PLASMA WAKEFIELD ACCELERATION UTILIZING MULTIPLE ELECTRON BUNCHES

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Abstract

We investigate various plasma wakefield accelerator schemes that rely on multiple electron bunches to drive a large amplitude plasma wave, which are followed by a witness bunch at a phase where it will sample the high acceleration gradient and gain energy. Experimental verifications of various two bunch schemes are available in the literature; here we provide analytical calculations and numerical simulations of the wakefield dependency and the transformer ratio when M drive bunches and one witness bunch are fed into a high density plasma, where M is between 2 and 10. This is a favorable setup since the bunches can be adjusted such that the transformer ratio and the efficiency of the accelerator are enhanced compared to single bunch schemes. The possibility of a five bunch ILC afterburner to accelerate a witness bunch from 100 GeV to 500 GeV is also examined.

INTRODUCTION

In this paper we investigate various plasma wakefield acceleration schemes using multiple electron bunches to drive the plasma wakes [1]. By adjusting the relative position of the bunches and/or the number of particles in each bunch, the accelerator can be tuned to either generate the maximum possible accelerating gradient or operate at maximum efficiency. Optimum efficiency is obtained when all the particles in the drive bunches experience the same retarding wakefield and lose energy at the same rate [2].

A plasma-based energy multiplier for ILC will probably utilize multiple bunches in order to achieve enhanced transformer ratios. Proof of principle experiments are currently underway at Brookhaven's Accelerator Test Facility (ATF) where multiple bunches can be produced either with a CO_2-driven IFEL [3] or by using an appropriately designed mask to partially block sections of an energy-dispersed single long bunch [4]. The next two sections provide 2D linear theory calculations [5] for trains of bunches that can be generated at the ATF, while the last section examines an efficient ILC afterburner design using 5 bunches and a 100 GeV linac to produce 500 GeV electrons.

The present discussion will be limited to longitudinally square-shaped bunches such as the ones ideally created by the mask technique, although the exact shape does not matter as long as the plasma wavelength is large enough so as not to resolve the bunch shape. The beam density n_b of the relativistic driving electron bunch train can be written in the following way:

$$\mathbf{n_b(\xi, r) = \sum_{m=1}^{M} \frac{Q_m}{2\pi e \sigma_r^2 w} rect\left(\frac{\xi - \xi_m}{w}\right) e^{-\frac{r^2}{2\sigma_r^2}}} \quad (1)$$

Here e is the electron charge, ξ = z-ct and r are the longitudinal and radial position in the relativistic bunch frame, M is the total number of bunches, σ_r = 100 μm is the transverse beam size, w is the width of each bunch, and Q_m and ξ_m are the charge and the relative position of the m-th bunch respectively. The function *rect* has a value of 1 for $|\xi-\xi_m| < w/2$ and 0 outside this range.

MAXIMIZING THE WAKEFIELD

Single Bunch

The on-axis wake amplitude E_0 generated by a single square bunch is (in SI units)

$$\mathbf{E_1 = \frac{Q_1}{\varepsilon_0 \pi \sigma_r^2 k_p w} \sin\left(\frac{k_p w}{2}\right) R(0) \sim \frac{n_{b1}}{n_p}\sqrt{n_p}} \quad (2)$$

Here $k_p = 2\pi/\lambda_p$ = sqrt($e^2 n_p/(\varepsilon_0 mc^2)$) is the plasma wavenumber, n_p is the plasma density, m is the electron mass, ε_0 is the vacuum permittivity and R(0) accounts for transverse effects [5]. Typically the maximum possible wake is obtained when the bunch is made as short as possible ($w \rightarrow 0$) while the charge is preserved. However, due to the special way that the bunches are formed at ATF by blocking parts of the initial beam profile, the number of particles that remain is proportional to the width w of the bunch. In that case the wake is optimized when $\sin(k_p w/2) = 1$ or $w = \lambda_p/2$ and all the particles of the bunch are in the first half of the plasma wave and they all contribute to the wake by losing energy. Then E_1 = 1.14MV/(m-pC) × R(0).

Multiple Bunches

When a train of M identical bunches is fed into the plasma, the maximum wakefield is obtained when the bunches are separated by one plasma wavelength, i.e., $\xi_m = m\lambda_p$ (m = 1, 2,…M). In this case the individual wakes add up and the final wake amplitude scales linearly with the number of bunches, $E_{final} = M \times E_1$, where E_1 is the wake of the first drive bunch.

Figure 1 shows an example where five 30-pC drive bunches with w = 125 μm separated by 250 μm excite a 136 MV/m wake when the plasma density is tuned to 1.8 × 10^{16} cm^{-3} (λ_p = 250 μm). In fact, in this scenario the final amplitude does not depend on the actual distribution

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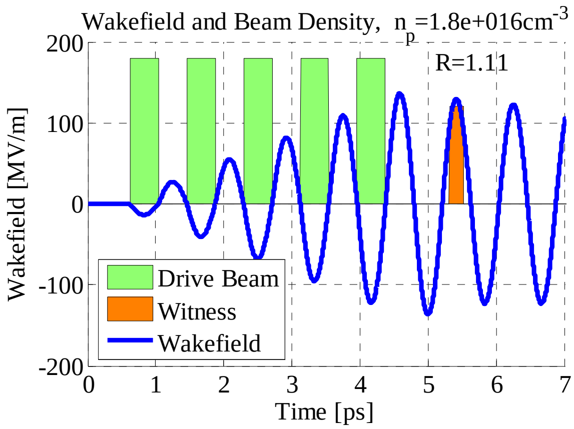

Figure 1: (Max Wakefield Scheme) Bunch distribution for maximum wake generation. A witness bunch 180^o out of phase samples the accelerating wakefield.

of the charge among the bunches as long as the total charge of all the bunches remains the same. The accelerating field is then sampled by a trailing witness bunch.

MAXIMIZING THE TRANSFORMER RATIO

A plasma accelerator operated in a maximum wakefield configuration as shown above is inherently inefficient because the trailing bunches transfer their energy to the plasma at a much higher rate than the early bunches since they experience the decelerating wake of every previous bunch, and as a result the maximum energy gain is limited. The efficiency is increased when every bunch experiences the same retarding wake. The transformer ratio, the ratio of the maximum accelerating wake after the beam to the maximum decelerating wake inside the beam, is a measure of the maximum energy gain acquired by a trailing bunch and can be maximized. For the scenario described in the previous section, the transformer ratio approaches its minimum value of 1 as the number of bunches increases, despite the fact that the peak wakefield increases linearly at the same time.

Single Bunch

The transformer ratio of a *single* and *symmetric* bunch has a maximum of 2 when the maximum retarding wake occurs in the center of the bunch [6]. For perfectly square bunches this condition is satisfied when $k_p w = \pi$ (see Figure 2). For Gaussian bunches this is true when $k_p\sigma_z = \pi/2$ [7]. This upper limit can be overcome either by using a single asymmetric bunch [8] or by introducing more than one drive bunches. We are going to examine the second approach in more detail.

Multiple Bunches: Ramped Bunch Train

Using equidistant bunches, the maximum possible transformer ratio can be obtained when the bunches are placed at the accelerating phase of the wakefield (half-integer plasma wavelengths apart) and at the same time the charge in each bunch is increased so that the wake inside each bunch equals the decelerating wake of the first bunch. This scheme has been demonstrated experimentally in a dielectric wakefield accelerator [2] using the following scenario:

$$\xi_{m\geq 2} = \xi_1 + m\frac{3\lambda_p}{2},\ Q_m = \left(\left(m-1\right)R_1 + 1\right)Q_1 \quad (3)$$

Here R_1 is the transformer ratio of the first bunch. If the above conditions are met, the transformer ratio increases linearly with the number of bunches ($R = MR_1$), while the peak accelerating field is only E_{final}=M × E_1 (at the expense of the total charge which scales as M^2Q_1 for R_1 = 2). E_1 is the wake excited by the first bunch. From an energy standpoint, each bunch transfers as much energy to the accelerating wake as the first bunch does, and the rest of its energy is transferred through the plasma to the subsequent bunches to prevent them from decelerating at higher rate. All the bunches then lose energy at the same rate.

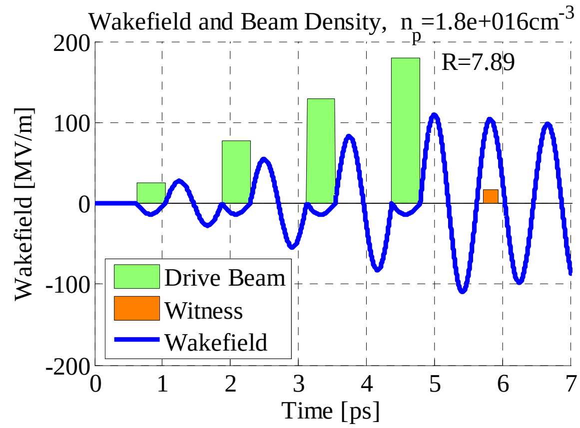

Figure 2: (Ramped Bunch Train Scheme) Bunch distribution for maximum transformer ratio. The drive bunches are separated by 1.5 plasma wavelengths.

Figure 2 shows an example of a case where 4 drive bunches with $k_p w=\pi$ whose charges scale as 15:45:75:105 pC drive a 110 MV/m wake when they are placed 1.5 plasma wavelengths apart. The wake amplitude is small given the total charge provided, but the transformer ratio is almost quadrupled. Notice that the phase of the wakefield is reset after each drive bunch.

Multiple Bunches: Phased Bunch Train

Another scenario where the transformer ratio can be enhanced with increasing number of bunches is to use identical bunches with careful positioning. Each bunch needs to be placed at a specific decelerating phase of the wakefield, such that, for a given bunch charge, the total wakefield inside each bunch will be brought exactly to the level of the first drive bunch. Since the wakefield phase will shift when a new bunch is inserted, the relative position of this subsequent bunch needs to be adjusted accordingly. Optimal transformer ratio is obtained when the bunch positions follow [9]:

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$$\xi_{m\geq 2} = \xi_1 + (m-1)\lambda_p + \frac{\lambda_p}{2\pi}\sum_{n=1}^{m}\tan^{-1}\left(\frac{1}{\sqrt{n-2}}\right) \quad (4)$$

If the bunches are appropriately short ($k_p w \ll 1$), then the above scenario yields both a transformer ratio and a wakefield amplitude that scale as the square root of the number of bunches. Compared to the ramped bunch train scenario, the transformer ratio is not enhanced as much, but the accelerating wakefield is now larger.

Figure 3 shows an example of the above scenario using four 30-pC short drive bunches in a 1.8×10^{16} cm^{-3} density plasma. The first bunch is made longer ($k_p w = 2\pi/3$) in order to enhance its own transformer ratio, however the following drive bunches must be short ($k_p w = \pi/4$ in this figure) so that their proper phasing is effective. In this case the transformer ratio is enhanced (compared to single bunch schemes) to 2.67 and the wake amplitude reaches 86MV/m using only half the charge of the ramped bunch train scheme.

A PLASMA-BASED AFTERBURNER DESIGN FOR ILC

Table 1 summarizes the main aspects of the three aforementioned schemes assuming M drive bunches. Q_1 is the charge of the first drive bunch, E_1 is the maximum wake generated by that first bunch, and E_0 is the wake generated by a single bunch with unit charge. The transformer ratio of the first drive bunch is assumed to be maximum, $R_1 = 2$. The table shows that there is always a tradeoff between the maximum wakefield and the transformer ratio.

An afterburner from 100 GeV to 500 GeV for ILC can be designed by extending the ramped bunches scheme (Figure 2) within the limits of linear regime; an exact design will have to account for the nonlinear effects at ILC. A small witness bunch can gain 400 GeV (per particle) by sampling a ~15 GV/m accelerating wakefield over ~27 m of plasma. This wakefield can be excited by a train of 4 ramped drive bunches if the plasma density is

Scheme	Total Beam Charge	Max Field Amplitude	Field per unit total charge*	Transf. Ratio
Max Field	$M \cdot Q_1$	$M \cdot E_1$	E_0	$\frac{M}{M-1/2}$
Ramped Bunches	$M^2 \cdot Q_1$	$M \cdot E_1$	$\frac{E_0}{M}$	2M
Phased Bunches	$M \cdot Q_1$	$\sqrt{M} \cdot E_1$	$\frac{E_0}{\sqrt{M}}$	$2\sqrt{M}$

Table 1 : Comparison of the theoretical main parameters for each multibunch scheme for $R_1 = 2$. *Values for point-charge bunches.

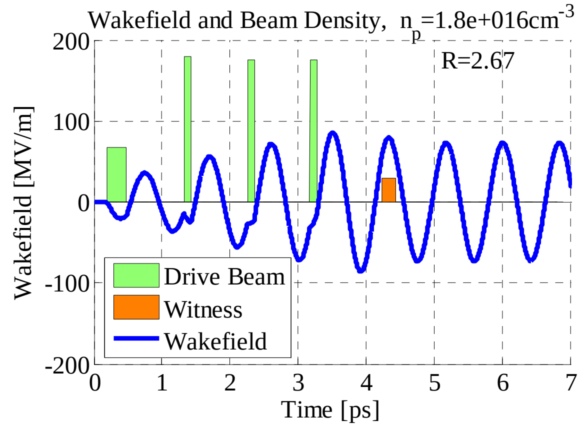

Figure 3: (Phased Bunch Scheme) Four bunches with equal charge placed properly (Eq.4) enhance the transformer ratio beyond 2.

increased to $n_p = 2 \times 10^{17}$ cm^{-3} and the beam charge is increased 5 times, while the focused beam size is $\sigma_r \sim 1/k_p \sim 10$ μm (see Equation 2). The total charge is 1.2 nC and needs to be distributed as 75:225:375:525 pC while the bunches are separated by $1.5\lambda_p$ = 112μm (or any half-integer number of plasma wavelengths). Finally, in this model, if the witness bunch only loads the wake by 30% in order to minimize the energy spread, its charge would have to be $0.3^*Q_{total}/R = 0.3 \times 1.2nC/8 = 45$ pC [10].

If the same amount of charge was distributed into 4 identical equidistant bunches (or one single drive bunch), then the maximum accelerating wakefield could be 4 times higher, ~60 GV/m, but the fourth drive bunch would lose all its energy in less than 2 m in the plasma, thus limiting the possible energy gain of the witness particles to 120 GeV. However, using the 4 ramped bunches, the transformer ratio in principle can be close to R = 8 and the decelerating wakefield inside any drive bunch is only (15 GV/m)/R = 1.875 GV/m.

This example demonstrates the advantages of using transformer ratio enhancement techniques in plasma accelerators [11].

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