

FOURIER SPECTRAL SIMULATIONS FOR WAKE FIELDS IN CONDUCTING CAVITIES *

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Abstract

We investigate Fourier spectral time-domain simulations applied to wake field calculations in two-dimensional cylindrical structures. The scheme involves second-order explicit leap-frogging in time and Fourier spectral approximation in space, which is obtained from simply replacing the spatial differentiation operator of the YEE scheme by the Fourier differentiation operator on nonstaggered grids. This is a first step toward investigating high-order computational techniques with the Fourier spectral method, which is relatively simple to implement.

FORMULATIONS

We study beam dynamics in two-dimensional conducting cavity structures. The governing equations and the numerical scheme are as follows.

Maxwell's Equations

We begin with the Maxwell equations:

$$\mu \frac{\partial H}{\partial t} = -\nabla \times E, \quad \epsilon \frac{\partial E}{\partial t} = \nabla \times H - J \quad (1)$$

$$\nabla \cdot E = \frac{\rho}{\epsilon}, \quad \nabla \cdot H = 0, \quad (2)$$

where the current source J is defined for an on-axis Gaussian beam moving in the x -direction:

$$J = ce_x \rho(y) \rho(x - ct), \quad \rho(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{\left(-\frac{x^2}{2\sigma_x^2}\right)}. \quad (3)$$

Numerical Scheme

We define the computational domain on $[-L_x, L_x] \times [-L_y, L_y]$ and the grid points as follows.

$$x_i = -L_x + \frac{2L_x i}{N_x} \quad (i = 0, \dots, N_x - 1) \quad (4)$$

$$y_j = -L_y + \frac{2L_y j}{N_y} \quad (j = 0, \dots, N_y - 1) \quad (5)$$

We approximate solutions to Maxwell's equations based on Fourier interpolation polynomials [3, 4, 5] by defining the

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approximate solution as

$$\bar{E}_{l,k} = \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} E_{i,j} L(x_i) L(y_j), \quad (6)$$

where

$$L(x) = \frac{1}{N_x} \sin \left[N_x \frac{x - x_i}{2} \right] \cot \left[\frac{x - x_i}{2} \right]. \quad (7)$$

Then, the Fourier differentiation matrix is given as

$$(\bar{D}_x)_{l,i} = \frac{d}{dx} L_i(x) |_{x_i} = \frac{(-1)^{i+l}}{2} \cot \left[\frac{(x - x_i)}{2} \right] \quad (8)$$

for $i \neq l$ and $(\bar{D}_x)_{l,i} = 0$ for $i = l$. In a similar manner, \bar{D}_y can be defined. If we use the tensor product to define the two-dimensional spatial derivatives $D_x = I \otimes \bar{D}_x$ and $D_y = \bar{D}_y \otimes I$, where I represents the identity matrix, and if we represent $\bar{E}_x^n = [(E_x)_{00}, (E_x)_{10}, \dots, (E_x)_{ij}, \dots, (E_x)_{N_x-1, N_y-1}]^T$ at the time level $t^n = n\Delta t$ with material properties, $\bar{\epsilon}$ and $\bar{\mu}$, on grids, our scheme is

$$\bar{\epsilon} \frac{\bar{E}_x^{n+\frac{1}{2}} - \bar{E}_x^{n-\frac{1}{2}}}{\Delta t} = D_y \bar{H}_z^n - \bar{J}_x^n \quad (9)$$

$$\bar{\epsilon} \frac{\bar{E}_y^{n+\frac{1}{2}} - \bar{E}_y^{n-\frac{1}{2}}}{\Delta t} = -D_x \bar{H}_z^n \quad (10)$$

$$\bar{\mu} \frac{\bar{H}_z^{n+1} - \bar{H}_z^n}{\Delta t} = D_y \bar{E}_x^{n+\frac{1}{2}} - D_x \bar{E}_y^{n+\frac{1}{2}}. \quad (11)$$

Initial Conditions

To describe the electromagnetic fields in the presence of the Gaussian beam for the initial time step, we first solve the Poisson equation in one dimension at the cross section of the initial beam position

$$\nabla^2 \Phi^{1D}(y) = -\frac{\rho^{1D}(y)}{\epsilon} \quad (12)$$

and get the one-dimensional electric field at the cross section

$$E^{1D} = -\nabla \Phi^{1D}(y). \quad (13)$$

Then, the initial electric field E in two dimensions is assigned along the x -direction by using the one-dimensional electric field E^{1D} scaled by the initial Gaussian distribution $\rho(x)$ as

$$E(y, x) = E^{1D}(y) \rho(x). \quad (14)$$

Boundary Conditions

We apply the uniaxial perfectly matched layer (UPML) boundary condition in the x -direction and the perfectly electric conducting (PEC) boundary condition in the y -direction.

UPML formulations in 3D [8] are defined as follows:

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \frac{\partial D_x}{\partial t} + \frac{1}{\epsilon} \sigma_y D_x \quad (15)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \frac{\partial D_y}{\partial t} + \frac{1}{\epsilon} \sigma_z D_y \quad (16)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \frac{\partial D_z}{\partial t} + \frac{1}{\epsilon} \sigma_x D_z, \quad (17)$$

where $\sigma_x = -(x/d)^m(m+1)\ln(R)/2\eta d$, denoting d , x , m , R , and η for PML size, PML depth, polynomial grading, reflection error, and impedance, respectively. In UPML, the components of E are updated by

$$\epsilon \left[\frac{\partial E_x}{\partial t} + \frac{\sigma_x}{\epsilon} E_x \right] = \frac{\partial D_x}{\partial t} + \frac{\sigma_x}{\epsilon} D_x \quad (18)$$

$$\epsilon \left[\frac{\partial E_y}{\partial t} + \frac{\sigma_x}{\epsilon} E_y \right] = \frac{\partial D_y}{\partial t} + \frac{\sigma_y}{\epsilon} D_y \quad (19)$$

$$\epsilon \left[\frac{\partial E_z}{\partial t} + \frac{\sigma_y}{\epsilon} E_z \right] = \frac{\partial D_z}{\partial t} + \frac{\sigma_z}{\epsilon} D_z. \quad (20)$$

A similar formula is used in UPML to update the components of H . In our simulations we apply UPML only in the x -direction by choosing $\sigma_y = \sigma_z = 0$.

PEC boundary conditions are assigned at the boundaries in the y -direction by setting the values for the E and H components as zeros.

COMPUTATIONAL RESULTS

We demonstrate the profiles of wake fields in cylindrical tube and pillbox cavity structures in two dimensions. We then discuss the problems we encounter and possible solutions for accurate simulations.

Wake Fields

Figure 1 shows the electric field profile for the y -component on $[-7.5, 7.5] \times [-2, 2]$ at a time step=60 with $\Delta t = \Delta y/10$ and initial beam position at $x = -3.5$. As we expect for the cases with no change in the structures, we observe no significant reflection from the conducting boundary, and the beam is moving with no significant distortion. However, we observe dissipation around the conducting boundary as the beam moves along the positive direction in x .

We carried out simulations of the pillbox cavity, as shown in Figure 2. Figure 3 shows the electric field profile for the y -component on $[-7.5, 7.5] \times [-2, 2]$ for the pillbox configuration in Figure 2. We observe strong oscillations at the corner of the cavity as soon as the beam enters the cavity. These oscillations remain until the beam passes throughout the cavity. This result was not observed from the results reported in [1, 2, 7].

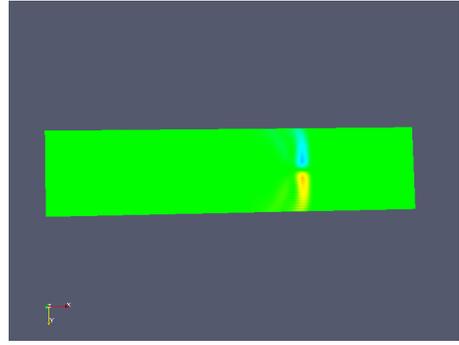


Figure 1: Electric field for the y -component on a tube mesh in 2D.

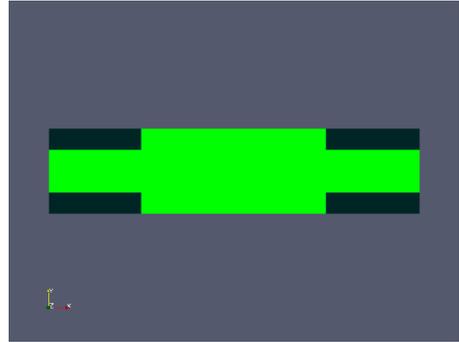


Figure 2: Pillbox cavity structure with PEC region (darker area) and vacuum (brighter area) on $[-7.5, 7.5] \times [-2, 2]$; ingoing and outgoing tube radius=1.

Discussion

We plan to continue the study of the dissipation and oscillation using the Fourier spectral scheme. We have a way to resolve the problems arising with oscillations when using Fourier spectral time domain simulations. In particular, one can apply the Gegenbauer or the Padé [5, 6] reconstruction techniques on the Fourier simulation data to remove the unphysical oscillations. Considering wake potential calculation, however, we need to carry out the reconstruction procedures every time step when wake po-

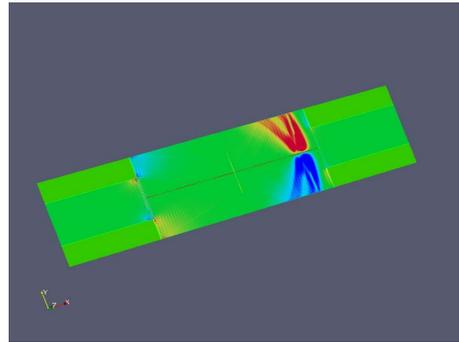


Figure 3: Electric field for the y -component on a pillbox configuration in 2D.

tential calculations are carried out over the time integrations. The reason is that accurate field values at each time to calculate the wake potential and these avoid using contaminated data from the oscillations. In order to reduce the computational cost for the reconstructions at every step where one has to provide reasonable field values, one can carry out reconstructions locally around the line path where one has to obtain the wake potential. For the pillbox configuration in Figure 2, one can get wake potential along the path $y = 1$ with two-dimensional reconstructions on $[-s, s] \times [1 - \Delta, 1 + \Delta]$ for $s = 5\sigma_x$ and some small Δ .

CONCLUSIONS

We demonstrated Fourier spectral time domain simulations for wake field calculations on cylindrical tube and pillbox cavity structures. We observe dissipations and unphysical oscillations depending on the structures in the Fourier spectral simulation data. Further study will be carried out on how to overcome the difficulties with dissipations and numerical oscillations. We also plan to investigate possible enhancement of its performance on the same grid base with the 2D wake field calculation code ABCI [2].

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