THE IMPEDANCE OF THE CERAMIC CHAMBER IN J-PARC

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Abstract

We calculate the impedance of the ceramic chamber inside of which is coated with TiN and outside of which is Cu shielded.

INTRODUCTION

Space charge impedance of rf-shielding wires with external ceramic and conducting pipes were studied by T. F. Wang et. al.[1]. This system corresponds to the beam duct adopted in ISIS.

In J-PARC[2] the ceramic chamber is surrounded by Cu-stripes due to technical reasons. The inner surface of the chamber is coated with TiN to suppress the secondary electron emission. Tsutsui and Lee studied this system by replacing Cu-stripes to the conducting beam pipe[3,4]. In this paper we deal with Cu-stripes as rf-shield wires, and compare the results with the previous results. The resistivity of these rf-shield wires and the TiN coating is considered.

THE LONGITUDINAL IMPEDANCE

In this section we calculate the longitudinal impedance of the ceramic chamber. We use the cylindrical coordinate(r,θ,z). A source particle q moves along the z-direction with velocity βc at an offset of $r=r_b$, $\theta=\theta_\beta$. The charge density ρ is expressed as,

$$\rho = \frac{q}{r_b}\delta(r-r_b)\delta_p(\theta-\theta_b)\delta(z-\beta ct)$$
$$= \sum_{m=0}^{\infty}\int\frac{dk}{2\pi}\frac{i_m\cos m(\theta-\theta_b)}{\pi r_b^{m+1}(1+\delta_{m0})}\delta(r-r_b)e^{ik(z-\beta ct)}, \tag{1}$$
$$i_m = qr_b^m.$$

Here we introduce multipole charge density ρ_m:

$$\rho_m = \frac{i_m\cos m(\theta-\theta_b)}{\pi r_b^{m+1}(1+\delta_{m0})}\delta(r-r_b)e^{ik(z-\beta ct)}. \tag{2}$$

The field driven by the charge density ρ is obtained by the superposition of the field come from this multipole charge density.

The system we consider is described in Fig.1.The field is calculated by solving the Helmholz equations for the scalar potential except the TiN coating region. When we deal with the TiN coating region, we solve the equation for the vector potential. The solutions of this system where $e^{ik(z-\beta ct)}$ is omitted are written as,

$$E_z = -\frac{ikb_m}{\gamma^2}\left(\cos(\theta-\theta_b)\frac{K_m(\bar{k}r_b)}{I_m(\bar{k}r_b)}I_m(\bar{k}r)\right.$$
$$\left.+\sum_{p=-\infty}^{\infty}K_{m-pN(k}\bar{k}aF_{m-pN}(k)\cos[(m-pN)\theta-m\theta_b]I_{m-pN}(\bar{k}r_b)\right) for \quad r<r_b, \tag{3}$$

$$E_z = -\frac{ikb_m}{\gamma^2}\left(\cos(\theta-\theta_b)K_m(\bar{k}r)\right.$$
$$\left.+\sum_{p=-\infty}^{\infty}K_{m-pN}(\bar{k}a_I)F_{m-pN}(k)\cos[(m-pN)\theta-m\theta_b]I_{m-pN}(\bar{k}r)\right)$$
$$for \quad r_b<r<a_I, \tag{4}$$

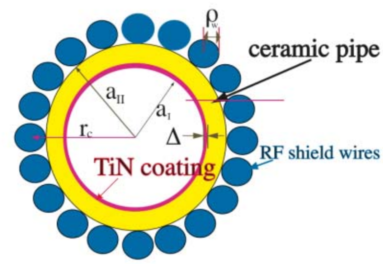

Fig.1: The ceramic chamber outside of which is surrounded by rf-shield wires. The inner surface is coated with TiN.

$$E_z = ikb_m\beta c\sum_{p=-\infty}^{\infty}(K_{m-pN}(\kappa_{TiN}(a_I+\Delta))A_{m-pN}I_{m-pN}(\kappa_{TiN}r)$$
$$+I_{m-pN}(\kappa_{TiN}a_I)B_{m-pN}K_{m-pN}(\kappa_{TiN}r))\cos[(m-pN)\theta-m\theta_b]$$
$$for \quad a_I<r_b<a_I+\Delta, \tag{5}$$

$$E_z = -\frac{ikb_m}{\gamma_1^2}\left(\frac{1}{\varepsilon_1}\sum_{p=-\infty}^{\infty}I_{m-pN}(\bar{k}_1a_I)F_{m-pN}^{(1)}(k)\cos[(m-pN)\theta-m\theta_b]K_{m-pN}(\bar{k}_1r)\right.$$
$$\left.+\frac{1}{\varepsilon_1}\sum_{p=-\infty}^{\infty}K_{m-pN}(\bar{k}_1a_{II})F_{m-pN}^{(2)}(k)\cos[(m-pN)\theta-m\theta_b]I_{m-pN}(\bar{k}_1r)\right)$$
$$for \quad a_I+\Delta<r_b<a_{II}, \tag{6}$$

$$E_z = -\frac{ikb_m}{\gamma^2}\left(\sum_{p=-\infty}^{\infty}I_{m-pN}(\bar{k}_1a_{II})F_{m-pN}^{(3)}(k)\cos[(m-pN)\theta-m\theta_b]K_{m-pN}(\bar{k}r)\right.$$
$$\left.-\frac{1}{2}\sum_{n=-\infty}^{\infty}\frac{I_n(\bar{k}\rho_W)}{K_n(\bar{k}\rho_W)}NK_{m+n}(\bar{k}r_c)(e^{im(\bar{m}\Delta-\theta_b)}Q_n(r)P_n+e^{-im(\bar{m}\Delta-\theta_b)}Q_n^*(r)P_n)\right)$$
$$for \quad a_{II}<r, \tag{7}$$

where

$$Q_n = \sum_{p=-\infty}^{\infty}K_{n+m-pN}(\bar{k}r_c)I_{m-pN}(\bar{k}r)e^{i(m-pN)(\theta-\bar{m}\Delta)}, \tag{8}$$

for $r<r_c$, r and r_c are exchanged for $r>r_c$, r_c is the position of rf-wire, ε_1 is the relative dielectric constant of ceramic, μ_1 is the relative permeability of ceramic, the wave number k and the frequency f is related by k=2πf/βc, $\bar{k}=k/\gamma$, $\gamma_1=1/\sqrt{1-\beta^2\varepsilon_1\mu_1}$, σ_{TiN} is the conductivity of TiN, σ_{RF} is the conductivity of rf-wire, $b_m=i_mI_m(\bar{k}r_b)/\varepsilon_0\pi r_b^m(1+\delta_{m0})$, $\delta_{m,0}$ is Kroneker δ, Δ is the width of TiN coating, $a_I+\Delta$ is the inner radius of the ceramic, a_{II} is the outer radius of the ceramic, ε_{RF} is the relative dielectric constant of rf-wire which is given by $1+i\sigma_{RF}Z_0/k\beta$[5,6], ε_{TiN} is the relative dielectric constant of TiN which is given by $1+i\sigma_{TiN}Z_0/k\beta$[5,6], $\kappa_{RF}=\sqrt{k^2-\varepsilon_{RF}\mu_{RF}k^2\beta^2}$, $\kappa_{TiN}=\sqrt{k^2-\varepsilon_{TiN}\mu_{TiN}k^2\beta^2}$, and F_{m-pN}, $F_{m-pN}^{(1)}$, $F_{m-pN}^{(2)}$, $F_{m-pN}^{(3)}$, P_n, A_{m-pN} and B_{m-pN} are arbitrary coefficients.

Let us consider the field inside of the rf-shield wire. Following T. F. Wang, we use local coordinate $(\rho,\phi_{\bar{m}},z)$, where $\bar{m}$ denotes the index of the ordering of rf-wire. Since the field inside of the rf-wire satisfies

$$\left(\frac{\varepsilon_{RF}\mu_{RF}}{c^2}\frac{\partial^2}{\partial t^2}-\Delta\right)\vec{A}=0, \tag{9}$$

where $\Delta=\nabla\cdot(\nabla\cdot)-\nabla\times(\nabla\times)$, A_z can be written as,

$$A_z^{RF}=\frac{b_m}{2}\sum_l\left(c_l e^{im(\bar{m}\Delta-\theta_b)}+c_{-l}e^{-im(\bar{m}\Delta-\theta_b)}\right)I_l(\kappa_{RF}\rho_{\bar{m}})e^{il\phi_{\bar{m}}} \tag{10}$$

where c_l are the arbitrary coefficients. According to matching conditions, P_n, c_l and $F_m^{(3)}$ are related as follows,

$$\sum_{n=-\infty}^{\infty}\frac{I_n(\bar{k}\rho_W)}{K_n(\bar{k}\rho_W)}NK_{m+n}(\bar{k}r_c)P_n d_{n,l}$$
$$=\sum_{p=-\infty}^{\infty}I_{m-pN}(\bar{k}a_{II})F_{m-pN}^{(3)}K_{m+l-pN}\left(\bar{k}r_c\right)I_l\left(\bar{k}\rho_W\right)+\gamma^2\beta cI_l(\kappa_{RF}\rho_W)c_l, \tag{11}$$

$$\frac{\beta}{c}\left(1-\frac{1}{\varepsilon_{RF}\mu_{RF}\beta^2}\right)\sum_{p=-\infty}^{\infty}I_{m-pN}(\bar{k}a_{II})F_{m-pN}^{(3)}K_{m+l-pN}\left(\bar{k}r_c\right)\frac{1}{2}\left(I_{l+1}\left(\bar{k}\rho_W\right)+I_{l-1}\left(\bar{k}\rho_W\right)\right)$$
$$-\frac{\beta}{c}\left(1-\frac{1}{\varepsilon_{RF}\mu_{RF}\beta^2}\right)\sum_{n=-\infty}^{\infty}\frac{I_n(\bar{k}\rho_W)}{K_n(\bar{k}\rho_W)}NK_{m+n}(\bar{k}r_c)P_n d'_{n,l}(\rho_W)$$
$$=\frac{1}{\mu_{RF}}c_l\frac{\kappa_{RF}}{2}\left(I_{l+1}(\kappa_{RF}\rho_W)+I_{l-1}(\kappa_{RF}\rho_W)\right), \tag{12}$$

where ρ_W is the radius of rf-wire, $d_{n,l}$ is the Fourier coefficients of Q_n:

$$Q_n=\sum_{l=-\infty}^{\infty}d_{n,l}(\rho_{\bar{m}})e^{il\phi_{\bar{m}}}, \tag{13}$$

prime at $d_{n,l}$ in Eq.(12) means taking the derivative of $\rho_{\bar{m}}$.

All arbitrary coefficients: F_{m-pN}, $F_{m-pN}^{(1)}$, $F_{m-pN}^{(2)}$, $F_{m-pN}^{(3)}$, P_n, c_l, A_{m-pN} and B_{m-pN} are decided by Eqs.(11),(12) and the matching conditions at $r=a_I$, $r=a_I+\Delta$, $r=a_{II}$: E_z, H_θ and the normal component of D are continuous. In our calculation, we only consider the lowest order.

Since we find arbitrary coefficients, we can calculate the longitudinal impedance. According to Eqs.(3) and (4), E_z for the cylindrical beam whose radius is σ is expressed as,

$$E_z=-\frac{cZ_0}{\pi\sigma^2}\frac{ik}{\gamma^2}\left(\int_r^\sigma dr_b r_b K_0(\bar{k}r_b)I_0(\bar{k}r)+\int_0^r dr_b r_b I_0(\bar{k}r_b)K_0(\bar{k}r)\right)$$
$$-\frac{cZ_0}{\pi\sigma^2}\frac{ik}{\gamma^2}\int_0^\sigma drr_b I_0(\bar{k}r_b)\sum_{p=-\infty}^{\infty}K_{pN}(\bar{k}a_I)F_{pN}(k)\cos[pN\theta]I_{pN}(\bar{k}r)$$
$$=-\frac{cZ_0}{\pi\sigma^2}\frac{ik}{\gamma^2}\left(\frac{1}{\bar{k}^2}-\frac{\sigma I_0(\bar{k}r)K_1(\bar{k}\sigma)}{\bar{k}}\right)-\frac{cZ_0}{\pi\sigma^2}\frac{ik}{\gamma^2}\frac{\sigma I_1(\bar{k}\sigma)}{\bar{k}}\sum_{p=-\infty}^{\infty}K_{pN}(\bar{k}a_I)F_{pN}(k)\cos[pN\theta]I_{pN}(\bar{k}r). \tag{14}$$

By taking an average for r, we obtain

$$\frac{Z_\parallel}{n}=2\frac{Z_0}{\sigma^2}\frac{i}{\gamma\sqrt{\gamma^2-1}}\frac{1}{\bar{k}^2}(1-2K_1[\bar{k}\sigma]I_1[\bar{k}\sigma]+2I_1^2[\bar{k}\sigma]K_0[\bar{k}a_I]F_0(k)). \tag{15}$$

Space charge impedance is automatically included in Eq.(15). In order to define the impedance of the ceramic chamber, we have to subtract the space charge part. By considering the system that the perfect conducting pipe exists at $r=a_I$ the space charge impedance is calculated as follows,

$$\frac{Z_\parallel}{n}=-2\frac{Z_0}{\sigma^2}\frac{i}{\beta k^2}\left(1-2K_1[\bar{k}\sigma]I_1[\bar{k}\sigma]-2\frac{K_0[\bar{k}a_I]}{I_0[\bar{k}a_I]}I_1^2[\bar{k}\sigma]\right). \tag{16}$$

Here we should notice that Eq.(16) reproduces

$$\to-\frac{iZ_0}{2\beta\gamma^2}\left(\frac{1}{2}-2\log\left[\frac{\sigma}{a_I}\right]\right),$$

for large γ[7]. From now on, we assume that space charge part included in Eq.(15) is given by Eq.(16). We define the impedance for the ceramic chamber by subtracting Eq.(16) from Eq.(15).

Previously, Lee obtained the analytic formula of the longitudinal impedance per a unit length for the system described in Fig.2[4]. This can be rewritten as follows,

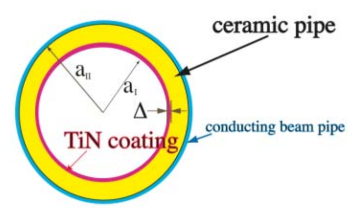

Fig.2: The ceramic chamber outside of which is surrounded by the conducting beam pipe.

$$\frac{Z}{n}=\frac{Z_0}{k\beta a_I\left(-\frac{\varepsilon_1}{(\varepsilon_1\beta^2-1)ka_I\ln a_{II}/a_I}i-Z_0\Delta\sigma_{TiN}\right)}, \tag{17}$$

where small k approximation is applied. In Fig.3 we present the results calculated by Eq.(17), and the rigorous numerical results where small k approximation is not applied. We find that these two results are good agreement as γ becomes larger.

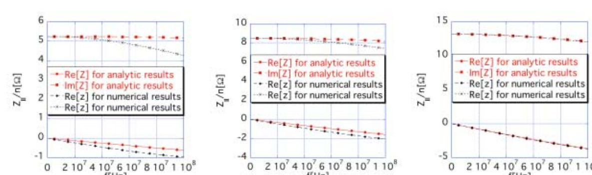

Fig. 3: The longitudinal impedance of the ceramic chamber for Fig.2 is shown. The analytic results and rigorous numerical results are compared. The left figure corresponds to γ=1.181, the middle figure γ=1.4, the right figure γ=4. r_c=0.1945[m], ε_1=10, μ_1=1, a_2=0.189[m], a_1=0.1815[m], $\sigma_{TiN}=6\times10^6[\Omega\cdot m]$, $\sigma_{RF}=5\times10^7[\Omega\cdot m]$, the beam size $\sigma=\sqrt{216\pi\times10^{-6}\times10}$, Δ=10[nm], N_W=110. $Re[Z_\parallel]<0$ means deceleration.

Let us compare our results of the longitudinal impedance with the previous results. In Fig.4, we compare the longitudinal impedances for Fig.1 with

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numerical results for Fig.2. These two results are good agreement as γ becomes larger.

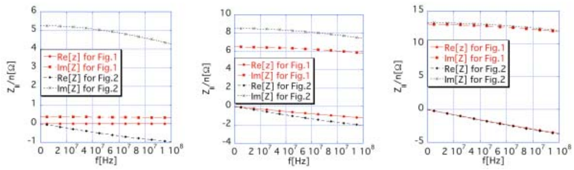

Fig.4: The longitudinal impedance of the ceramic chamber is shown. The results for Fig.1 and the rigorous numerical results for Fig.2 are compared. The left figure corresponds to γ=1.181, the middle figure γ=1.4, the right figure. The parameters: r_c, σ_{TiN}, ε_1, μ_1, a_1, a_2, Δ and σ_{TiN} are the same as those described in Fig.3. Other parameters are given by $\sigma_{RF}=5\times10^7[\Omega\cdot m]$,$\rho_W$==2.75[mm],$N_W$=110. Re[$Z_{\|}$]<0 means deceleration.

THE TRANSVERSE IMPEDANCE

The transverse force $F_{x,y}$ is calculated by Panofsky-Wenzel theorem using E_z, while the transverse force can be written by the transverse impedance Z_1 as follows,

$$F_{x,y} = ic\beta i_1\left(\frac{dr}{d\vec{r}}\cos(\theta-\theta_b) - r\frac{d\theta}{d\vec{r}}\, sin(\theta-\theta_b)\right)\frac{Z_1}{2\pi R}, \tag{18}$$

where $2\pi R$ is the circumference of our ring[8]. Therefore, we obtain

$$Z_{1,withSP} = i\frac{I_1[\bar{k}r_b]Z_0kR}{\beta\gamma^3 r_b}\left(\frac{K_1[\bar{k}r_b]}{I_1[\bar{k}r_b]} + K_1[\bar{k}a_1]F_1\right). \tag{19}$$

As in the previous section, the space charge impedance is also included in this impedance. Since the transverse space charge impedance for the system that the perfect conducting pipe exists at r=a_1, can be written as,

$$Z_{1,SP} = i\frac{I_1[\bar{k}r_b]Z_0kR}{\beta\gamma^3 r_b}\left(\frac{K_1[\bar{k}r_b]}{I_1[\bar{k}r_b]} - \frac{K_1[\bar{k}a_1]}{I_1[\bar{k}a_1]}\right), \tag{20}$$

we define the transverse impedance for the ceramic chamber as follows,

$$Z_1 = i\frac{I_1[\bar{k}r_b]Z_0kR}{\beta\gamma^3 r_b}\left(\frac{K_1[\bar{k}a_1]}{I_1[\bar{k}a_1]} + K_1[\bar{k}a_1]F_1\right), \tag{21}$$

by subtracting Eq.(20) from Eq.(19). Here we should notice that Eq.(20) reproduces

$$\to\to iZ_0\frac{R}{\beta\gamma^2}\left(\frac{1}{r_b^2}-\frac{1}{a_1^2}\right),$$

for large γ[8].

Let us compare the transverse impedances of the ceramic chamber for Fig.1 with those for Fig.2 as in the longitudinal case. Results are represented in Fig.5. Two results are good agreement as γ becomes larger.

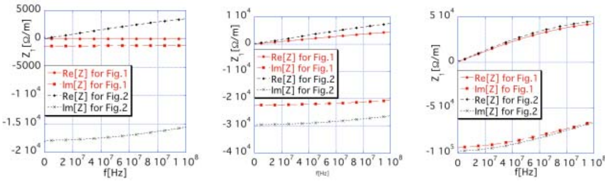

Fig.5: The transverse impedance of the ceramic chamber is shown. The results for Fig.1 and those for Fig.2 are compared. The left figure corresponds to γ=1.181, the middle figure γ=1.4, the right figure γ=4. The parameters : r_c, σ_{TiN}, ε_1, μ_1, a_1, a_2, Δ, σ_{TiN}, σ_{RF}, ρ_W and N_W are the same as those described in Fig.4.

CONCLUSIONS

The longitudinal and transverse impedance of the ceramic chamber inside of which is coated by TiN and outside of which is Cu-shielded are calculated. Previously, the impedance for this system was calculated by simplifying the system: replacing the Cu-stripes to the perfect conducting pipe. The results obtained here are compared with those previously obtained results. As γ becomes larger, those two results coincide each other; this means that the approximation that replacing the rf-shield wires to the conducting beam pipe is quite appropriate for large γ.

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