as

$$
\begin{equation*}
L\left(\mathrm{~cm}^{-2} s^{-1}\right)=2.17 \times 10^{34} \frac{\xi_{y}(1+r)}{\beta_{y}^{*}(c m)} I_{b}(A) E(G e V) \tag{1}
\end{equation*}
$$

where $I_{b}$ is current per beam and $r=\sigma_{y} / \sigma_{x}$ defines the beam aspect ratio.

As the bunches have a finite length, the $\beta$ functions at the positions where the different parts of the bunches meet are different ("hour glass" effect). Furthermore, crossing angle brings another geometrical effect to the luminosity reduction. The luminosity reduction due to the final bunch length and crossing angle is given by [3]

$$
\begin{equation*}
\frac{L_{g}}{L_{0}}=\sqrt{\frac{2}{\pi}} a e^{b} K_{0}(b) \tag{2}
\end{equation*}
$$

where $K_{0}$ is the modified Bessel function, $a$ and $b$ are defined as

$$
\begin{equation*}
a=\frac{\beta_{y}^{\star}}{\sqrt{2} \sigma_{z}} \cos \phi_{c} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
b=a^{2}\left[1+\left(\frac{\sigma_{z}}{\sigma_{x}} \tan \phi_{c}\right)^{2}\right] \tag{4}
\end{equation*}
$$

The horizontal flat beam ( $\sigma_{y} \ll \sigma_{x}$ ) is assumed in eq.(2). In the case of the vertical flat beam ( $\sigma_{x} \ll \sigma_{y}$ ), the subscript " $y$ " in eq.(3) should be replaced with " $x$ ". On the other hand, the eq.(4) is applied to horizontal crossing scheme, while in the case of vertical crossing, the subscript "x" should be replaces with " $y$ ".

Above discussion on the beam-beam issues is based on a simplified model and linear approximation. In reality, the beam-beam effects are much more complicated. The computer simulation is necessary in order to study the complexity of the nature for beam-beam interactions. The simulation is performed by taking advantage of the computer code BBC (Beam-Beam interaction with a Crossing angle) developed by K.Hirata.

The algorithm applied in the code is detailed in the reference [3]. The simulation takes the machine parameters listed in Table 1. In our weak-strong simulation, the distribution of weak beam changes due to the beam-beam interaction, while the strong beam remains Gaussian. The distribution of the weak beam is obtained by simulation as the sum of $\delta$-functions, which represent the ensemble of particles. The simulated luminosity is computed as a convolution of the distribution function of both weak and strong beams.

## 3 BEAM-BEAM TUNE SCAN

The purpose of beam-beam tune scan is to examine the design luminosity and to optimize the tunes. Figure 1 gives the simulated luminosity scanned on the (fractional) tune plane $\delta Q_{x} \in(0,1), \delta Q_{y} \in(0,1)$ for $\phi_{c}=2.6 \mathrm{mrad}$. The mash size is 0.025 , which is smaller than the synchrotron tune $Q_{s}=0.069$. In the figure, the magnitude of luminosity is presented with the gray scale. The darker the gray
is, the higher the luminosity gets. The contour spacing is $5 \times 10^{31} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$. The luminosity reduction due to the beam-beam driven resonances of $\nu_{x} \pm \nu_{s}=k, 2 \nu_{x} \pm \nu_{s}=$ $k, \nu_{y} \pm \nu_{s}=k, 2 \nu_{y} \pm \nu_{s}=k, 2 \nu_{x} \pm 2 \nu_{y} \pm \nu_{s}=k, k$ being integer, is indicated in Figure 1. The simulation is also done when $\phi_{c}=0$, and the results show that it is hard to find the difference between them. This means that the effect of the crossing angle of $2 \times 2.6 \mathrm{mrad}$ is small enough.


Figure 1: Tune scan for $\phi_{c}=2.6 \mathrm{mrad}$.

## 4 BEAM-BEAM PARAMETERIZATION

### 4.1 Beam-beam parameter

It can be found from eq.(1) that the larger the beam-beam parameter $\xi_{y}$ reaches, the higher the luminosity can be obtained. However, the maximum value of $\xi_{y}$ is limited by the beam-beam interaction. The simulation shows that the vertical beam blow-up takes place for $\xi_{y}$ above 0.02 . The blow-up factor is $1.1 \sim 1.2$ for the design value of $\xi_{y}=0.04$. The relative luminosity keeps above the analytic values of the luminosity for $\xi_{y} \in(0,0.05)$ at the tunes of $\delta Q_{x}=0.53$ and $\delta Q_{y}=0.60$. This is understood as the dynamic $\beta$ effect. The simulation confirms the design value of $\xi_{y}=0.04$ is reasonable with certain safety margin.

### 4.2 Crossing angle

A crossing angle of $2 \times 2.6 \mathrm{mrad}$ is chosen for the BTCF in order to make it possible to increase the bunch number. However, the collision with the crossing angle will not only cause the geometric luminosity reduction, referring to eq.(2)-(4), but also influence on the synchro-betatron coupling. The question remains how large the crossing angle is acceptable. Figure 2 displays the simulated luminosity and vertical beam size as functions of crossing angle. It is revealed in Figure 2 that the vertical beam size increases rapidly when the crossing angle is larger than 4 mrad and the luminosity reduction occurs. However, the slope of the luminosity reduction is smaller than the beam blow-up rate. This is explained as the non-Gaussian tail, which may
largely influence the rms beam size, while the luminosity is still dominated by the beam core. The simulation shows that for the design crossing angle $\phi_{c}=2 \times 2.6 \mathrm{mrad}$ the luminosity reduction is only a few percent.


Figure 2: Luminosity and vertical beam size vs. $\phi_{c}$.

### 4.3 Bunch length

The simulated luminosity as a function of bunch length is shown in Figure 3. It can be found from the figure that the simulated luminosity is close to the analytic values calculated with eq.(4) for $\beta_{y}^{*} / \sigma_{z} \approx 1\left(\beta_{y}^{*}=1 \mathrm{~cm}\right)$. At the design value of bunch length $\sigma_{z}=0.76 \mathrm{~cm}$, the luminosity $L \approx L_{0}$ is expected. However, if the bunch length gets to 1.25 cm , the luminosity reduces to $0.8 L_{0}$ according to the simulation. In order to maintain the good luminosity, the bunch lengthening for various reasons should be avoided.


Figure 3: Luminosity and vertical beam size vs. $\sigma_{z}$.

## 5 MONOCHROMATOR

The monochromator mode provides a narrow center-ofmass energy spread in the collider. This is achieved by means of opposite orbit dispersion of electron and positron beams. The vertical dispersion functions at the IP is chosen as $D_{y}^{e+, e-}=0.35,-0.35 \mathrm{~m}$ for the BTCF monochromator. The large dispersion function makes the vertical beam size dominated by the orbit dispersion $\sigma_{y} / \sigma_{y \beta} \simeq 11$ and vertical flat beam $\sigma_{y} / \sigma_{x} \simeq 13$ in the monochromator. A particle with the betatron amplitude $y$ and momentum error of $\delta$ receives a dipole kicker of $-4 \pi \xi_{y}\left(y+D_{y}^{*} \delta\right) / \beta_{y}^{*}$ from the opposite beam. This momentum dependent beam-beam kick may drive the synchro-betatron resonance.

The tune scan is done based on the parameters listed in Table 1. The mesh size of 0.025 is smaller than $Q_{s}=0.057$, the vertical beam-beam parameter is chosen as 0.015 . Figure 4 pictures a 3-dimensional vertical betatron beam size
on the $Q_{x}-Q_{y}$ plane. The resonance lines of $Q_{y} \pm Q_{s}=k$, $2 Q_{y} \pm Q_{s}=k, 3 Q_{y} \pm Q_{s}=k, 4 Q_{y} \pm Q_{s}=k$ are clearly seen in the figure, where $k$ is integer.


Figure 4: Vertical beam size scan on $Q_{x}-Q_{y}$ plane.
The beam size blow-up can be reduced by a careful choice of the tunes. As the design tunes are in the region of ( $0,0.5$ ), we choose $Q_{x}=0.08$ and $Q_{y}=0.11$ for the further study.

Figure 5 displays the luminosity and beam size vs. the vertical beam-beam parameter. Although the vertical beam size increases about linearly with $\xi_{y}$, the luminosity does not reduce obviously at the chosen tunes of $Q_{x}=0.08$ and $Q_{y}=0.11$. The design beam-beam parameters $\xi_{y}=0.015$ and $\xi_{x}=0.014$ sit at a quite comfort region where no significant blow-up takes place and the luminosity is close to the analytic value.


Figure 5: Luminosity and beam size vs. $\xi_{y}$ for $D_{y}^{*}=0.35 \mathrm{~m}$.
The luminosity and beam size as functions of vertical dispersion are shown in Figure 6. The simulation has shown a possibility to increase the vertical dispersion, say to 0.5 m , in the case of the optimized tunes and other parameters from the viewpoint of beam-beam interaction. The primary study on beam-beam effects shows that it is possible to get the goal luminosity of monochromator mode.

## 6 PARASITIC BEAM-BEAM EFFECTS

The multi-bunch collision will cause the parasitic beambeam interaction. In the BTCF, there are seven and two parasitic crossings symmetrically located on either sides of the IP in the crossing angle scheme and monochromator respectively. The tune shift $\Delta Q_{x}$, and $\Delta Q_{y}$ experienced by


Figure 6: Luminosity and beam size vs. $D_{y}^{*}$.
a particle at the center of the bunch from a single parasitic IP are given by [4]:

$$
\begin{equation*}
\left|\Delta Q_{x, y}\right|=\frac{n_{b} r_{e} \beta_{x, y}^{p c}}{2 \pi \gamma(2 d)^{2}} \tag{5}
\end{equation*}
$$

where $n_{b}$ is particles per bunch, $d=\left(d_{x}^{2}+d_{y}^{2}\right)^{1 / 2} \gg \sigma_{x, y}$, $d_{x}$ and $d_{y}$ being the half distances between two beams in the horizontal and vertical planes respectively. Parasitic beam-beam force may also disturb closed orbits. As a zeroth order evolution of the effect, the Temnykh parameter $B$ is derived from the measurement at CESR [5] with some modifications.

$$
\begin{equation*}
B=\frac{10372}{\gamma} \frac{10 n_{b}}{1.6 \times 10^{11}} \sqrt{\sum_{i}^{n_{p c}}\left(\frac{\beta_{y}^{p c} \sigma^{2}}{(2 d)^{2}}\right)_{i}^{2}} \tag{6}
\end{equation*}
$$

where $\sigma$ is the relevant beam size.
The parasitic beam-beam parameters for the crossing angle scheme and monochromator are given in Table 2. According to the experience of CESR, $B \leq 10$ is considered to be safe. The parameters shown in Table 2 are believed to be conservative. However, there is only a little experience in this aspect, we intend to carry on relevant simulation and machine studies with BEPC.
Table 2: Parameters for the parasitic beam-beam interaction.

| Parameter | Crossing Angle | Monochromator |
| :---: | :---: | :---: |
| $E(\mathrm{GeV})$ | 2.0 | 1.55 |
| $I_{\text {bunch }}(\mathrm{mA})$ | 6.6 | 7.1 |
| $N_{b} 10^{10}$ | 5.3 | 5.7 |
| $2 \phi_{c}(\mathrm{mrad})$ | $2 \times 2.6$ | 0 |
| $\sum \Delta Q_{x}\left(10^{-3}\right)$ | -3.44 | 8.44 |
| $\sum \Delta Q_{y}\left(10^{-3}\right)$ | -0.46 | -10.8 |
| B | 7 | 6 |

## 7 REFERENCES

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