EFFICIENT MODELING OF NONLINEAR BEAM OPTICS USING PARAMETRIC MODEL INDEPENDENT ANALYSIS*

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Abstract

Based on precision beam orbit measurements, Model Independent Analysis has been used successfully to build a computer model that matches the linear optics of the real accelerator. We report a parametric extension of model independent analysis that will allow efficient modelling of the beam optics in the presence of beam energy dispersions. A simulation study is presented where the nonlinear dependency of lattice parameters on beam energy is captured by constrained training of a universal nonlinear approximator. Simulation results are presented that demonstrate the improved accuracy of beamline model verification and diagnosis with parametric model independent analysis. Improved optics models are expected to positively impact model-based beam operation and control.

INTRODUCTION

The accuracy of the constructed accelerator beamlines (compared to the designed lattice) directly determines accelerator performance. Therefore, the algorithms to verify and diagnose accelerator optics have long been of interest to accelerator physicists. Model-independent analysis is an analysis technique that employs statistical methods to verify the beamline model given turn-by-turn Beam Position Monitor (BPM) measurements [1, 2]. SLAC scientists have been able to use model-independent analysis to identify quadrupole strengths and sextupole feed-downs in the lattice model, as well as BPM gains and BPM cross-plane couplings, such that phase advance and coupling ellipses among BPMs calculated from the constructed lattice model match those derived from orbit measurements [3]. The lattice model so fitted to orbit measurements acts as a *virtual accelerator* that matches the real accelerator in linear optics, and hence may be used to identify new settings that could improve the performance of the real accelerator[1].

Model-independent analysis, however, is a linear analysis technique. While attempts to extend the analysis to verify nonlinear properties of the lattice have been of interest, the reported research on this topic has been scarce [2].

For this study, we used **P**arametric **U**niversal **N**onlinear **D**ynamics **A**pproximator (PUNDA) models [4], as a framework for building accurate and computationally efficient models for beamline optics (see Figure 2). The PUNDA model in this case is formed by a series connection of a Neural Networks (NN) model block and a Parametric Nonlinear Model (PNM) block. The NN model captures the functional dependency of beam invariants, Q_{12} and Q_{34} described by Eqs. (3) and (4), on beam operating conditions, given available BPM measurements. The PNM block embodies the **N**on-**L**inear **P**rogramming (NLP) problem for model independent analysis, described in the next section, and as such its outputs (*i.e.* normal and skew quad errors, and BPM gains and cross-plane couplings in our simulations) are implicit (*i.e.* not directly measured). The NN model is fully determined by the NN weights and biases. Constrained optimization is used to determine the NN weights and biases in this series structure.

NLP PROBLEM FOR MODEL INDEPENDENT ANALYSIS

In this section, we simply state the constrained NLP problem that lies at the heart of model independent analysis to emphasize the fact that unknown lattice parameters are estimated by an explicit nonlinear optimization[2]:

$$
\begin{aligned}
\min_{\{\Omega\}} \Big[& \tfrac{x_2^a x_1^b - x_1^a x_2^b}{Q_{12}} + \tfrac{x_4^a x_3^b - x_3^a x_4^b}{Q_{34}} + \left(g_x^b R_{12}^{ba} + \theta_{xy}^b R_{32}^{ba}\right) g_x^a \\
& \qquad + \left(g_x^b R_{14}^{ba} + \theta_{xy}^b R_{34}^{ba}\right) \theta_{xy}^a \Big]^2 + \\
\Big[& \tfrac{x_2^b y_1^a - x_1^b y_2^a}{Q_{12}} + \tfrac{x_4^b y_3^a - x_3^b y_4^a}{Q_{34}} - \left(g_x^b R_{12}^{ba} + \theta_{xy}^b R_{32}^{ba}\right) \theta_{yx}^a \\
& \qquad + \left(g_x^b R_{14}^{ba} + \theta_{xy}^b R_{34}^{ba}\right) g_y^a \Big]^2 + \\
\Big[& \tfrac{x_2^a y_1^b - x_1^a y_2^b}{Q_{12}} + \tfrac{x_4^a y_3^b - x_3^a y_4^b}{Q_{34}} + \left(\theta_{yx}^b R_{12}^{ba} + g_y^b R_{32}^{ba}\right) g_x^a \\
& \qquad + \left(\theta_{yx}^b R_{14}^{ba} + g_y^b R_{34}^{ba}\right) \theta_{xy}^a \Big]^2 + \\
\Big[& \tfrac{y_2^a y_1^b - y_1^a y_2^b}{Q_{12}} + \tfrac{y_4^a y_3^b - y_3^a y_4^b}{Q_{34}} + \left(\theta_{yx}^b R_{12}^{ba} + g_y^b R_{32}^{ba}\right) \theta_{yx}^a \\
& \qquad + \left(\theta_{yx}^b R_{14}^{ba} + g_y^b R_{34}^{ba}\right) g_y^a \Big]^2
\end{aligned}
$$

$\mathrm{s.t:}$ Physically meaningful constraints.

where:

a. $x_i^a/y_i^a, i = 1, 2, 3, 4$ are x and y measurements for 4 independent linear orbits at "BPM-a",

b. $x_i^b/y_i^b, i = 1, 2, 3, 4$ are x and y measurements for 4 independent linear orbits at "BPM-b",

c. $R_{12}^{ba}, R_{14}^{ba}, R_{32}^{ba}$, and R_{34}^{ba} are the elements of the symplectic transfer matrix between the two "BPM-a" and "BPM-b",

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[1] An interesting example is reported in [3] where model-independent analysis helped PEP-II to achieve a peak luminosity above $6.5 \times 10^{33}\ cm^{-2} s^{-1}$ for the first time by bringing the Low Energy Ring (LER) working tune to near half integer and simultaneously fixing the beta beat, which would have been difficult otherwise, due to LER coupling effect.

[2] An outline for the derivation of this NLP problem is presented in Appendix A. A more detailed derivation may be found in [5] and the references therein.

d. $\{\Omega\}$ includes the decision variables of the optimization problem:

   i. The gains, $g_x^a/g_y^a, g_x^b/g_y^b$, and cross-plane couplings, $\theta_{xy}^a/\theta_{yx}^a, \theta_{xy}^b/\theta_{yx}^b$ for the two BPMs,

   ii. Normal and skew quad errors, q_n/q_s, for any quadrupole magnet between the two BPMs that is not fully known,

   iii. Invariants Q_{12}/Q_{34} (defined in appendix B).

SIMULATION SCENARIO

The simulation study was carried out as follows:

1. A section of the storage ring between two BPMs (designated as "BPM-a" and "BPM-b") was selected for our simulation study. Each element in this section was modelled with at most quadratic nonlinearity as described in [6, 7].
2. We assumed that except for one quadrupole magnet (for which normal and skew quad errors, q_n and q_s, were not known) all the elements between "BPM-a" and "BPM-b" were fully known.
3. For BPMs, we treated BPM gains, g_x/g_y, and cross-plane couplings, θ_{xy}/θ_{yx}, as unknown parameters.
4. We assumed that four independent linear orbit measurements are available at the two BPMs.
5. With no energy dispersion, model-independent analysis produced accurate estimates of the unknown lattice parameters. With an energy dispersion, however, model-independent analysis produced erroneous estimates for the unknown parameters.
6. We showed that accounting for energy dispersion via a PUNDA model improved the estimate of the unknown beam parameters by a factor of 10.

The training of this PUNDA model was particularly challenging because none of the PUNDA model outputs (*i.e.* beamline model parameters in this case) were explicitly measured.

The Effect of Beam Dispersion

To carefully study the effect of beam dispersion, we configured our simulations as follows:

1. All components of the beam between the two BPMs were fully known and the exact values for $R_{12}^{ba}, R_{14}^{ba}, R_{32}^{ba}$, and R_{34}^{ba} were available.
2. We associated normal and skew quad errors with only one of the quadrupole magnets. Without loss of generality, this setup simplified the analysis of the simulation results.
3. We used unity BPM gains and zero roll parameters.

We then simulated the following two scenarios with a linear model for all elements of the beam:

1. *Zero energy dispersion:* Results of the model-independent analysis indicate accurate estimate of all the decision variables, $\{\Omega\}$.
2. *Non-Zero beam energy dispersion:* In this case we considered two different optimization problems:

   (a) Solved the optimization problem for all decision variables, $\{\Omega\}$ (see Fig. 1). The estimates for normal and skew quad errors deviated from 0.

   (b) We computed the beam invariants exactly (using Eqs. (3) and (4) in Appendix), and solved the optimization problem to only estimate BPM parameters and the normal and skew quad errors. The estimate for all parameters improved. In particular, the estimate for normal quad error improved with a factor of 8, while the estimate for skew quad error improved by a factor of 2.

Motivated by this observation, we developed a PUNDA model in which the NN block captured the functional dependency of beam invariants on the orbits and energy dispersion. A description of this PUNDA model and our simulation results are presented next.

PUNDA Models for Beamline Model Verification

The block diagram of the PUNDA model for beamline model verification is shown in Fig. 2. Note that in this PUNDA model

1. The NN block was trained to capture the functional dependency of the beam invariants on the independent orbit measurements and energy dispersion.
2. The PNM block was in fact the NLP problem described earlier, in which all outputs of the PUNDA model (*i.e.* the lattice parameters) are implicit (*i.e.* none are directly measured).

The implicit nature of the PNM block in the PUNDA model allowed us to investigate the additional challenges introduced by the implicitness of the PUNDA model outputs. Our simulation study clearly demonstrated that the use of constraints is critical to the meaningful training of the PUNDA model. For example, we used a known relationship between the two invariants [5] as a constraint on the outputs of the NN model.

We varied δP from 0 by $\pm\%20^3$, and verified that accounting for the changes in the beam invariants as a function of energy dispersion would consistently result in more-accurate estimates of BPM gains and normal and skew quad errors. On average, the PUNDA-based approach improved the estimate for normal quad error by a factor of 10, while the estimate for skew quad error was improved by a factor of 5 [5].

[3] In practice, energy dispersion is limited to only a few percent in most storage rings. The simulation here is designed to examine the effectiveness of a parametric approach to model independent analysis by exaggerating beam energy dispersion.

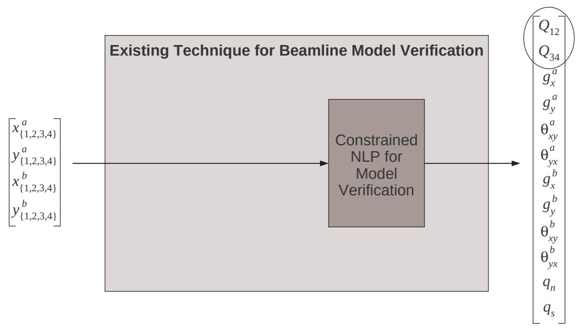

Figure 1: Pictorial representation of existing model independent analysis approach for beamline model verification.

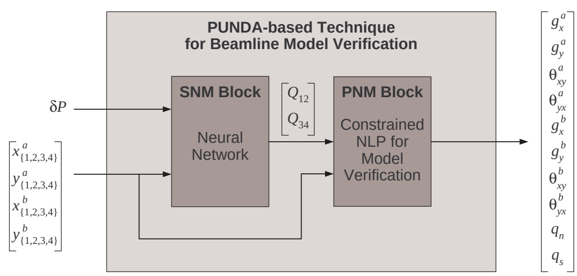

Figure 2: PUNDA model for beamline model verification.

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APPENDIX

To calculate the local Green's functions and the global invariants, model independent analysis uses 4 independent linear orbits. These linear orbits may be extracted from a complete set of turn by turn X and Y measurements by the BPMs throughout the ring as the result of one horizontal excitation and one vertical excitation [3, 8]. Having 4 independent orbits, one can conceptually form a non-singular matrix at "BPM-a" consisting of phase-space coordinates as follows:

$$Z^a = \begin{bmatrix} x_1^a & x_2^a & x_3^a & x_4^a \\ v_1^a & v_2^a & v_3^a & v_4^a \\ y_1^a & y_2^a & y_3^a & y_4^a \\ w_1^a & w_2^a & w_3^a & w_4^a \end{bmatrix} \tag{1}$$

where x_i^a is the horizontal displacement of the particle with respect to the central trajectory for the i-th orbit, y_i^a is the vertical displacement of the particle with respect to the central trajectory for the i-th orbit, $v_i^a = \frac{dx_i^a}{ds}$ is the particle's angle with the horizontal plane for the i-th orbit, and $w_i^a = \frac{dy_i^a}{ds}$ is the particle's angle with the vertical plane for the i-th orbit. The variable s indicates the position of the particle along the central longitudinal trajectory. Note that only x_i^a and y_i^a, $i = 1, 2, 3, 4$, are directly measured. The measurements at "BPM-b" can then be related to Z^a using beam transfer matrix $\mathbb{R}^{ba}$ as follows:

$$Z^b = \overbrace{\begin{bmatrix} R_{11}^{ba} & R_{12}^{ba} & R_{13}^{ba} & R_{14}^{ba} \\ R_{21}^{ba} & R_{22}^{ba} & R_{23}^{ba} & R_{24}^{ba} \\ R_{31}^{ba} & R_{32}^{ba} & R_{33}^{ba} & R_{34}^{ba} \\ R_{41}^{ba} & R_{42}^{ba} & R_{43}^{ba} & R_{44}^{ba} \end{bmatrix}}^{\mathbb{R}^{ba}} Z^a \tag{2}$$

Under symplectic conditions [1], *i.e.* when damping in the ring is offset by the excitation to an equilibrium state, beam invariants (*i.e.* constants around the ring) are represented by an anti-symmetric matrix Q:

$$Q = \left(Z^a\right)^T S Z^a = \left(Z^b\right)^T S Z^b, \; S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

that in general contains 6 independent invariants. With the assumption of only one horizontal and one vertical excitation, however, only two of these invariants are non-zero:

$$Q_{12} = -v_1^a x_2^a + v_2^a x_1^a - w_1^a y_2^a + w_2^a y_1^a \tag{3}$$

$$Q_{34} = -v_3^a x_4^a + v_4^a x_3^a - w_3^a y_4^a + w_4^a y_3^a \tag{4}$$

The equations for model independent analysis are then derived by noting that the transfer matrix $\mathbb{R}^{ba}$ is symplectic, *i.e.* $\left(\mathbb{R}^{ba}\right) S \left(\mathbb{R}^{ba}\right) = S$, and that a cleverly selected measurement matrix,

$$\widehat{Z} = \overbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ R_{11}^{ba} & R_{12}^{ba} & R_{13}^{ba} & R_{14}^{ba} \\ 0 & 0 & 1 & 0 \\ R_{31}^{ba} & R_{32}^{ba} & R_{33}^{ba} & R_{34}^{ba} \end{bmatrix}}^{\widehat{\mathbb{R}}} Z^a \tag{5}$$

only contains the available measurements and satisfies:

$$\left(\widehat{Z}\right) Q^{(-1)} \left(\widehat{Z}\right)^T = -\left(\widehat{\mathbb{R}}\right) S \left(\widehat{\mathbb{R}}\right)^T \tag{6}$$

Expanding Eq. (6), and transforming the left hand side to the measurement frame results in the NLP problem for model independent analysis.

