COPING WITH POWER-LIMITED TRANSVERSE STOCHASTIC COOLING SYSTEMS

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Abstract

We present the formulas relevant to the behavior of (transverse) stochastic cooling systems which operate under the the not uncommon condition that performance is limited by available output power, and contrast the operation of such systems with non-power-limited ones. In particular, we show that for power-limited systems, the two most effective improvements are the use of pickup/kickers which operate in both planes simultaneously [1] and/or plunging of the cooling system electrodes. We apply our results to the proposed upgrade of the Fermilab P source.

Introduction

Conventional analyses of stochastic cooling systems assume that performance is not limited by available electronic gain, and that the latter quantity can be set to maximize the cooling rate. Under these conditions, one can expect an improvement of as much as a factor of 2 in the cooling time by doubling the model operating frequency of the cooling system. In practical systems, power-limited limitations on the maximum available output power may restrict the maximum attainable gain to be less than its optimal value: such is the case for the beams from antiproton sources at both CERN and Fermilab. We show that the criteria for improving such power-limited systems are rather different from those for systems for which one can optimize the gain, in particular, the maximum expected improvement resulting from doubling the operating frequency of such a power limited system is less than a factor of 2.

In the following sections we first review the formulas relevant to the behavior of power-limited cooling systems; we limit our treatment throughout to the case of systems which cool the transverse phase space of the beam. We then discuss the implications of our results for the upgrade of such cooling systems, contrasting this case with that for systems in which the electronic gain can be optimized. Finally, we apply our results to the specific case of the Fermilab debuncher ring.

Formulary for Power-Limited Systems

The cooling rate of a stochastic cooling system is given by [2]

\[ \frac{1}{\tau} = \frac{W}{N} \left( 2g - g^2 \right) \]  

where \( W \) is the signal bandwidth of the cooling system. \( M \) is the so-called mixing factor, \( U \) is the noise to signal ratio, and \( G \) is usually referred to as the system gain; in a transverse cooling system, it represents the fraction of the beam-signal centroid error corrected in a single pass through the pickup and kicker. One can formally express the system gain as

\[ G = g' G \]  

where \( G = E^* G / E \) (2)

\[ g = g' G \]  

where \( G \) represents the electronic (voltage) amplification, and \( G' \) includes everything else (i.e., pickups, kickers, external circuit losses, etc.). Expressing \( G' \) in terms of the various system parameters, we have

\[ g' = \alpha \beta n L K L \frac{f o}{E} \frac{e}{\sqrt{2 \pi E/} \ln (E/Z)} \]  

where

\[ N = \text{total number of particles} \]
\[ \alpha = \text{voltage attenuation in the pickup and kicker circuitry located between the electrodes and the amplifier circuits} \]
\[ \beta = \text{(geometric) mean of beta functions at pickup and kicker} \]
\[ \nu = \text{proton charge} \]
\[ f o = \text{particle revolution frequency} \]
\[ c = \text{velocity of light} \]
\[ L K = \text{number of kickers/pickup loop pairs} \]
\[ Z L = \text{single loop pair (transverse) transfer impedance} \]
\[ K L = \text{single loop-pair (transverse) kicker constant = \alpha Z U/\nu} \]
\[ f N = \text{mid band signal (s) frequency} \]
\[ Z C = \text{characteristic impedance of external signal lines} \]

For non-power limited systems, one minimizes \( \tau \) by setting \( g = 1/(M+U) \), and hence yielding the familiar result

\[ \frac{1}{\tau} = \frac{W}{N} \left( M+U \right) \]  

We now consider the results for power limited systems. If we define \( G_{lim} \) as the maximum available (i.e., power limited) electronic gain, and \( G_{opt} \) as the gain required to yield \( g_{opt} = 1/(M+U) \). We can then write

\[ \frac{1}{\tau_{lim}} = \frac{1}{\tau_p} \left( 2 - \frac{G_{lim}}{G_{opt}} \right) \]  

where, for analyzing power-limited systems, it is convenient to introduce the quantity

\[ \frac{1}{\tau_p} = \frac{1}{\tau_{opt}} \frac{G_{lim}}{G_{opt}} \]  

which can also be written in the form

\[ \frac{1}{\tau_{lim}} = \frac{W}{N} \frac{G_{lim}}{G_{opt}} \]  

We can express \( G_{lim} \) in terms of \( T_x \) and \( T_y \), the equivalent noise temperatures of the input circuit and preamplifier, respectively, and the electronic bandwidth \( W \).

\[ G_{lim} = \sqrt{\frac{P_{opt}}{1 + U}} \]  

The quantity \( \tau_{opt} \) is then given by

\[ \frac{1}{\tau_{opt}} = \frac{\nu}{2 \pi E/} \frac{f o}{e} \frac{\sqrt{P_{opt} W}}{1 + U} \]  

If one has already calculated \( G_{lim} \) and one now use Eq. 6 to obtain

\[ \frac{G_{lim}}{G_{opt}} = \frac{1}{\tau_p} \frac{\nu}{2 \pi E/} \frac{f o}{e} \frac{\sqrt{P_{opt} W}}{1 + U} \]  

To calculate the ratio directly, we can write

\[ G_{lim} = \frac{e \nu \beta n L (Z L)^2}{2 \pi E/} \frac{f o}{e} \frac{\sqrt{P_{opt} W}}{1 + U} \]  

To evaluate either Eq. 4 or 8, we use the expression

\[ U = 2 \pi E/ \frac{f o}{e} \frac{\sqrt{P_{opt} W}}{1 + U} \]  

where \( R \) is the voltage attenuation factor for the external pickup electronics, and the average emittance \( \gamma \) is defined by the relation

\[ \frac{< x^2 >}{< x_0^2 >} = \frac{R \gamma}{\gamma_0} \]  

\[ R = \frac{< x_0^2 >}{< x_0^2 >} \]  

where \( \gamma_0 \) is the beta function at the pickup, and for a (2-dimensional) Gaussian emittance distribution, \( \gamma = \gamma_0 \). Finally, to evaluate the ability

\[ 1 - \frac{< x^2 >}{< x_0^2 >} \]
of a system to cool a beam from an initial emittance $e_i$ to a final emittance $e_f$, we calculate the total cooling time $T_{tot}$:

$$T_{tot} = \int_{e_i}^{e_f} \frac{\pi(t) \, dt}{e_i}$$  \hspace{1cm} (15)$$

**General Conclusions**

We begin by reviewing the situation for systems which are not power limited. Let us assume for definiteness that we have a cooling system which operates over a one-octave frequency range. Eq. 4 shows that doubling the mid-band frequency doubles the cooling rate due to the doubling of $W$. If the system is mixing-limited, an additional factor of two results from halving $M$. A similar additional factor of two is usually obtained for noise-limited systems as well. Under the combined assumptions that the length of individual pickup elements is proportional to the operating frequency, it is possible to preserve the same pickup impedance for the higher frequency electrodes, and that the total space available for electrodes remains unchanged, doubling the operating frequency permits a doubling of the number of electrodes, and hence a halving of $U$ and a doubling of the cooling rate. In practice, this gain is partially offset by the increases in the preamplifier noise temperature and external circuit attenuation which accompany an increase in operating frequency. Hence overall, the cooling rate increases proportional to something between the first and second power of $f_0$.

Let us now consider the power-limited system. From Eq. 5, we see that the quantity which best characterizes the performance of such a system is $T_{opt}$, which is defined by Eq. 6. For $G_{opt} / G_{opt} < 1$, the power-limited cooling rate $T_{opt}^{-1}$ is simply given by $2 \cdot T_0^{-1}$, as the beam cools, the gain ratio approaches unity, and the cooling rate falls by a factor of $1 / 2$ $T_0^{-1}$, while at the same time $G_{opt}$ approaches $T_0$. As the ratio exceeds unity, the system is of course no longer power limited, and the maximum cooling rate is determined by $T_{opt}$ from Eq. 4. The situation is illustrated in Fig. 1, where we have replaced $T_{opt}$ by $T_{lim}$ in the region where the gain ratio would exceed unity.

Using $T_0^{-1}$ as our basic figure of merit, we see from Eq. 11 that most, if not all, the advantage in going to higher frequency is lost when the system is power-limited. The doubling of $n_A$ made possible by the reduced electrode length is offset by the factor of $T_{0 \pi}$ in the denominator, which arises from the $1 / f$ dependence of the piezoelectric constant (this is based on the reasonable assumption that it is the transfer impedance, rather than the mass constant, which one can preserve when raising the frequency). Also, because $G_{opt}$ decreases as $W^{1 / 2}$ due to the increased noise bandwidth at higher frequency, the explicit $W$-dependence of $T_0^{-1}$ is as the one-half power, rather than the usual linear one. Moreover, this improvement is likely to be at least partly offset (possibly even more often offset) by increases in attenuation and amplifier noise which usually characterize a frequency increase.

As noted above, cooling of the beam may cause the system's operating range to span both the power-limited and noise-limited regimes. As might be surmised, (and as is shown explicitly in Ref. 3), such a system exhibits a greater than $\sqrt{2}$ improvement with a doubling of the operating frequency even in the region prior to it emerges from its power-limited condition.

An additional distinction between power-limited and non-power-limited systems concerns their scaling with beam aperture. Assuming that the pickup impedance $Z_L$ scales as the reciprocal of the gap width, it is straightforward to show that for the latter type of system, the time to cool to a given fraction of the initial emittance is independent of the initial gap. However, as shown in Ref. 3, for a power-limited system, that time increases as the gap increases.

To improve the performance of power-limited systems, in most cases one must either increase the available amplifier power, or increase the number of output TWTs in the transverse cooling system. The first is the introduction of a notch filter in the low-level electronics to suppress the noise signal in between the betatron sidebands. The effect of this filter is ideally to reduce the noise bandwidth in the expression for $G_{opt}$ by a factor of a few. Our calculations assume such ideal performance. Note that because the filter suppresses the noise only at frequencies at which the noise does not heat the beam, it leaves the value of $U$ unaffected (however, the signal power term in Eq. 9 must now be changed from $1/2$ to $2/3$).

Our calculations include the effects of two improvements in the existing electronics which are currently being undertaken to ameliorate the severely power-limited condition of the present system but which, by themselves, will not suffice to meet the above goals. The first is a straightforward increase in the maximum power available by doubling the number of output TWTs in the transverse cooling system. The second is the improvement in the performance of the low-level electronics to suppress the noise signal in between the betatron sidebands. The effect of this filter is ideally to reduce the noise bandwidth in the expression for $G_{opt}$ by a factor of $2/3$. Our calculations assume such ideal performance. Note that because the filter suppresses the noise only at frequencies at which the noise does not heat the beam, it leaves the value of $U$ unaffected (however, the signal power term in Eq. 9 must now be changed from $1/2$ to $2/3$).

For reasons which will become apparent, we consider both 4-8 GHz systems at maximum power levels of 2.5 kW (the comb output power capability as the 2-4 GHz system), and 5 kW; the effects of the notch filter are included for all systems. We consider each system at intensities of $N=4, 8$, and $16 \times 10^3$. The remaining system parameters used in our calculations are listed in Table 1.

We made two sets of calculations, one for both fixed electronics, and one for so-called unplanned electronics, where the electronics are moved inward to follow the envelope of the beam as the beam cools. For these calculations, we made the conservative assumption that the pickup impedance increased as the reciprocal of the electrode gap.

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3 An alternate specification is $30 \pi/3$ to $3\pi/3$; the ramifications of this alternative are discussed below.

4 A detector capable of sensing motion in both transverse planes simultaneously.

5 The p-source group at Fermilab has recently developed a design for a 4-8 GHz detector [4]; employing striplines arranged in two parallel arrays in order to achieve adequate lateral coverage of the beam, which possibly can also be used as a bi-planar detector, although its performance appears inferior to the corner detector of Ref. 1. We have adopted the "p1-planar" designation as a way of avoiding the separate issue of which design makes for a superior bi-planar detector.

6 Our initial models of the bi-planar corner detector appear to show that the longitudinal loop separation can be reduced to the point that the longitudinal loop density can be increased by as much as 40%; however to keep our estimates conservative, we have neglected this factor in our calculations.
For all of the cooling scenarios we list the cooling times from an initial (full) performance of an additional doubling of the output power to 5 kW.

4-8 GHz systems remain power-limited down to nearly the smallest emittances. More detailed results, showing all of the calculated quantities at a number of intermediate emittances, are presented in Ref. 3. Because the 2.5 kW 4-8 GHz systems remain power-limited down to nearly the smallest emittance, we felt it reasonable to calculate the effect on their performance in both the power-limited and non-power limited regimes.

A summary of the results of the calculations is presented in Table 2; for all of the cooling scenarios we list the cooling times from an initial (full) performance, and for power-limited situations, such as that which exists at the Fermilab debrischider ring, after performance improvements greater than those resulting from an increase in operating frequency. The cooling times for fixed-electrode uni-planar systems are compared to those of a bi-planar design which can be plunged, such a detector will clearly outperform an unplunged one. For the 4-8 GHz system doubles the available output power is less efficacious than either bi-planarity or plunging, and its efficacy decreases in precisely those regions for high intensity. The lower emittance where the demands on the cooling system are greatest. Furthermore, it would be of even less benefit at 2-4 GHz, where (as one can see from the table) one is less severely power-limited.

In conclusion, bi-planarity and plunging offer comparable improvements in performance, and for power-limited situations, such as that which exists at the Fermilab debrischider ring, after performance improvements greater than those resulting from an increase in operating frequency. Moreover, the first two approaches permit one to utilize the electronics associated with the existing cooling system, thereby giving them a decided advantage in both time and cost. If one can arrive at a bi-planar electrode design which admits of plunging, it would seem to be worthwhile to implement it. Otherwise, it is not clear that the mechanical complexity and expense involved in a plunged system is warranted.

As anticipated, the bi-planar 4-8 GHz system outperforms the uni-planar system by roughly a factor of two throughout, by virtue of having twice as many electrodes (which, as noted above, doubles its performance in both the power-limited and non-power limited regimes). What is perhaps more surprising, is that for all but the highest intensity and low emittance, where the demands on the cooling system are greatest.

Looking at the results for plunged detectors, it is clear that plunging is considered as an alternative to bi-planarity, for a given frequency range and power level, a plunged uni-planar detector gives cooling times comparable to those of a fixed-electrode bi-planar one, not performing it only at the very smallest emittances. On the other hand, if one can arrive at a bi-planar design which can be plunged, such a detector will clearly outperform an unplunged one; the bi-planar design in Ref. 1 does not easily admit of plunging, and perhaps an alternate design, for example, utilizing a pair of side-by-side parallel plates [4], could be made to work.

Finally we should note that, as expected, for the 4-8 GHz system doubling the available output power is less efficacious than either bi-planarity or plunging, and its efficacy decreases in precisely those regions for high intensity. The lower emittance where the demands on the cooling system are greatest. Furthermore, it would be of even less benefit at 2-4 GHz, where (as one can see from the table) one is less severely power-limited.

Acknowledgements

In writing this paper, we have benefited from discussions with the P source group at Fermilab. We are particularly indebted to John Martin for his ready availability to answer questions about the operation of the Fermilab P source.

References


Table 2: Cooling Times for Fixed Uni- and Bi-Planar Arrays

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<tr>
<th>No. of Electrons</th>
<th>Envelope Emittance</th>
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<th>2.5 kW 4 to 8 GHz</th>
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<td></td>
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<td>Plunged</td>
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<td>30π</td>
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<tr>
<td>15π</td>
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