PRACTICAL DEFINITIONS OF BEAM LIFETIMES IN AN ELECTRON STORAGE RING

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Abstract

This paper derives simple and accurate formulas breaking down a measured beam lifetime into its partial lifetimes of Touschek and beam-gas scattering. These formulas use only the measured data of beam lifetime and are as accurate as the lifetime measurement. These are useful as practical means, because the existing definitions of the two partial lifetimes often fail to give numerically accurate numbers due to the lack of accurate information of necessary parameters.

INTRODUCTION

The beam lifetime in an electron storage ring is mainly determined by the two effects, the intra-beam Touschek scattering [1] and (elastic [2] and inelastic [2, 3]) electron-gas scattering, if we assume the absence of several instabilities affecting the beam lifetime. Another potential lifetime affecting factor is the aperture limitations for the transverse or longitudinal electron motions [4]. But this effect is negligible for most electron storage rings and we will not consider it in this paper. The above effects have been analyzed and evaluated to define partial lifetimes, Touschek lifetime and (elastic and inelastic) beam-gas scattering lifetime, which give the total beam lifetime when added. The addition is given as

$$\frac{1}{\tau} = \frac{1}{\tau_t} + \frac{1}{\tau_v}, \tag{1}$$

where τ is the total beam lifetime, τ_t is the Touschek lifetime, and τ_v collectively denotes the elastic and inelastic beam-gas scattering lifetimes. While it is easy and clear to define the two partial lifetime theoretically, they are mixed up in real measurements. It is not easy to separate them. The well known existing formulas for the partial lifetimes, which describe the relevant scattering analytically, are obviously valid. However, they often give numerically inaccurate values. For example, calculated τ_t and τ_v may fail to give a number fairly close to the measured beam lifetime τ in Eq. (1). The inaccuracy comes from the lack of the exact knowledge of various machine and beam parameters in the definitions. Especially, the Touschek lifetime formula is complicated and even depends on the radiative polarization of the electron beam that is not measured easily. Polarization dependence of the Touschek lifetime has not been analyzed extensively in literatures. Hence the formulas are useful in estimating the beam lifetimes in a design stage, but may not be practical in the analysis of a measured beam lifetime into partial lifetimes, which is a necessary step for achieving the design goal and improving the total lifetime. Improving the lifetime may not be important anymore with the advent of the top-up operation, but the analysis itself is a useful diagnostic means of machine and beam parameters. The partial lifetimes reflect different parameters of a storage ring and are sensitive monitors of whatever may be happening on those parameters. Usually we do the analysis by direct measurement; we can measure one partial lifetime by minimizing the other effect (maximizing the other partial lifetime). But since it is almost impossible to increase a partial lifetime to a negligible level, direct measurement does not guarantee accuracy.

In this paper, we devise methods (formulas) to break down the measured beam lifetime into partial lifetimes. We give formulas for the partial beam lifetimes, which use only the measured data of τ. The derivation is based on the different dependence of τ_t and τ_v on the beam parameters, especially on the number of electrons per bunch and degree of polarization. The accuracy is as good as that of the beam lifetime measurement. These formulas can serve as practical and numerical definitions of τ_t and τ_v.

PROPERTIES OF THE TOUSCHEK AND BEAM-GAS SCATTERING LIFETIMES

The beam-gas scattering lifetime τ_v is inversely proportional to the gas density as in

$$\frac{1}{\tau_v} = \beta c \sigma_{loss} \rho, \tag{2}$$

where ρ is the gas density and σ_{loss} is the sum of beam loss cross sections for elastic and inelastic electron-gas scattering. σ_{loss} is time independent and depends on machine parameters such as β-function values, vertical aperture of the ring, and momentum acceptance of the ring (see, e.g., [2, 5]). Although calculated σ_{loss} may not be vary accurate, the inverse proportionality of τ_v on ρ is an exact one. Note that τ_v has no explicit time dependence. But in general it depends on time implicitly through the gas density ρ, because it depends on the stored current, which decays as time goes on, if desorption effect is still relevant. Including the desorption effect, we write for ρ

$$\rho = \rho_0 + GmN, \tag{3}$$

where G is a desorption coefficient, m is the number of bunch, and N is the number of electrons in a bunch. Hence τ_v depends on the total number of electrons, mN, (total stored beam current), while τ_t depends on the number of electrons in a bunch, N, (stored bunch current) as shown below. This difference is the basis of the derivation of new formulas in the next section.

The Touschek lifetime applies to a bunch or a bunch string with equal number of electrons per bunch. If N is the number of electrons in a bunch, the loss rate due to the the Touschek scattering is proportional to N^2. Neglecting the polarization effect, we write it as

$$\frac{dN}{dt} = -bN^2, \tag{4}$$

where b is the proportionality constant and the Touschek lifetime is proportional to N

$$\frac{1}{\tau_t} = -\frac{1}{N}\frac{dN}{dt} = bN. \tag{5}$$

Derivation shows that b can be written as $aC(\epsilon)$ and the parameter a is given by

$$a = \frac{\sqrt{\pi} c r_e^2}{\gamma^3 V \sigma_{x'} (\Delta p_m/p)^2}, \tag{6}$$

where r_e is the classical electron radius, $\Delta p_m/p$ is the momentum acceptance, γ is the Lorentz factor, $V = (4\pi)^{3/2}\sigma_x\sigma_y\sigma_l$ is the bunch volume, and that the function $C(\epsilon)$ is defined by [5]

$$\begin{aligned} C(\epsilon) &= \epsilon \int_\epsilon^\infty \frac{1}{u^2}\left\{\left(\frac{u}{\epsilon}\right) - \frac{1}{2}\ln\left(\frac{u}{\epsilon}\right) - 1\right\} e^{-u} du, \\ \epsilon &= \left(\frac{\Delta p_m}{\gamma \sigma_{p\perp}}\right)^2 \end{aligned} \tag{7}$$

where $\sigma_{p\perp}$ denotes the r.m.s. relative momentum distribution. $C(\epsilon)$ does not allow analytic integration but numerical integration shows that $C(\epsilon) \leq 0.3$. The bunch volume V is not easy to measure accurately. The momentum acceptance $\Delta p_m/p$ is determined not only by the longitudinal RF acceptance but also the transversal dynamic or physical aperture, and it is also difficult to determine accurately. The inaccuracy of the Touschek lifetime comes from the difficulty to determine a accurately. Furthermore the Touschek lifetime depends on the radiative polarization of the electron beam through the polarization dependence of the Möller scattering, which makes the Touschek lifetime impossible to calculate without knowledge of the polarization state. However, the inverse proportionality to N of τ_t (Eq. (5)) is an exact relation, upon which derivation of formulas in this paper are based. Since N decays as time goes on for a stored beam, the Touschek lifetime varies with time. We will use τ_t to denote a Touschek lifetime at the instant the number of electrons per bunch is N.

Stored electron beam gets polarized transversely. The radiative polarization for an initially unpolarized electron beam proceeds according to the formula [6, 7]

$$S = S_0(1 - e^{-t/T}), \tag{8}$$

where S is the degree of polarization and its saturation value S_0 can have the maximum value 0.924. Without depolarizing effects, the characteristic time constant T is given by

$$T \simeq 98 \times \frac{R_{bnd}^2 R_{avg}}{E^5} \text{ [sec]}, \tag{9}$$

where E is the beam energy in GeV, and R_{bnd} and R_{avg} are the bending radius and the mean radius of the ring in meters respectively. The Touschek lifetime depends on the beam polarization and thus it should be modified from Eq. (5) to

$$\frac{1}{\tau_t(S)} = -\frac{1}{N}\frac{dN}{dt} = a\Big[C(\epsilon) + F(\epsilon)S^2\Big]N, \tag{10}$$

where the new parameter $F(\epsilon)$ gives the polarization dependent contribution to the Touschek lifetime [8].

DERIVATION OF NEW FORMULAS

Consider a stored beam of m bunches with equal number of electrons. At the instant each bunch has N electrons, suppose that we measure the total lifetime τ. Compare this with another measurement in which we use a different beam of pm bunches with an appropriate rational number p. At the instant each bunch has N/p electrons, we measure the total lifetime τ'. How different is τ' from τ? The beam-gas scattering lifetime will not change in the second case because the total number of electrons is still mN. However, since the Touschek lifetime is inversely proportional to the number of electrons in a bunch, it will now change to a different value $\tau_t' = p\tau_t$. The new total lifetime τ' is related to τ as

$$\begin{aligned} \frac{1}{\tau'(S)} &= \frac{1}{\tau_v} + \frac{1}{p\tau_t(S)} \\ &= \frac{1}{\tau_v} + \frac{1}{p}\left(\frac{1}{\tau(S)} - \frac{1}{\tau_v}\right) \\ &= \frac{1}{\tau_v}\frac{\tau_v + (p-1)\tau(S)}{p\tau(S)}, \end{aligned} \tag{11}$$

which gives

$$\tau'(S) = \frac{p\tau_v}{\tau_v + (p-1)\tau(S)}\tau(S). \tag{12}$$

Now we use q to denote the ratio of the two total lifetimes,

$$q(S) \equiv \frac{\tau'(S)}{\tau(S)} = \frac{p\tau_v}{\tau_v + (p-1)\tau(S)}. \tag{13}$$

Since $q(S)$ is also measurable, we can solve this equation for τ_v as

$$\tau_v = \frac{(p-1)q(S)}{p - q(S)}\tau(S). \tag{14}$$

Therefore, τ_v is easily obtained from the two measured quantities, τ and q. The accuracy of the obtained beam-gas scattering lifetime is that of the measured τ and q. Since τ_v is independent of the polarization S, the right side of Eq.

(14) is actually independent of S and we can choose any S appropriate. Now the Touschek lifetime of the first beam configuration is obtained from

$$\frac{1}{\tau_t(S)} = \frac{1}{\tau(S)} - \frac{1}{\tau_v} = \frac{1}{\tau(S)} \frac{p[q(S)-1]}{(p-1)q(S)}, \qquad (15)$$

which gives

$$\tau_t(S) = \frac{(p-1)q(S)}{p[q(S)-1]}\tau(S), \qquad (16)$$

and the Touschek lifetime of the second beam configuration is given by

$$\tau_t'(S) = \frac{(p-1)q(S)}{[q(S)-1]}\tau(S). \qquad (17)$$

The unpolarized Touschek lifetime is given by

$$\tau_t(0) = \frac{(p-1)q(0)}{p[q(0)-1]}\tau(0). \qquad (18)$$

APPROXIMATE FORMULAS

In order to use the above formulas, measurements with two separate beam configurations are needed. However, in the case that the desorption is negligible and thus τ_v is effectively time-independent, it is possible to obtain τ_v and τ_t only by using the recorded graph of τ versus t. Although these formulas were discussed previously [9], we will describe them here briefly for the completeness. We use the fact that τ_t is time dependent while τ_v is not. The starting point is again Eq. (1), which we rewrite as

$$\frac{1}{\tau(t)} = \frac{1}{\tau_v} + \frac{1}{\tau_t(t)} \qquad (19)$$

where we denoted the time dependence of τ_t and τ explicitly. Suppose that we measure τ again at a later time $t' = t + \Delta t$ and find that τ is increased to $\tau + \Delta\tau$. Since τ_v is time independent, $\Delta\tau$ is caused entirely by the increase of τ_t to $\tau_t + \Delta\tau_t$. At $t' = t + \Delta t$, we are left with the following relation

$$\frac{\Delta\tau}{\tau^2} \simeq \frac{\Delta\tau_t}{\tau_t^2}. \qquad (20)$$

An important relation to be used here is

$$\Delta\tau_t = \Delta t, \qquad (21)$$

which is valid provided the beam polarization S saturates to S_0 and thus the only time dependence of τ_t comes from its N dependence. Using Eqs. (4) and (5), we see that

$$\frac{d\tau_t}{dt} = \frac{d\tau_t}{dN}\frac{dN}{dt} = \frac{-a(C + FS_0^2)N^2}{-a(C + FS_0^2)N^2} = 1. \qquad (22)$$

For example, with the PLS parameters, $E = 2.5$ GeV, $R_{bnd} = 6.30$ m, and $R_{avg} = 44.65$ m, T is approximately 0.5 h. Equation (9) shows that as the beam energy increases, T decreases rapidly. For higher energy rings, polarization saturates much more rapidly and Eq. (21) holds except only the initial a few minutes. Hence it is legitimate to use Eq. (21) in Eq. (20) to get

$$\tau_t(S_0) \simeq \frac{\tau(S_0)}{\sqrt{\Delta\tau/\Delta t}}, \qquad (23)$$

which shows how τ_t is determined from the measured values of τ, $\Delta\tau$, and Δt. Then τ_v is simply determined by

$$\frac{1}{\tau_v} = \frac{1}{\tau} - \frac{1}{\tau_t} \simeq \frac{1}{\tau}\left(1 - \sqrt{\frac{\Delta\tau}{\Delta t}}\right), \qquad (24)$$

which is just

$$\tau_v \simeq \frac{\tau(S_0)}{1 - \sqrt{\Delta\tau/\Delta t}}. \qquad (25)$$

The above approximate equalities become exact equalities when $\Delta t \to dt$,

$$\tau_t(S_0) = \frac{\tau(S_0)}{\sqrt{d\tau/dt}}, \qquad (26)$$

$$\tau_v = \frac{\tau(S_0)}{1 - \sqrt{d\tau/dt}}. \qquad (27)$$

These are exact and practical formulas defining τ_t and τ_v in terms of easily measurable quantities. You need to know only τ and $d\tau/dt$. In Eq. (27) τ_v is time independent and actually it is straightforward to show $d\tau_v/dt = 0$. It is practical to use Eqs. (23) and (25) instead of exact relations. It is easy to determine both τ_t and τ_v from the measured values of τ, $\Delta\tau$, and Δt. This method is simple and as accurate as the measured values of τ and $\Delta\tau$. However, it has the limitation that it can not be used to determine the unpolarized Touschek lifetime $\tau_t(0)$ and the beam polarization.

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