

## THE MEASUREMENT OF TUNE AND PHASE SPACE AT HLS \*

Y.L.YANG<sup>#</sup>, J.H.Wang, Z.P.Liu, J.H.Liu, B.G.Sun, K.ZHENG

NSRL, University of Science and Technology of China, Hefei, Anhui 230029, P.R.China

### Abstract

Tune and phase space online monitor at the electronic storage ring of Hefei Light Source (HLS) has been implemented by using Turn-by-turn Beam Position Monitor System (TBT System). In this paper, we compare a number of methods employed to compute tune and deals with choosing the best fitting method for our online tune computing. We can compute and display tune online, at the same time, record beam tracks on the transverse phase space by using turn-by-turn beam position data at two differently-located beam-position-monitor electrodes. With these instruments we can precisely and attractive study machine instabilities.

### TURN-BY-TURN BEAM POSITION MONITOR SYSTEM INTRODUCTION

The Turn-by-turn (TBT) Beam Position Monitor (BPM) system at Hefei Light Source (HLS) has been implemented. HLS is an electron storage ring with a circumference of 66m. So it requires that the TBT System operates with reliability and precision. HLS Turn-by-turn system consists of front end pick-up electrodes mounted in a skew 45°, Log-ratio electronics, timing system, and data acquisition system. The log-ratio processor works at 408MHz which is  $2 * f_{RF}$  of HLS. The functional block diagram is represented in Figure 1.

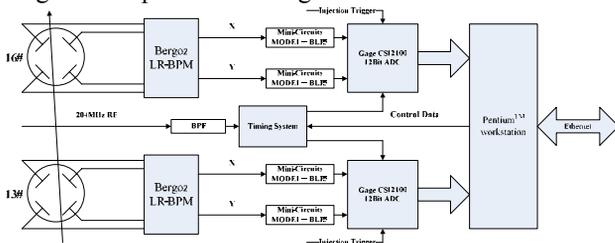


Figure 1: Turn-by-turn Function Block Diagram

The data acquisition system includes two GaGe<sup>TM</sup> 12Bit ADCs in the ADVANTEC<sup>TM</sup> Industrial Computer 610. The Timing System divides down the RF signal and provides two differing in phase TTL signal (have same frequency of 4.533MHz) for each of the ADC as the external clock. Further more, the time delay between two square signals can be finely adjusted by computer over network and the delay range is from 0.5ns to 220ns.

### INSTANTANEOUS TUNE EXTRACTION BY NAFF METHOD

The tune extraction algorithm methods and its intrinsic error have been discussed [1]. By using two standard algorithms we can obtain the tune from a time series of N

consecutive values of the particle trajectory: the average phase advance (APA) in phase space and the fast Fourier transform (FFT) of turn-by-turn position data. However, both methods have intrinsic error proportional to  $1/N$  [1]. In this case, we adopt NAFF as our instantaneous tune extraction algorithm. NAFF is based on J.Laskar [1] proposed theorem characterized by searching the maximum of the continuous Fourier transform namely the numerical analysis of fundamental frequency method. We apply NAFF to a subset of obtained turn-by-turn beam position data starting at turn m containing N turns.

$$f_n, \text{ where } n=m, \dots, m+N-1,$$

$f_n$  represents turn-by-turn beam position data for one of the canonical variables. We search for the fundamental frequencies  $\nu_{m,N}^{(k)}$  that maximize the absolute value of the correlator:

$$I(\nu_{m,N}^{(k)}) = \sum_{n=m}^{m+N-1} f_n \exp(-i2\pi \cdot \nu_{m,N}^{(k)} n) \chi_{m-n} \quad (1)$$

Where  $\chi_{m-n}$  is the weighting function to improve the precision of extracted tune:

$$\chi_{m-n} = \sin\left(\frac{\pi \cdot (m-n)}{N}\right) \quad (2)$$

These frequencies extracted from the numerical tracking data asymptotically converge for  $N \rightarrow \infty$  to the tunes associated with invariant surfaces in the phase space.

With a subset of the beam position data such as 128 samples, NAFF picks out the instantaneous tunes  $\nu_{m,N}^{(k)}$ ,

this way chi square fit the amplitude  $a_{m,N}$  in the fitting function

$$f_n = \sum_k a_{m,N}^{(k)} \cos(2\pi \cdot \nu_{m,N}^{(k)} n + \phi_{m,N}^{(k)}) \chi_{m-n} \quad (3)$$

The error associated with the interpolation and with the Laskar method is proportional to  $1/N^2$  [2]. Additional, in the case of the Hanning filter we have an estimate of the error proportional to  $1/N^4$ . However, this algorithm is excluded because it takes too much computing time.

### Instantaneous Tune Measurement Experiment

These days the online tune display program based upon NAFF have been successfully applied to study of various nonlinear beam dynamics at HLS. In these experiments, transverse betatron oscillations of a single bunch are excited by a pair of striplines. Meanwhile, positions of the bunch are recorded every turn by TBT system. The result is shown in Figure 2 and Figure 3. Considering online computing tune, we pitch on  $N=256$  as the length of subset of beam position data. Experiment result show that the tune error is  $<0.00005$ .

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<sup>#</sup>ylyang@mail.ustc.edu.cn

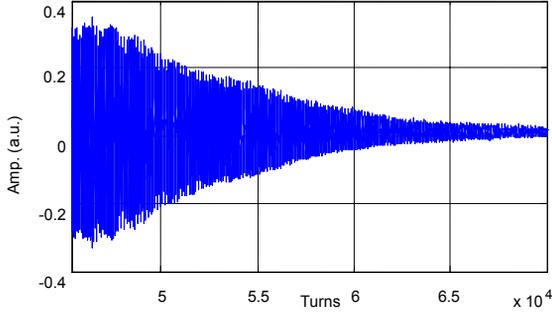


Figure 2: Beam horizontal turn by turn position vs. turn number.

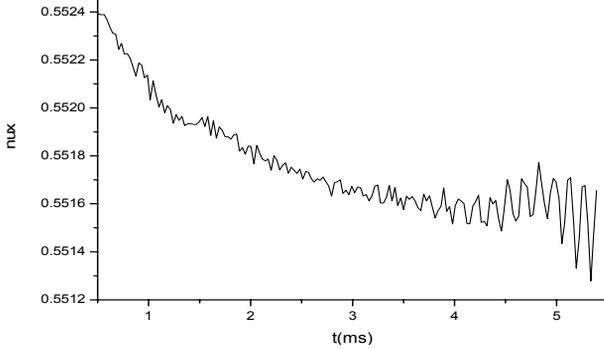


Figure 3: NAFF Extracts Horizontal Tune vs. Time.

### TBT PHASE SPACE MONITOR TUNE EXTRACTION ALGORITHM

Considering particle motion in a circular accelerator, the horizontal deviation from the closed orbit,  $x(s)$ , satisfies Hill's equation (4):

$$\frac{d^2x}{ds^2} + K(s)x = \frac{\Delta B_z}{B\rho} \quad (4)$$

Where  $K(s)$  is a function of the quadrupole strength,  $B\rho = p/e$  is the magnetic rigidity, and  $s$  is the longitudinal particle coordinate. From Hill's equation, Eq.(5) can be solved using the Floquet transformation to obtain the solution [3]:

$$x = \sqrt{2\beta_x J} \cos \varphi$$

$$x' = \frac{1}{\beta_x} \{-\alpha_x x - \sqrt{2\beta_x J} \sin \varphi\} \quad (5)$$

By defining normalized momentum  $p_x$ , we can get

$$p_x = \alpha_x x + \beta_x x' \quad (6)$$

From Eq.(5), we get

$$p_x = -\sqrt{2\beta_x J} \sin \varphi \quad (7)$$

It is convinced that when linear motion is plotted in  $x - p_x$  space, it is a circle defined by the equation,

$$p_x^2 + x^2 = 2\beta_x J \quad (8)$$

The variables that are practically measured are  $x_1$  and  $x_2$  at the two BPMs which are separated in betatron phase by  $\varphi_{12}$ . So, Eq.(6) for  $p_{x1}$  becomes

$$p_{x1} = -x_1 \cot \varphi_{12} + \frac{\sqrt{\beta_{x1} / \beta_{x2}}}{\sin \varphi_{12}} x_2 \quad (9)$$

On the basis of the above equations, we can get the transform matrix (10):

$$\begin{bmatrix} p_{x1} \\ p_{x2} \end{bmatrix} = \begin{bmatrix} -\cot \varphi_{12} & \frac{\sqrt{\beta_1 / \beta_2}}{\sin \varphi_{12}} \\ -\frac{\sqrt{\beta_2 / \beta_1}}{\sin \varphi_{12}} & \cot \varphi_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (10)$$

### Timing in Phase Space Data Acquisition

The phase space monitor system is based on turn-by-turn BPM system. Beam tracks are obtained on the transverse phase space by using turn-by-turn beam position data at two differently-located beam-position-monitor electrodes.

To measure positions of one bunch in the same turn, usually two techniques are applicable. One is using delay line on signal transport line and the other is using delay sampling at ADC. In this case, we use the latter. We use injection kicker as the two ADCs' external trigger. And through network computer, we can adjust the time delay of the ADCs' external clock to obtain optimal acquisition of position corresponding to the same bunch. Figure 4 shows the relationship between ADC sampling clock and the sampled signal.

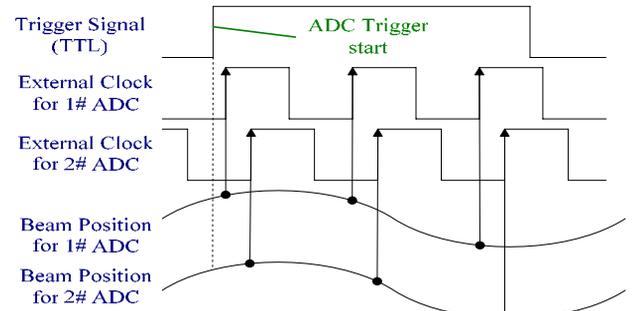


Figure 4: External Clock for two ADCs.

To validate the reliability of the system, we use a sine wave signal generator and a power splitter to produce two same sine wave with no phase delay. By sampling these two signals, we can know whether the ADC work as expected. Figure 5 and Figure 6 show the origin signal and the sampled signal. After computing the sampled signal, we can conclude that the method using delay sampling at the two ADC work well.

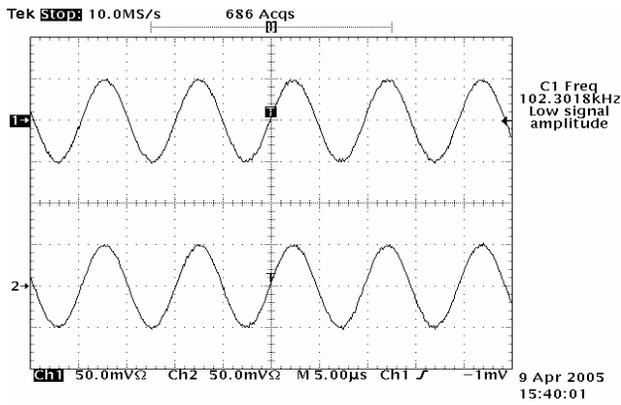


Figure 5: The in-phase origin sampled signal generated by signal generator and spitted by a power splitter.

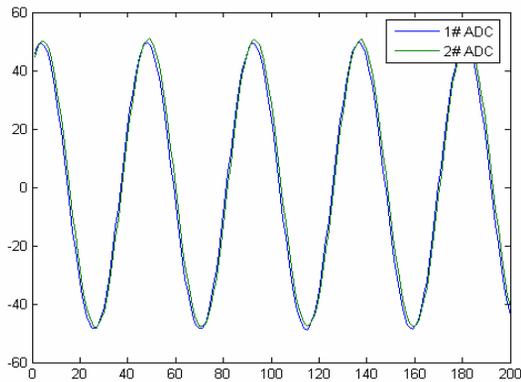


Figure 6: The sampled signal by two ADC with out-phase external clock.

In our TBT system, the ADC's sampling rate is just the revolution frequency of bunch (4.533MHz). Here, we use the Lattice shown as Table 1 in our experiment:

Table 1: The Lattice parameter

	BPM13#	BPM16#
BETX	10.07368276	10.07376894
ALFX	-3.608354324	3.608355155
MUX	2.662911552	2.707070453

With the transform matrix (10), we can compute the map of phase space. Figure 7 and Figure 8 show the map of phase space of each ADC's sample data.

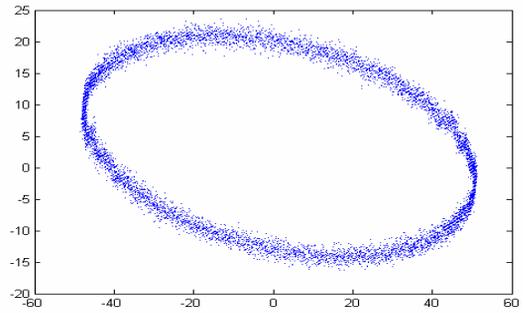


Figure 7: The map of phase space at 1#ADC with turn by turn data of Figure 6 shown.

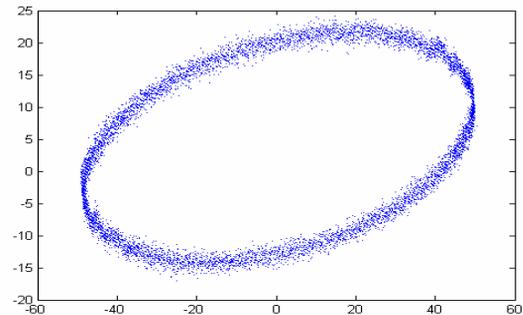


Figure 8: The map of phase space at 2#ADC with turn by turn data of Figure 6 shown.

### ACKNOWLEDGEMENTS

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