STUDIES OF THE INJECTION SYSTEM IN THE DECAY RING OF A BETA-BEAM NEUTRINO SOURCE

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Abstract

After their acceleration, the beta radioactive ions are accumulated in a decay ring [1]. The losses due to their decay are compensated with regular injections in presence of filled bucket. Without a damping mechanism, the new ions are injected at a different energy from the stored beam energy, then the old and the new buckets are merged with RF manipulation. This type of injection has to be done, in a dispersive region, in presence of a closed orbit bump and a septum magnet. The sizes of the injected beam and of the stored beam have to be adjusted in order to minimize the losses on the septum and to maximize the stored intensity keeping small beam sizes. The dispersion has to be large enough in order to decrease the energy difference.

INJECTION PRINCIPLE

The injection is « off-momentum »: the injected beam energy (E+ΔE) is different from the stored beam one (E). The injection principle appears in Fig. 1:

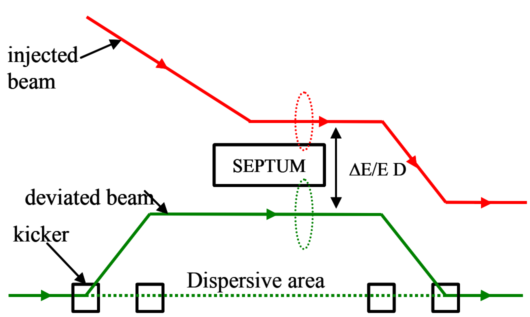

Figure 1: Injection layout

The injection is realized in a high dispersion (D) area. At the time of the injection, the stored beam is pushed near the septum blade by kickers. At the same time, a fresh beam is deviated by the septum into the chamber. The two beams are synchronous but do not have the same energy. The fresh beam is injected on its chromatic orbit. The kickers are switched off before the end of the first turn. The injected and stored beams pass then under the septum blade. The two beams are merged by using two RF cavities. The first cavity is used to make the injected beam realize a quarter of a turn in the longitudinal phase space. The second, at twice the RF frequency, is used to merge the two beams that are at the same energy but are not synchronous [2][3].

The energy difference ΔE/E and the bump depend on the distances between the two beam axis. The aim is to calculate precisely this difference and the bump to realize.

At each injection, a part of the stored and injected beams is lost on the septum. The admissible losses determine the beam envelope sizes, on which the energy difference and the bump values depend.

DETERMINATION OF THE DEPOSIT LOSSES ON THE SEPTUM BLADE

The number of ions lost directly on the septum depends on the number of stored ions in the ring. Between two injections, there are two loss sources: the decay and the deposit on the septum. In fact, there is a third one: the merging is not completely stable and part of the beam is lost after several injections. This effect is not taken in account here. The number of stored ions after n injections is:

$$N_{n+1} = a2^{\frac{-T}{\gamma\tau}} N_n + N_{inj}$$

with N_n the stored ion number after n injections

N_{inj} the injected ion number

γ the relativistic factor

τ the half-life time of the ion at rest

T the injection period

a the transmission coefficient at the septum. It is directly linked to the number of rms beam size by the relation: $a = \int_{-\infty}^{n_m\sigma} f_\sigma(x)dx$

f_σ the beam distribution

σ the rms beam size

n_m the number of rms beam size

When the number of injections tends to the infinite, the stored ion number tends to: $N_{stored} = \dfrac{N_{inj}}{1-a2^{\frac{-T}{\gamma\tau}}}$

The average lost power on the septum at each injection by the stored beam is then:

$$P_{déposited}^{stored} = (1-a)N_{stored}(\gamma-1)E_0/T$$

where E_0 is the ion energy at rest

The lost power by the injected beam is then:

$$P_{deposited}^{injected} = (1-b)N_{inj}(\gamma-1)E_0/T$$

where b is the transmission coefficient for the injected beam at the septum.

For the beta-beam decay ring, the parameters used above [4] are given in the Table 1. The injection cadence T is equal to 8 s.

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Table 1: Beta beam project parameters

Ion type	$^{6}He^{2+}$	$^{18}Ne^{10+}$
γ	100	100
Energy (GeV)	555	1669
τ at rest (s)	0.8	1.67
ε_{rms} (Pi mm.mrad)	0.233	0.465
$N_{Injected}$ (ions/batch)	$0.9\ 10^{13}$	$4.9\ 10^{11}$
N_{Stored} (ions/batch)	$1.3\ 10^{14}$	$1.5\ 10^{13}$

The beam distribution is supposed Gaussian. We assume that the distribution is 1D. Also, we obtain the deposited power on the septum at each injection for the stored beam (Fig. 2) and for the injected one (Fig. 3).

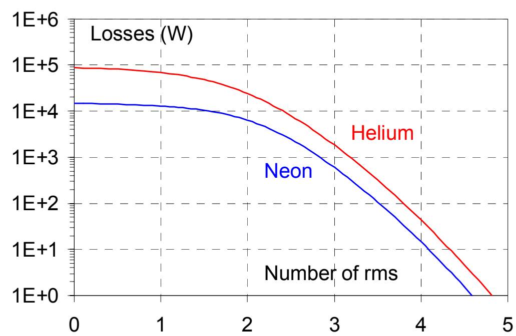

Figure 2: Power lost by the stored beam on the septum blade

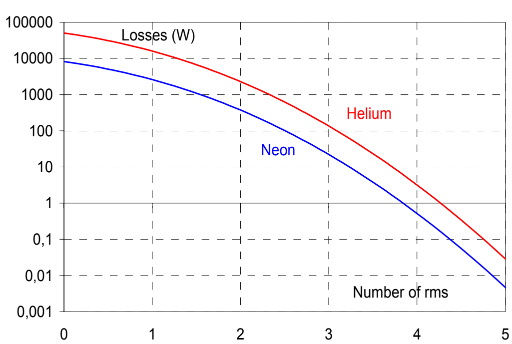

Figure 3: Power lost by the injected beam on the septum blade

In order to keep the total deposited power below 20 W, it is necessary to preserve 4.1 rms for the stored beam and 3.3 rms for the injected beam. These results are valid for Gaussian distributions.

MOMENTUM DIFFERENCE BETWEEN THE TWO BEAMS

If we know the optical functions at the injection point, we are able to calculate the momentum difference between the stored and injected beams. Indeed, the number of rms gives the envelope sizes. So, we can establish the deviation to realize and the difference between the stored beam and the injected beam axis.

In fact, at the time of the injection, the injected beam may not be adapted in dispersion. Because of the momentum spread of the injected beam δ_i, there will be a blow-up. Indeed, the betatron oscillations around its reference orbit will increase the emittance. We have to take in account this effect. In that section, we are going to write the relations between the emittance, the energy difference (renamed here Δ), the optical functions, the number of rms and the deviation to realize (Fig. 4).

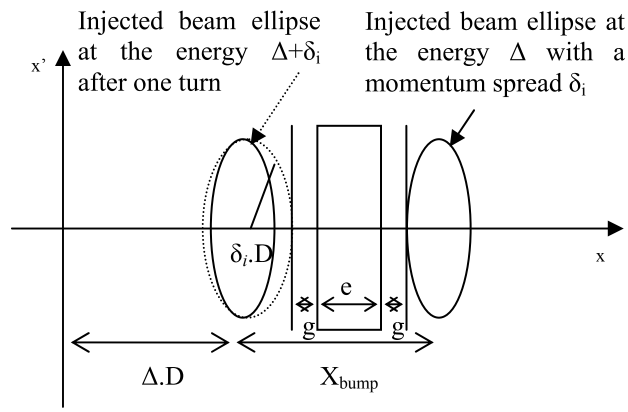

Figure 4: Representation in the phase space of the injection of a fresh beam in a dispersive area

The ellipse equation of the injected beam in the chamber is (by using as an origin the injected beam axis):

$$\varepsilon_i = \beta_i.x'^2 + 2\alpha_i.x.x' + \gamma_i.x^2$$

α_i, β_i and γ_i are the Twiss parameters at the injection line output. ε_i is the injected beam emittance.

The ellipse equation of the beam at the energy $\Delta+\delta$ after one turn is then:

$$\varepsilon(\delta_i) = \beta_m.x'^2 + 2\alpha_m.(x-\delta_i.D).x' + \gamma_m.(x-\delta_i.D)^2$$

α_m, β_m and γ_m are the Twiss parameters at the decay ring injection. D is the dispersion function in the decay ring. We do the variable change: $X' = x' + \frac{\alpha_i}{\beta_i}x$ and $X = x$.

If $\alpha_i = \alpha_m$ and $\beta_i = \beta_m$, then the variables X et X' are not correlated.

$$\varepsilon(\delta) = \beta_m.X'^2 + \frac{1}{\beta_m}(X-\delta.D)^2 - 2\alpha_m.\delta.D.X' + \frac{\alpha_m{}^2}{\beta_m}.(\delta.D)^2$$

We use $\varepsilon_i = \beta_i.X'^2 + \frac{X^2}{\beta_i}$. In fact, the generated ellipse depends on the initial conditions: some ions run a larger ellipse than others. Thus, the emittance depends on the initial position of the ion. That gives:

$$\varepsilon(\delta_i,(X,X')) = \frac{1}{\beta_m}.\left[\beta_m.\varepsilon_i - 2\delta.D.X + 2.|\alpha_m.\delta.D|..\sqrt{\beta_m.\varepsilon_i - X^2} + (1+\alpha_m{}^2)(\delta.D)^2\right]$$

X maximizes the emittance if:

$$\frac{\partial\varepsilon(\delta_i)}{\partial X}=\frac{1}{\beta_m}\left[-2\delta_i.D.-2.\frac{|\alpha_m.\delta_i.D|}{\sqrt{\beta_m.\varepsilon_i-X^2}}X.\right]=0\Rightarrow X=-\frac{|\delta_i.D|}{\delta_i.D}\sqrt{\frac{\varepsilon_i}{\gamma_m}}$$

The maximal emittance value is then:

$$\varepsilon(\delta_i)=\left(\sqrt{\varepsilon_i}+\sqrt{\gamma_m}|\delta_i.D|\right)^2$$

Figure 4 shows that:

$$X_{bump}=\delta_i.D+n_i\sqrt{\beta_i\varepsilon(\delta_i)}+2g+e_S+n_i\sqrt{\beta_i\varepsilon_i}$$

Now, we can determine the momentum difference between the two beams. We have calculated it so that the stored beam envelope passes exactly under the septum blade when the kickers are switched on. That condition (Figure 5) enables to minimize the momentum difference.

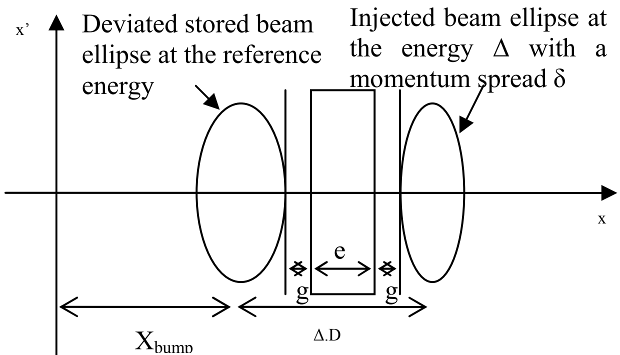

Figure 5: Representation in the phase space of the deviation of the stored beam

The Twiss parameters for the injected beam and for the stored beam are slightly different because of the chromatic effects. Moreover, we have supposed that the dispersion is null in the injection line. Therefore, it is not true for the stored beam and we cannot neglect the momentum spread δ_m. That increases the distribution rms size. The system below takes in account that effect:

$$\begin{cases}X_{bump}=\delta_i.D(\Delta)+n_i\sqrt{\beta_m(\Delta)\varepsilon(\delta_i,\Delta)}+2g+e_S+n_i\sqrt{\beta_m(\Delta)\varepsilon_i}\\ \Delta.D(\Delta)=n_m\sqrt{\beta_m(0)\varepsilon_m+(\delta_m.D(0))^2}+2g+e_S+n_i\sqrt{\beta_m(\Delta)\varepsilon_i}\end{cases}$$

TRACKING SIMULATIONS

In order to illustrate the effects of a dispersion mismatch, we have realised a tracking in the phase space by using the code BETA [5]. The decay ring structure appears on Figure 6. According to the above system, the higher the dispersion, the lower the momentum difference between the two beams. That is why the injection section structure is a low beta, high dispersion area.

We have calculated the bump and the momentum difference for a 1 mm guard at each septum side. On Fig.7, the injected beam is adapted for the optical functions α_x and β_x but it is mismatched for the dispersion (8.25 m). At the injection point, the β_x function is 21.24 m whereas the α_x function is -0.55. Calculation gives a momentum difference equal to 0.46 % and a 36.8 mm bump. We use two kicker families to realize the bump. Their integrated fields are 0.84 T.m and 0.24 T.m.

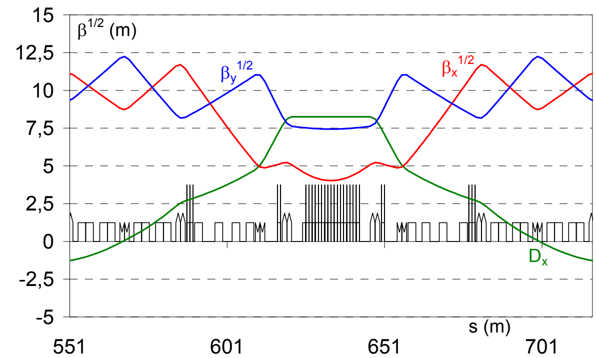

Figure 6: Injection section structure

We actually verify that this is in agreement with the simulation. Indeed, Figure 7 shows that the distance between the entering beam envelope and the stored beam one is 1.2 cm. We have the same distance between the fresh beam envelopes before and after its injection.

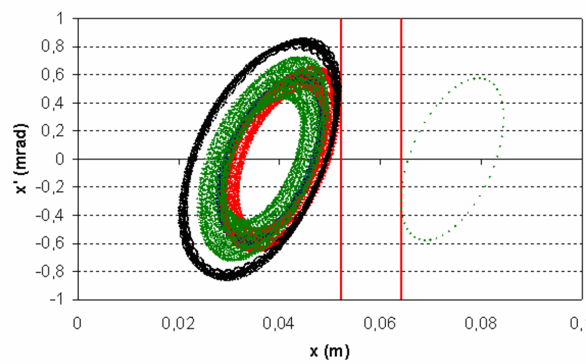

Figure 7: Beam representation in the phase space. On the right, injected beam before entering into the ring. The injected beam is respectively at the energy (1+Δ-δ) E (in green) and at (1+Δ+δ) E (in red). In black, deviated stored beam. The vertical red lines represent the septum blade plus its guard.

REFERENCES

[1] P.Zucchelli, "A novel concept for a neutrino factory: the beta-beam", Phys. Let. B, 532 (2002) 166-172

[2] M. Benedikt, S. Hancock and J.-L.Vallet, "A proof of principle of asymmetric bunch pair merging", http://documents.cern.ch//archive/electronic/cern/others/ab/ab-note-2003-80.pdf

[3] M. Benedikt and S. Hancock "A Novel Scheme for the Injection and Stacking of Radioactive Ions at High Energy", to be published in Nuclear Instruments and Methods in Physics Research A (NIM A)

[4] M. Benedikt, S.Hancock and M.Lindroos, "Baseline Design for a Beta-Beam Neutrino Factory", EPAC, 2004, Lucerne, Switzerland

[5] Code BETA, J. Payet

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