THE CODE MBIM1 FOR THE CALCULATION OF THE MULTIBUNCH BEAMS COHERENT OSCILLATIONS STABILITY (IN APPROACH OF SHORT BUNCHES)

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Abstract

The code MBIM1 for the calculation of the coherent oscillations stability for multibunch beams in storage rings is presented. The multibunch beams with arbitrary charges of bunches are considered, including counterrotating beams (in approach of short bunches in comparison with minimal wavelength of considered environment RF spectrum), with the account of beams coupling with the environment (i.e. RF cavities or/and smooth vacuum chamber with walls of finite conductivity). The code uses the approach of small shifts of coherent frequencies, when different multipole types of synchrotron oscillations can be treated as independent from each other.

INTRODUCTION

One of the ways to analyze the stability of coherent oscillations is the frequency domain solution, with formulation of the selfconsistent equation system for the whole beam as the eigenvalue problem and with defining the stability of the motion by its eigenvalues. Most authors using this method deal only with symmetric beams and use for unsymmetric an upper estimation of its growth rates via the maximal growth rate of the symmetric beam for which this unsymmetric beam could be a constituent [1]. But the current of such symmetric beam is higher than that of the considered unsymmetric beam, therefore, this approximation can exceed significantly actual growth rates of the unsymmetric beam and actual requirements to the feedback system.

In this paper we present the formulation of the eigenvalue problem for arbitrary multibunch beams, (possibly with different charges, including the case of the counterrotating beams), in the approach of short bunches for which the order of the eigenvalue problem is equal to the number of bunches. The Landau damping is taken into account.

We follow here the method developed in [2] for symmetric beams using a continuum model, with the same restrictions: the sinusoidal oscillations in the absence of excitation are small; the perturbations of the distribution functions due to the interaction are small (as compared to the undisturbed distribution); the interaction only with the cavity modes with the wavelength greater than the bunch length is considered; the dependence of the synchrotron or/and betatron frequency on the amplitude is taken into account in the first approach of small amplitudes; the unperturbed distribution functions of all bunches are supposed to be identical (gaussian) (in particular, all bunches have the same length), but their currents can be different; smooth focusing with the same betatron tune ν_x is considered.

The details of all derivations are given in [3] and [4].

THE SYSTEM OF INTEGRAL EQUATIONS

Starting from the linearized Vlasov equation (in terms of action J and phase ψ) for the perturbations of distribution functions of all bunches and assuming that all multipole modes of synchrotron oscillations could be considered separately, for the multibunch beam, one can write the system of integral equations (the RHS is equal to zero for the eigenvalue problem):

$$F_n^l(J,s) - \sum_{j=1}^{N_b} \int K_{nn}^{lj}(J,J',s) F_n^j(J',s)\,dJ' = 0,$$

$$l = 1,\dots,N_b,$$

where $F_n^l(J,s)$ is the n-th multipole harmohic of synchrotron oscillations for the perturbation distribution of the l-th bunch; in the case of the transverse oscillations denotation J defines a combination of two variables, J_x and J_z, for which $\int dJ = \int dJ_x dJ_z$.

The kernel of the system for the multipole synchrotron oscillations is

$$K_{nn}^{lj}(J_z,J_z',s) = e\frac{\partial f_0^l}{\partial J}\frac{I_j}{(s+in\Omega_z(J_z))}\times$$

$$\times\sum_m\left\{\frac{n}{m}Z(s-im\omega_0)e^{im(\theta_l-\theta_j)}\times\right.$$

$$\left.\times J_n\left(m\phi_0\sqrt{J/J_0}\right)J_n\left(m\phi_0\sqrt{J'/J_0}\right)\right\},$$

where $f_0^l(J)$ is the undisturbed distribution function of the l-th bunch, independent from time and phase, which is supposed to be the same (gaussian) for all bunches: $f_0^l(J) = \frac{1}{2\pi J_0}\exp(-J/J_0)$; $\phi_0 = \sigma_b\sqrt{2}/R$, R is the radius of the storage ring, σ_b is the r.m.s. bunch length (the same for all bunches); $Z(s-im\omega_0)$ is the total impedance of the cavity reduced to the narrow gap; ω_0 is the revolution frequency; I_j is the average current of the j-th bunch; $\Omega(J)$ in the frequency of synchrotron oscillations (at zero current).

For the transverse oscillations (taking into account their possible dependence on the n-th harmonic pf synchrotron oscillations),

$$K_{nn}^{lj}(J_x,J_z,J'_x,J'_z,s)=\frac{e\dfrac{\pi\omega_0 R}{E_s\nu_x}\dfrac{\partial f_0^l}{\partial J_x}\sqrt{J_xJ'_x}\cdot I_j}{(s+in\Omega_z(J_z)+i\Omega_x(J_x))}\times$$
$$\times\sum_m\Big\{Z_{tx}(s-im\omega_0)e^{im(\theta_l-\theta_j)}\times$$
$$\times J_n\Big((m-\nu_x+\kappa)\phi_0\sqrt{J/J_0}\Big)J_n\Big((m-\nu_x+\kappa)\phi_0\sqrt{J'/J_0}\Big)\Big\},$$

where $\kappa=(p_0/\alpha)\partial\nu_x/\partial p_z$ describes the dependence of the betatron tune on the longitudinal momentum via momentum compaction factor α, p_0 being the whole longitudinal momentum; E_s is the energy of the synchronous particle.

In the approach of short bunches, using the first approximation for Bessel functions of small arguments, the system of integral equations can be reduced to the system of linear algebraic equations:

$$(A_0\hat{Z}\hat{N}-g(s)\hat{E})\vec{X}=0,$$

where $\vec{X}$ is the vector of complex amplitudes of perturbation functions of all bunches, $\hat{N}_{ij}=\delta_{ij}I_i$ is the diagonal matrix describing the charges of all bunches, the matrix $\hat{Z}$ describes the interaction of the bunches with environment and thereby with each other, the function $g(s)$ is a dispersion function describing relation $\lambda_j=g(s_j)$ between the system eigen values λ_j and eigen frequencies of coherent modes $\Omega_j=is_j$, where real parts of s_j determine the stability.

For multipole synchrotron oscillations (with $n>0$), in view of the amplitude dependence of the synchrotron frequency $\Omega_z(J_z)=\Omega_{z0}(1-\xi_zJ_z/J_{z0})$

$$\hat{Z}_{lj}=\sum_m m^{2n-1}e^{im(\theta_l-\theta_j)}Z(-in\Omega_{z0}-im\omega_0),\qquad(1)$$

$$A_0=\frac{n}{n!}\left(\frac{\sigma_b^2}{2R^2}\right)^{n-1}\frac{\Omega_{z0}}{2q_{rf}V_{rf}sin(\phi_s)},$$

$$g(s)=\frac{\int\dfrac{\partial f_0}{\partial J_z}(J_z/J_{z0})^n\,dJ_z}{\int\dfrac{\partial f_0/\partial J_z}{s+in\Omega_z(J_z)}(J_z/J_{z0})^n\,dJ_z},\qquad(2)$$

where q_{rf}, V_{rf} and ϕ_s are the RF harmonic number, the RF voltage amplitude and the synchronous phase, θ_j are the angular positions of the bunches.

For large complex coherent shifts $|\Delta s_j|=|s_j+in\Omega_0|>>\xi\Omega_0$ one can neglect the amplitude dependence of the synchrotron frequency and get $\Delta s_j=\lambda_j$. At small $|g(s)|\le\xi\Omega_0$ the eq.(2) could have no solution, i.e. no coherent motion at all. The solutions for $n<0$ are complex conjugate to those for $n>0$.

For the transverse oscillations, in view of the influence of different multipole modes of synchrotron oscillation and taking into account the amplitude dependence of synchrotron and betatron frequencies, we have

$$\hat{Z}_{lj}=\sum_m(m-\nu_x+\kappa)^{2|n|}e^{im(\theta_l-\theta_j)}(Z_{tx})^+_{m,n},\qquad(3)$$

$$(Z_{tx})^+_{m,n}=Z_{tx}(-im\omega_0-i\Omega_{x0}-in\Omega_{z0}),$$

$$g(s)=\frac{\int\dfrac{\partial f_0}{\partial J_x}J_xJ_z\,|n|\,dJ_xdJ_z}{\int\dfrac{(\partial f_0/\partial J_x)J_xJ_z\,|n|}{s+i\Omega_x(J_x)+in\Omega_z(J_z)}dJ_xdJ_z},\qquad(4)$$

$$\Omega_\alpha(J_\alpha)=\Omega_{\alpha0}\big(1-\xi_\alpha J_\alpha/J_{\alpha0}\big),\quad \alpha=x,z.$$

These equations are written for solutions near $s=in_x\Omega_{x0}+in\Omega_{z0}$, for $n_x=1$ and any integer n_z. The solutions for $(-n_x,-n_z)$ are complex conjugate to those for (n_x,n_z).

The dispersion function (4) for the transverse oscillations is a somewhat more complicate two-dimensional analog of this function for longitudinal oscillations (2). The main difference consists in dependence on two parameters, $n\xi_z\Omega_{z0}$ and $\xi_x\Omega_{x0}$.

COUNTERROTATING BEAMS

Dealing with counterrotating electron and positron beams, which interact with an RF cavity placed at the angular distance θ_c from the point of meeting of bunches, we will replace (for convenience of derivations) the positron bunches with equivalent electron bunches whith the same currents, rotating in opposite direction and passing the cavity at the same moments as the original positron bunches. If the angular distances between neighbour bunches (including, if necessary, bunches with the zero charge) is equal to $2\pi R/N_b$, then we can write the angular position of all bunches as

$\theta_j=\dfrac{2\pi}{N_b}(j-1)$ $j=1,...,N_b$ - for bunches of the original electron beam,

$\theta_j=\dfrac{2\pi}{N_b}(j-N_b-1)+2\theta_c$ $j=1+N_b,...,2N_b$ - for bunches of the additional electron beam, equivalent to the original positron beam.

In view of this difference, one can denote

$$\vec{X}=\begin{pmatrix}\vec{X}_e\\ \vec{X}_p\end{pmatrix},\ \hat{Z}=\begin{pmatrix}\hat{Z}_{ee} & \hat{Z}_{ep}\\ \hat{Z}_{pp} & \hat{Z}_{pp}\end{pmatrix},$$

where $\hat{Z}_{ee,pp}$ describe the interaction between the bunches of the same beam and $\hat{Z}_{ep,pe}$ describe the interaction between the bunches of the counterrotating beams. Therefore, in (1) and (3) one should take

$\theta_i-\theta_j=\dfrac{2\pi}{N_b}(i-j)$ for $\hat{Z}_{ee,pp}$ and

$\theta_i-\theta_j=\dfrac{2\pi}{N_b}(i-j)\mp2\theta_c$ for $\hat{Z}_{ep,pe}$.

Note, that if the RF system consists of N_{cav} cavities, then the elements of $\hat{Z}$ should be summed up for all the cavities with their angular distances from the point of meeting: $\hat{Z} = \sum_{c=1}^{N_{cav}} \hat{Z}_c(\theta_c)$.

One should mark that at the interaction of the counterrotating beams with the resistive wall impedance of the longitudinally homoheneous vacuum chamber, the fields induced by each beam have only harmonics propagating in the same direction as the beam itself and have no reflected waves which could interact with counterrotating beam. As a consequence, in this case the elements of $\hat{Z}_{ep,pe}$ are equal to zero.

THE CODE MBIM1

The method given above is realized in the computer code MBIM1 (MultiBeam Instability, Multipole oscillations, version 1 for short bunches). The code solves a problem of longitudinal (or transverse) coherent oscillations of arbitrary multibunch beams in approach of short bunches and small coherent frequency shifts, in view of Landau damping.

The interaction with cavities of RF system and with a resistive wall impedance of the longitudinally-homogeneous vacuum chamber (for transverse oscillations) is taken into account, in the terms of the cavities impedance described as a table with resonant parameters of all modes of all cavities or as a given (tabulated) frequency dependence. For resonant modes of RF cavities spectrum, the method of analytical summation of the serieses over azimuthal harmonics given in [5] is applied. If the counterrotating beams are considered, one should define the angular position of cavities with respect to one of the points of meeting of bunches.

In the used approach of short bunches, the order of the problem (that is the order of the considered equation system) is equal to the number of bunches with a nonzero charge.

AN EXAMPLE

Considering only one resonant mode of the RF cavity (with resonant frequency ω_r), let us demonstrate dependence of the maximal growth rate of the multibunch beam with a gap on the gap width and quality factor. We consider $N_b = 1,...,N_{b0}$ bunches ($N_{b0} = 30$) following with distances between them $2\pi R/N_{b0}$; the gap length is $2\pi R(N_{b0} - N_b + 1)/N_{b0}$. Fig.1 shows the field map of equal maximal growth rates (in logarithmic scale) at the plane of variables N_b and $log(Q)$. The results are given for the same current of one bunch $I_1 = const$ (above) and for the same current of the whole beam $I_1 N_b = const$ (below). This example shows that the upper estimation of the growth rates with that of symmetric beam given in [1], though valid for $I_1 = const$, cannot predict the change of growth rates for the beam with a gap with the constant whole current of the beam.

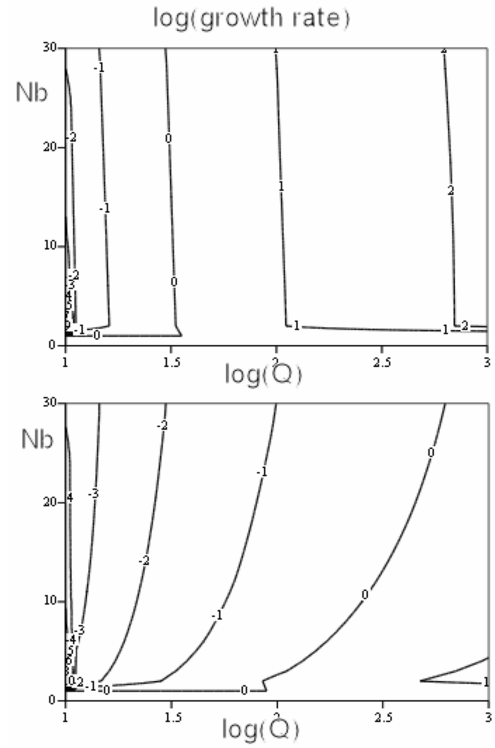

Figure 1: The field map of equal maximal growth rates (in logarithmic scale) in dependence on the number of bunches N_b and $log(Q)$ for the same current of one bunch $I_1 = const$ (above) and for the same current of the whole beam $I_1 N_b = const$ (below).

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