STIMULATED DIELECTRIC WAKEFIELD ACCELERATOR‡

T-B. Zhang,* J.L. Hirshfield,*,§ T.C. Marshall,† and B. Hafizi*
*Omega-P, Inc., New Haven CT, 06520
§Department of Physics, Yale University, New Haven CT 06511
†Department of Applied Physics, Columbia University, New York City 10027

Abstract

A dielectric-lined waveguide can be built that supports multi-mode, nearly non-dispersive propagation; wherein a superposition of TM_{0n} wakefields is shown to result in highly peaked axial electric fields localized on each driving bunch in a periodic sequence. This allows stimulated emission of wakefield energy to occur at a rate which is much larger than the coherent spontaneous emission from a single driving bunch of equal charge. This mechanism can make possible design of a stimulated wakefield accelerator that appears to have the potential of providing an acceleration gradient for electrons or positrons in the range of 50-100 MeV/m, taking a driving bunch charge of a few nC. We present calculations for such wakefields, and give an example with acceleration of a 30 MeV test bunch to 155 MeV in a 2-D dielectric waveguide 200 cm in length, using ten identical 2.0 nC/mm drive bunches.

1 INTRODUCTION

In the dielectric wakefield accelerator [1 - 5], a dielectric--loaded waveguide supports wakefields induced by the passage of an electron bunch of high charge number (the driving bunch). If a test bunch of low charge number is injected at a suitable interval after the driving bunch, it can move in synchronism with the wakefields and experience net acceleration [1-3]. Wakefield acceleration by multiple driving bunches has also been investigated both theoretically and experimentally [4,5]. In this paper, we analyze both spontaneous and stimulated wakefield emission in a waveguide from a train of driving bunches [6]. The multi-mode structure of the wakefields, and stimulated emission from the train of driving bunches, are shown to allow generation of high wakefield acceleration gradients, without need for bunches of exceptionally high charge.

The particular waveguide analyzed here enjoys two uncommon virtues: (a) Many waveguide modes can participate in wakefield formation; this leads to coherent superposition, with a net wakefield amplitude that can be larger than the amplitude of the lowest mode. (b) The near-periodic character of the wakefield allows constructive interference of field amplitudes from successive bunches.

A train of moderate-charge driving bunches is employed for the build-up of an intense wakefield. The bunches are assumed to be identical, and each bunch is injected to move initially with near-synchronism in the net wakefield of prior bunches. The stimulated emission from each trailing bunch can greatly exceed coherent spontaneous Cerenkov emission from a bunch moving alone. It is the spontaneous Cerenkov wakefield emission of one bunch plus the stimulated Cerenkov emission from a train of succeeding bunches that build up a substantial wakefield amplitude.

‡This research was supported by the US Department of Energy, Division of High Energy Physics.

2 WAVEGUIDE GEOMETRY AND MODES

The model analyzed here is simplified to bring out the essential physics. Thus a two-dimensional waveguide is considered, in which two parallel slabs of dielectric are separated by a small vacuum gap, and in which the outer surfaces of the slabs are sheathed in a lossless conductor. The relative dielectric constant $\kappa = \varepsilon/\varepsilon_o$ is assumed to be independent of frequency. The geometry is depicted in Fig. 1, and all quantities are taken to be independent of *y*.

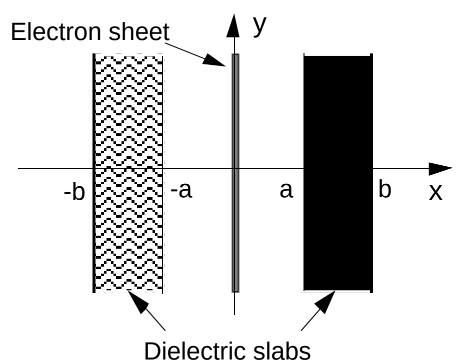

Fig. 1. Geometry for the 2D dielectric-lined waveguide.

The dispersion relation for modes having an axial electric field E_z in this geometry is found to be

$$p_m \tanh(k_m a) = \kappa\, k_m \cot\left[p_m(b-a)\right], \qquad (1)$$

where $k_m = \omega_m / c\beta\gamma = p_m \Big/ \gamma\sqrt{\kappa\beta^2 - 1}$, $\beta = v/c$, $\gamma = \left(1-\beta^2\right)^{-1/2}$, k_m is the (evanescent) transverse wavenumber in the vacuum, and p_m is the (real) transverse wavenumber in the dielectric. All the modes have the same phase velocity $v_{ph} = v$. Only the TM^x mode, also known as LSM modes [7], has an axial electric field; this is the mode considered here. It is noted that one can have eigenfrequencies with nearly equal spacing, since $p_m(b-a) \to (n+1/2)\pi$ as $\kappa \to \infty$. As $m \to \infty$ the asymptotic eigenfrequency spacing approaches

$\Delta\omega = \pi c\beta\left[(b-a)\sqrt{\kappa\beta^2-1}\right]^{-1}$. The wakefield is more strongly peaked and more nearly periodic in $z-vt$ as the eigenfrequency spacings become more nearly equal. Preliminary study indicates that similar wakefields can be generated in cylindrical dielectric-lined waveguide.

3 NEARLY-PERIODIC WAKEFIELD

The wakefield induced by an electron sheet bunch can be obtained by expanding in orthonormal modes the solution of the inhomogeneous wave equation. For a Gaussian sheet bunch of length Δz, one finds [6]

$$E_z(x,z,t) = -E_0 \sum_{m=0}^{\infty} \frac{f_m(x)}{\alpha_m} e^{-(\omega_m \Delta z/2v)^2} e^{-i\omega_m z_0/v} \tag{2}$$

where $$f_m(x) = \frac{1}{\sin p_m(b-a)} \times \begin{cases} \cosh k_m a \sin p_m(b+x), & -b \le x \le -a \\ \cosh k_m x \sin p_m(b-a), & -a \le x \le a \\ \cosh k_m a \sin p_m(b-x), & a \le x \le b \end{cases} \tag{3}$$

and $z_o = z - vt$; α_m is the normalizing constant. In Eq. 2, $E_o = -Ne/2\varepsilon_o a$ is a measure of the Coulomb field of the bunch. Eq. 2 has been evaluated for a waveguide with a = 0.30 cm, b = 1.147 cm, κ = 10.0, Δz = 0.3 cm, -Ne = Q = -2 nC/mm, and γ = 60.0. A relative dielectric constant of κ = 10.0 is close to the value of 9.6 for alumina. For these parameters, E_o = -37.7 MV/m. In this case, modes up to m = 12 in Eq. 2.are significant in evaluating the wakefield. The first frequency interval is

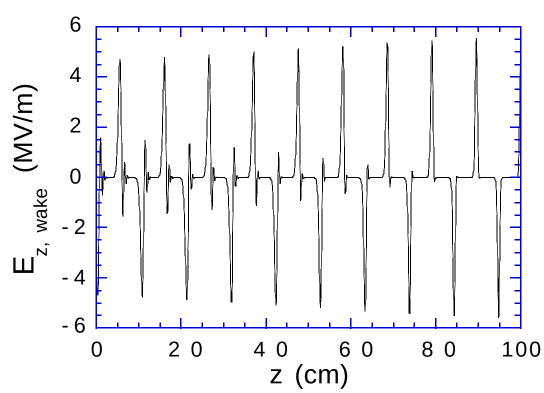

Fig. 2. Spontaneous wakefield from a -2 nC/mm, 30 MeV sheet bunch after it has traveled 100 cm left-to-right in the 2D waveguide depicted in Fig. 1.

5.70 GHz, while the asymptotic interval $\Delta\omega/2\pi$ is 5.88 GHz; eigenfrequency intervals differ by at most 3.1%. The computed spontaneous wakefield pattern of a single bunch is shown in Fig. 2 for $0 \le z_o \le 100$ cm. The wakefield peaks are seen generally to alternate in sign; to each be relatively concentrated in z_o; and to have a period of 10.5 cm, corresponding to the vacuum wavelength at 2.856 GHz, i.e. half the asymptotic frequency interval $\Delta\omega/2\pi$. The peak values of E_z for the first wake are -5.57 and +5.54 MeV/m, and later wakes develop oscillatory precursors. Consideration of only a few modes gives an incomplete wakefield.

4 BEAM DYNAMICS AND SIMULATION

A single-bunch wakefield as shown in Fig. 2 leads to a a drag field on a bare bunch that can be obtained from the energy flow into the fields [6]; here one finds E_{drag} = 1.18 MV/m. Now, when a second bunch is introduced into the waveguide at the first peak of the first bunch wakefield, it can be decelerated by up to 1.18 + 5.54 = 6.72 MV/m, the sum of the second bunch's bare drag field associated with spontaneous emission, plus the wakefield of the first bunch; this produces additional stimulated emission. Second bunch deceleration of 6.72 MV/m is 5.7 times that of a bare bunch. The second bunch adds its wakefield energy to that of the first bunch, for which we find the combined wakefield to be 14.33 MV/m. Successive amplitude enhancements from incremental addition to the net wakefield energy continues so long as synchronism is maintained between bunches and peak wakefields. This conceptual notion has been examined with greater accuracy in a numerical simulation. Particles in each 3.0-mm long bunch are injected each 350 psecs (10.5 cm) near the peak of the cumulative wakefield from all prior bunches. The initial energy of each bunch is chosen as about 30 MeV (γ = 60.0 and $v_o \approx c$).

In Figs. 3a, 3b and 4 are shown the results of injecting a test bunch of low charge into the accelerating phase of the wakefield set up by the passage of ten prior driving bunches in a structure 100 cm in length. The initial energy of the test electrons is also 30 MeV. No beam loading by the test bunch is taken into account. Fig. 3a shows the buildup of the wakefield from the ten driving bunches, while Fig. 3b shows locations of the electrons to be accelerated in the test bunch, which enter behind the tenth drive bunch at the accelerating phase, after the first drive bunch has gone 100 cm. As the drive bunches

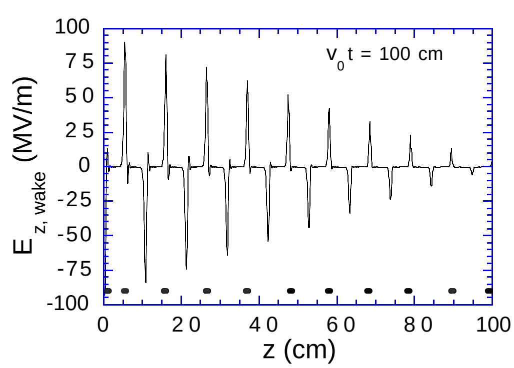

Fig. 3a. Cumulative wakefield set up by ten identical successive bunches, at the time the first bunch has moved 100 cm along the waveguide. The positions of the bunches are indicated by the heavy dots at the bottom.

proceed through the 100 cm long waveguide, the energy of the test electrons increases. In Fig. 4 we plot the energy of typical drive bunch electrons, as well as of the test electrons in bunch #11. After $v_o t$ = 200 cm, the 100 cm train of drive bunches has moved fully out of the wave-guide, and the test electrons are also ready to emerge. Their energy has been increased to γ = 200, which represents an average gradient of about 70 MV/m. The later drive bunches show energy depletion and, indeed by $v_o t$ = 120 cm, they decrease to an energy so low that there is first a slippage off the wakefield maximum, followed by a further slippage into the following accelerating phase, after which the drive bunch energy begins to increase again. As a result, the maximum wakefield is eroded downstream. Reflection of wakefields at the waveguide exit is neglected.

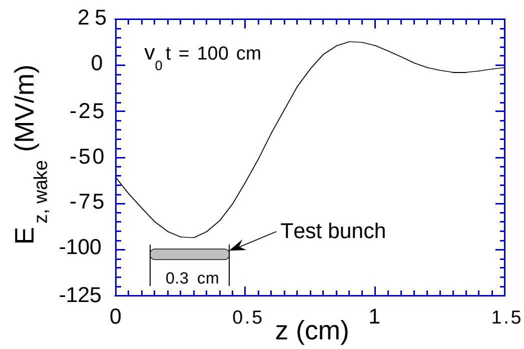

Fig. 3b. Location of the test bunch (bunch 11) in the accelerating wakefield near the entrance of the accelerator.

From the results shown in Fig. 4, it is apparent that one should endeavor to remove driving bunches from the system after their energy has fallen so much that they slip into the following accelerating phase. In Fig. 5, we show a result obtained from a 200 cm dielectric structure, where the ten drive bunches are assumed to be deflected out after traveling 100 cm, i.e. before their energy falls too low. (Discussion of means for achieving such a removal of the driving bunches is beyond the scope of this paper.) We find that the energy of the test bunch particles increases

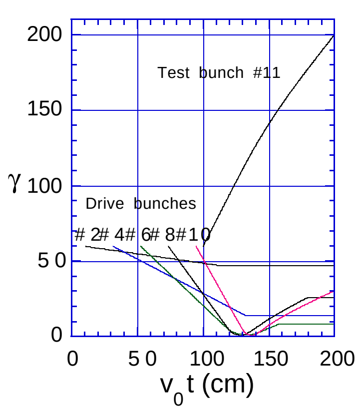

Fig. 4. History of the energy of selected drive bunches, and of the test bunch #11, up to the time the test bunch just emerges from the waveguide.

steadily to 155 MeV after traversing the system. This compares favorably with the case when the drive bunches stay in for the full 200 cm, where the test particles reach an energy of only 135 MeV. In another case, the energy of drive bunches does not fall below 3 MeV; no slippage was observed; six bunches survive to z = 65 cm; and the test electrons reach 126 MeV at $v_o t$ = 200 cm.

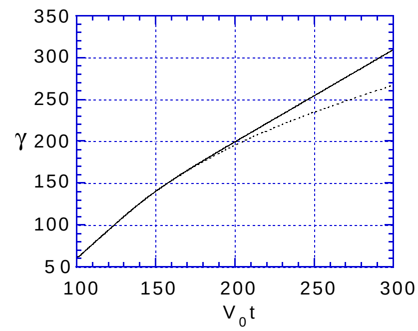

Fig. 5. Energy of test electrons versus $v_0 t$ for two cases. Dotted line is the case shown in Fig. 4, but carried out for a 200 cm long structure, and the bunches are left in the system its entire length. Solid line is the case where drive bunches are removed after traveling 100 cm.

5 CONCLUSIONS

A planar two-dimensional dielectric-lined waveguide is shown to be capable of supporting a large number of modes with nearly equal phase velocities. The waveguide can be designed so that these velocities are nearly equal to the velocity of a train of moderate charge drive bunches in a sheet beam. As a result, strongly-peaked cumulative wakefields can arise through stimulated emission that builds up from bunch to bunch. For ten 30 MeV driving bunches of 2 nC/mm each, an axial acceleration gradient of up to 70 MV/m is predicted, as compared with 5.6 MV/m for a single bunch of equal charge. A means for separating spent drive bunches from their wakefields is found to be necessary to avoid re-absorption of wakefield energy by driving bunches that slip into trailing accelerating zones. Acceleration of electrons and positrons to high energy appears feasible using a multi-stage, non-colinear, multi-bunch wakefield accelerator, based on the single-stage mechanism discussed in this paper.

6 ACKNOWLEDGMENT

Suggestions by A. Ganguly are gratefully acknowledged.

7 REFERENCES

[1] K.-Y. Ng, *Phys. Rev.* D **42**, 1819, (1990).
[2] M. Rosing and W. Gai, *Phys. Rev.* D **42**, 1829 (1990).
[3] W. Gai, et al., *Phys. Rev. Lett.* **61**, 2756 (1988).
[4] R. D. Ruth, et al., *Particle Accelerators* **17**, 171 (1985).
[5] I. N. Onishchenko, et al., *Proc. 1995 PAC*, pp. 782-3.
[6] T-B. Zhang, et al., *Phys. Rev.* E (to be published).
[7] R. E. Collin, *Field Theory of Guided Waves*, ch. 6, (IEEE Press, New York, 1991).