THE OPTIMUM CHROMATICITY SCHEME CORRECTION FOR MONOCHROMATIC AND NON-MONOCHROMATIC BEAM IN HESR

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Abstract

The High Energy Storage Ring (HESR) of FAIR project consists of two achromatic arcs and two dispersionless straight sections. Due to the multi-functional purpose of the straight sections their contribution into the total chromaticity of the first and second order exceeds the arc's contribution and can affect on the non-monochromatic beam dynamic aperture. We investigate the optimum sextupole and octupole scheme correction for monochromatic and non-monochromatic beam to reach the larger dynamic aperture.

INTRODUCTION

For High Energy Storage Ring we applied the "resonant" lattice with the controlled momentum compaction factor [1, 2]. It based on the resonantly correlated curvature and gradient modulations in arcs with the integer tunes in horizontal or both planes, and the straight sections are tuned to give the desired fractional tune for whole machine. The distinguishing features of this lattice are:

- the gamma transition variation in wide region from $\gamma_t \approx \nu_x$ to $\gamma_t \approx i\nu_x$ with quadrupole strength variation only;
- the dispersion free straight section;
- the independent optics parameters of arcs and straight sections;
- two families of focusing and one defocusing quadrupoles;
- the separated adjustment of gamma transition, horizontal and vertical tunes;
- the convenient chromaticity correction method by sextupoles;
- the first order self compensating scheme of multipole and large dynamic aperture;
- the low sensitivity to the multipole errors.

The lattice has distributed functions. In order to minimize the preparation procedure for each experiment the lattice is supposed to have the decoupled functions, which are responsible for the machine global parameters on the arcs like transition energy, tunes, zero chromaticity, dispersion suppressing and local parameters on the straight sections like beam luminosity on target, optimum parameters for cooling, injection system. The HESR lattice consists of two arcs and two straight sections for target and cooling facilities with circumference ~500 m [1]. In HESR we considered two types of lattice, and both have a racetrack shape with two arcs and two straight sections. In first option the arc has the four-fold symmetry with four super periods. In second option the arc has six-fold symmetry with six super periods. The phase advance per arc is chosen 3.0 and 5.0 in first and second options correspondingly.

CHROMATIC SEXTUPOLE SCHEME

Figure 1 shows the common view of HESR with a half superperiod of arc and indication of elements together with their functions.

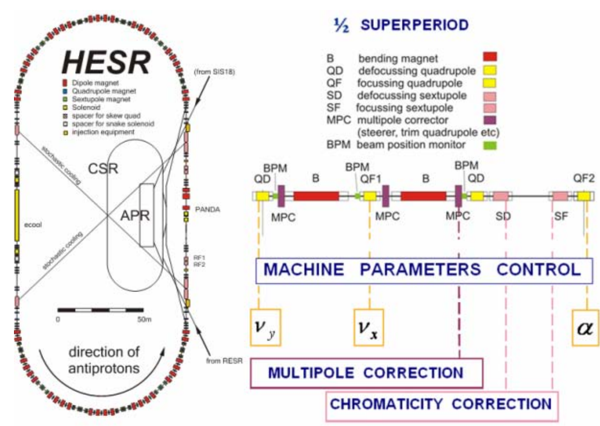

Figure 1: Schematic layout of the HESR lattice with the half superperiod of arc.

The chromaticity is created by the quadrupole and defined as the variation of the betatron tune $\nu_{x,y}$ with the relative momentum deviation $\delta = \Delta p / p$. The sextupoles are inserted in the lattice for chromaticity correction. Their integrated contribution over whole ring circumference C into the chromaticity is:

$$\frac{\partial \nu_{x,y}}{\partial \delta} = \pm \frac{1}{4\pi} \int_0^C \beta_{x,y}(s) \cdot D(s) \cdot S(s) ds \qquad (1)$$

Obviously to rise the sextupole efficiency they have to be allocated in the maximum dispersion and different β_x and β_y values to split the chromaticity correction in the horizontal and vertical planes. From last point of view the "resonant" lattice based on FODO structure is the most preferable in comparison with another lattices based the doublet or triplet structures. In the "resonant" lattices the empty space of free magnet cells is used for the sextupoles location (see Fig. 1). Two families of sextupoles, each one having two focusing and two defocusing sextupoles are used. In some project, for instance in JPARC, the focusing sextupole is inserted in the splintered central focusing quadrupole [3]. It makes sextupole efficiency higher, but decreases the effective quadrupole strength.

To prove independent controllability of chromaticity on both focusing and defocusing sextupoles we have done the numerical simulation of such control in the lattice with initially installed zero chromaticity $\xi_{x,y} = 0$. Figures 2 and 3 show the numerical simulation results how focusing and defocusing sextupoles SF and SD change the horizontal and vertical chromaticity correspondingly.

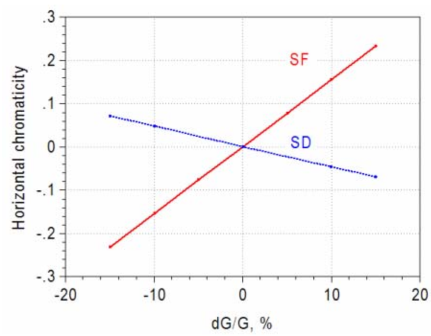

Figure 2: Horizontal chromaticity vs. focusing and defocusing sextupole gradient.

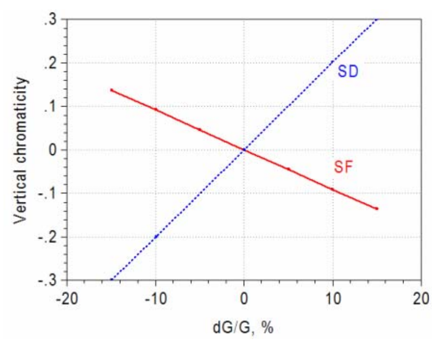

Figure 3: Vertical chromaticity vs. defocusing and focusing sextupole gradient.

From these results we see that the derivatives of horizontal ξ_x and vertical ξ_y chromaticities with gradient in the sextupoles are in relation:

$$\left|\frac{\partial \xi_x}{\partial G_{SF}}\right| > \left|\frac{\partial \xi_x}{\partial G_{SD}}\right| \quad \text{and} \quad \left|\frac{\partial \xi_y}{\partial G_{SD}}\right| > \left|\frac{\partial \xi_y}{\partial G_{SF}}\right| \qquad (2)$$

Thus, two sextupole families can control the horizontal and vertical chromaticities simultaneously.

SEXTUPOLE NON-LINEARITY COMPENSATION

In common case the optics of ring consists of the quadrupoles, the bend magnets, the sextupoles and the multipole correctors. Besides, due to the imperfections the multipoles errors are added in the lattice. Even in ideal optics and for the monochromatic beam each n-th multipole M_n in composition with the curvature h^m creates all higher multipoles M_{n+m}. In case of non-monochromatic beam $\delta \equiv \Delta p/p \neq 0$ each multipole of n-th order M_n gives all multipoles $M_{1\div(n-1)}$ of $1\div(n$-1)-th order in the place where $D\neq 0$. In case of the closed orbit distortion each n-th multipole M_n gives additionally all multipoles $M_{1\div(n-1)}$.

Usually the strongest contribution into the non-linearity is coming from the chromatic sextupoles. In order to investigate the non-linear optics we use the Hamiltonian formalism, which is presented with Hamiltonian function:

$$H(I_x, \vartheta_x, I_y, \vartheta_y) = \nu_x I_x + \nu_y I_y + \\ + \frac{1}{2} \sum_{j,k,l,m,p} h_{jklmp} \cdot I_x^{j/2} \cdot I_y^{k/2} \exp i(l\vartheta_x + m\vartheta_y - p\theta), \qquad (3)$$

where h_{jklmp} are the Fourier coefficients

$$h_{jklmp} = \frac{1}{2\pi} \int_0^{2\pi} E_{lm}^{jk}(\theta) \exp ip\theta \, d\theta. \qquad (4)$$

The coefficients $E_{lm}^{jk}(\theta)$ depend on the value and the distribution of the non-linear elements. They have the periodicity 2π with the new "time" coordinate $\theta = s/\overline{R}$, where $\overline{R}$ is average machine radius and s is the length of arc in curvilinear system.

In case, when we wish to exclude the resonance influence, we should minimize the harmonic amplitude h_{jklmp}. The only condition, which one cancels all coefficients E_{lm}^{jk} is the zero value of $h_{jklmp} = 0$ for all j,k,l,m. In particular, in case of the chromaticity correction on arcs with S_{arc} superperiods the sextupoles have to be placed with the phase advances μ_x, μ_y per one super-period, when the total multipole of third order is cancelled:

$$M_3^{total} = \sum_{n=1}^{S_{arc}} S_{x,y} \beta_x^{l/2} \beta_y^{m/2} \exp in(l\mu_x + m\mu_y) = 0\,, \qquad (5)$$

where $S_{x,y}$ is the sextupole gradient.

In one hand in order to have a free dispersion straight section, an arc consisting of S_{arc} superperiods must have an 2π integer phase advance. It means the phase advance per superperiod will be equal $2\pi \frac{\nu_{arc}}{S_{arc}}$, where ν_{arc} is the arc tune. On the other hand, for driving of the momentum-compaction factor the horizontal betatron tune must be less than the resonant harmonic of the perturbation, and the difference between them has to be minimum integer value; we take $\nu_{arc} - S_{arc} = -1$. It is realized at strictly fixed sets of S_{arc} and ν_{arc}: (4,3), (6,5), (8,7) and so on. As you see the arc superperiodicity S_{arc} is taken even and ν_{arc} is odd. Then the phase advance between any two cells, located in the different half arcs and separated by $S_{arc}/2$ number of superperiods, equals $\frac{\nu_{arc}}{S_{arc}} \cdot \frac{S_{arc}}{2} = \frac{\nu_{arc}}{2}$. It means we have an exact condition for compensating each sextupole's nonlinear action by another one in first approach. This remarkable feature is concern of the regular multipole errors in the bend magnets or quadrupoles as well, since for any element its twin exists on another half arc where non-linear kick is compensated.

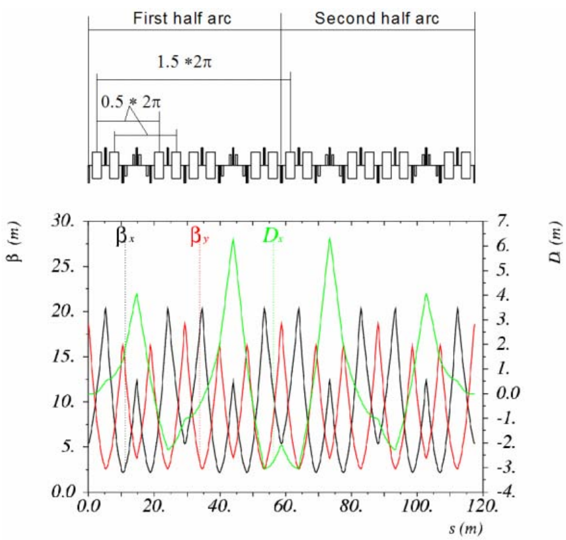

Figure 4: The HESR arc with elements location.

Figure 4 shows the scheme with explanation how the self compensating scheme works. We can see in case of the four fold symmetry arcs we have two compensating scheme: through half arc and one super period.

NON-LINEAR TUNE SHIFT CONTROL

In the first order of the perturbation theory the sextupoles can be cancelled. But in the second order the non-linear perturbation contains already the higher order of h_{jklmp}, which gives the non-linear tune shift, like octupoles.

Taking into account non-linear terms not higher than $I_{x,y}^2$ the average Hamiltonian in vicinity of resonance can be written as:

$$H(I_x, \vartheta_x, I_y, \vartheta_y) = \frac{(k_x^2 + k_y^2)^{1/2}}{k_x} \Delta \cdot I_x + \frac{(k_x^2 + k_y^2)^{1/2}}{k_y} \cdot$$
$$\Delta \cdot I_y + 2\left\langle h_{k_x, k_y, p} \right\rangle I_x^{k_x/2} I_y^{k_y/2} \cos\left(k_x \vartheta_x + k_y \vartheta_y\right) + \qquad (6)$$
$$\zeta_x I_x^2 + \zeta_{xy} I_x I_y + \zeta_y I_y^2,$$

where Δ is the detuning from p-th resonance. The coefficients $\zeta_x, \zeta_y, \zeta_{xy}$ determine the non-linear tune shift. In the first order of the perturbation theory the non-linear tune shifts arise due to the octupoles only, but in the second order the sextupoles give the non-linear tune shift as well: $\zeta_{x,xy,y} = \zeta_{x,xy,y}^{sex} + \zeta_{x,xy,y}^{oct}$. In principle, the sextupole non-linear tune is not under our control after the sextupole location fixing. Therefore the sign of total chromaticity is controlled by the octupoles, which are located in the multipole correctors. Thus, after the chromaticity correction we measure the non-linear tune shift and then using the correcting octupoles, we adjust the required sign and value of the non-linear tune shift.

SECOND ORDER CHROMATICITY

Since the chromatic sextupoles are located on the arcs only they have to correct chromaticity coming from the straight section quadrupoles as well. Unfortunately the target quadrupoles contribution into the total chromaticity of the first $\partial \nu_{x,y} / \partial \delta$ and second order $\partial^2 \nu_{x,y} / \partial \delta^2$ exceeds the arc's contribution and can affect on the non-monochromatic beam dynamic aperture. Since the second order chromaticity is the dependence chromaticity on the momentum spread, and the effective sextupole is determined by $S_{x,y} = S_{x,y} + \delta \cdot D \cdot O_{x,y}$ the octupoles allocated in the non-zero dispersion place can play the role of second order chromaticity correctors.

To compensate the second order chromaticity we use the octupole correctors incorporated into the multipole correctors. They are located near to each quadrupole and allow the separated tuning. Unfortunately if we need to correct both the non-linear tune and the second order chromaticity the additional octupoles on the straight section have to be foreseen. They will affect on the non-linear tune and not influence on the chromaticity.

CONCLUSION

We developed the lattice with possibility to compensate high order non-linearity up to third order. All mulipole correctors are decoupled each with other and have a high controlled efficiency. We are thankful to R. Maier for attention to our work.

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