# SIMULATION ANALYSIS OF HEAD-TAIL MOTION CAUSED BY ELECTRON CLOUD

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*Abstract*

Synchro-beta side band signal caused by electron cloud instability has been observed at KEK-B factory. The side-band appears above the threshold of beam size blow-up and disappears by exciting solenoid magnets which covers whole ring. The side-band is an evidence of strong head-tail instability caused by electron cloud. The side-band appears in tune spectrum with positive shift $\nu_\beta + a\nu_s$ ($1 < a < 2$), while general strong head-tail instability shows a negative shift $\nu_\beta - a\nu_s$ ($a > 0$). We study the synchro-beta spectrum using a code, PEHTS, which simulates single bunch electron cloud instability.

## INTRODUCTION

Electron cloud induces a short range wake field with a characteristic frequency of

$$\omega_e = \sqrt{\frac{\lambda_p r_e c^2}{\sigma_y(\sigma_x + \sigma_y)}}, \qquad (1)$$

where $\lambda_e$ is a line charge density of a positron bunch. The wake field is considered to cause a strong head-tail instability above a threshold of an electron cloud density and/or a bunch population [1]. Needless to say, since the instability is coherent single bunch effect, synchro-beta side band signal should be observed near the frequency of $\nu_\beta + m\nu_s$ in Fourier analysis of the bunch motion. In the strong head-tail instability, two synchro-betatron oscillations are coupled and merged each other due to the wake field: that is, the frequency of the sideband is shifted from original one with $m$=integer for increasing the bunch intensity and/or the wake strength.

Synchro-beta side band signal for the strong head-tail instability caused by electron cloud has been observed in KEKB [2]. Figure 1 shows a measured typical synchro-beta sideband spectrum. Two peaks are seen in the spectrum; lower and higher peaks correspond to betatron tune and its synchrotron sideband, respectively, where the synchrotron tune is 0.024.

We have to note several important features in the observed spectrum. One is the sideband peak is higher than the betatron peak. Second is that the sideband peak appears between $\nu_\beta + \nu_s$ and $\nu_\beta + 2\nu_s$, and two peaks are separated for increase of cloud density.

A toy model using a wake field with a focusing nature roughly reproduces the feature [2]. Simulations, which take into account the interactions between bunch and electron cloud, must reproduce these features. Main theme of this paper is whether the sideband spectrum is reproduced by such simulations.

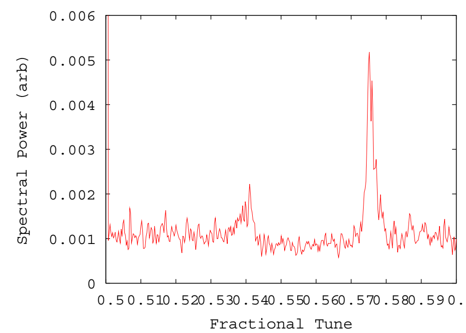


Figure 1: Typical synchro-beta sideband spectrum observed above the threshold of the beam size blow-up due to electron cloud (measurement). The vertical betatron tune is set $\approx 0.535$ in this measurement.

## SIMULATION AND RESULTS

We discuss the head-tail instability induced by ordinary wake field, $W_1(z) = az$, before electron cloud induced one. In general strong head-tail instability, modes with $\nu_\beta$ and $\nu_\beta - \nu_s$ couples each other, therefore instability peak is seen between $\nu_\beta$ and $\nu_\beta - \nu_s$. Figure 2 shows frequency spectrum of the dipole and $\langle yz \rangle$ motion of a bunch obtained by a simulation. The figure shows a typical behavior of the strong head-tail instability: that is, for increasing bunch intensity, synchrotron sideband $\nu_\beta - \nu_s$ peak shifts to higher, while betatron peaks shifts to lower, and the two peaks are merged above a threshold intensity as shown in the bottom picture. The simulation reproduces the behavior, which is consistent with analytic theory.

We now turn to the electron cloud induced head-tail instability. The simulation is performed by tracking a bunch and an electron cloud, which is represented by macro-particles, $\sim$ 300,000. Interactions between a bunch and electron cloud are evaluated by the particle in cell method on the transverse plane as is done for studying the beam-beam effect. A bunch is divided into 30 slices along the bunch length and motion of each slice interacting with electron cloud is calculated. Electron clouds are placed some positions in a ring.

Figure 3 shows the transverse beam centroid and size


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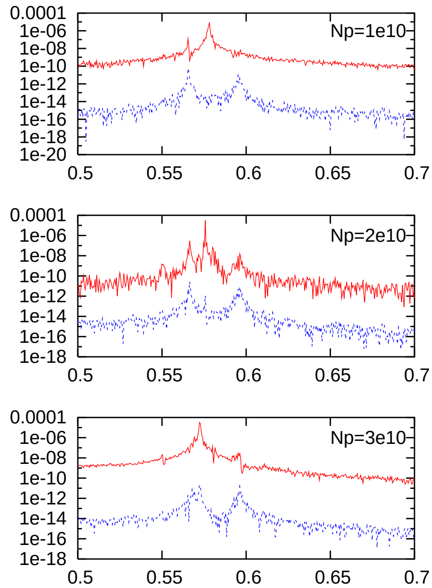


Figure 2: Typical frequency spectra for an ordinary wake, $W(z) = az$ (simulation with $\nu_\beta = 0.58$). Beam intensity are increased for lower picture. Red and blue lines are frequency spectra for $\langle y \rangle$ and $\langle yz \rangle$ motions, respectively.

of the bunch slices and electron cloud centroid along the longitudinal direction ($z/\sigma_z$). The centroids of bunch and cloud oscillate with the frequency of $\omega_e$. Integrated bunch centroid has a finite dipole moment as shown in the figure, if the dipole mode merges with another mode in the strong head-tail instability.

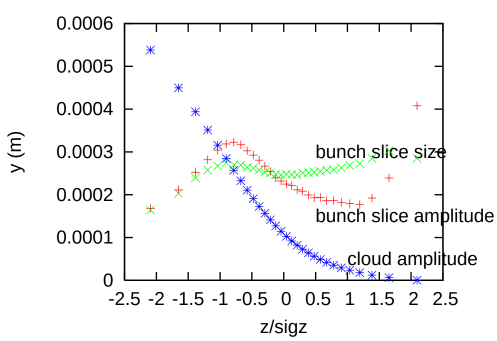


Figure 3: Typical bunch and electron cloud profile obtained by a simulation.

The dipole moment is damped by a bunch by bunch feedback system. The feedback system acts to suppress only the average dipole moment: i.e., each macro particle $(y_i, p_{y,i})$ is kicked in response to the averaged dipole moment $(\langle y \rangle, \langle p_y \rangle)$. The transformation, which is generated by the feedback system with active and reactive component is expressed by

$$\begin{pmatrix} \Delta y_i(n) \\ \Delta p_{y,i}(n) \end{pmatrix} = -(d_a + d_r S) M \begin{pmatrix} \langle y(n-1) \rangle \\ \langle p_y(n-1) \rangle \end{pmatrix} \quad (2)$$

where $M$ and $S$ are the revolution matrix and symplectic metric, respectively. The gains of active and reactive components are $d_a$ and $d_r$, respectively. The map includes one turn delay. Though actual feedback system is more complex, most of the essential feature will be included.

Chromaticity ($\xi_y$) may affect the spectrum of the bunch motion. Transfer map of the chromaticity is expressed by a Hamiltonian as follows,

$$H_c(y, p_y) = \frac{\gamma y^2 + 2\alpha y p_y + \beta p_y^2}{2} 2\pi \xi_y \delta. \quad (3)$$

where $\delta = \Delta E/E$.

The simulation gives centroid of each slice of a bunch in every turn. Fourier analysis was performed using the averaged centroid ($\langle y \rangle = \int y(z)\rho(z)dz$) and the correlation of y-z ($\langle yz \rangle = \int y(z)z\rho(z)dz$). Figure 4 shows frequency spectra for the averaged centroid ($\langle y \rangle$). Monitors measure the centroid position in turn by turn, therefore this figure can be compared with the experimental data in Figure 1. For fast feedback gain (top picture), three peaks are seen at low cloud density $2 \times 10^{11}$ m$^{-3}$. The peaks, $\sim 0.56$, $\sim 0.58$ and $\sim 0.605$, seems to correspond to $\nu_\beta - \nu_s$, $\nu_\beta$ and $\nu_\beta + \nu_s$, respectively. The peaks shift to higher for increasing cloud density. The peak, $\nu_\beta + \nu_s$, seems to behave the consistent with experiment. However we can not say the sideband peak is dominant in the figure. The behavior is not clear for low feedback gain (bottom picture). For high cloud density, bunch amplitude grows strongly, with the result that the peak tend to return the original value. Such behavior could be seen in strongly nonlinear system.

Figure 5 shows frequency spectra for the correlation function ($\langle yz \rangle$). Since the head-tail mode with $\nu_\beta + \nu_s$ hold the key of the experimental data, the spectra of $\langle yz \rangle$ may give us some information, though the beam monitor can not measure this mode. Top and bottom pictures depict spectra for fast (50 turn) and slow (200 turn) feedback damping time, respectively. We have sideband peak around $\nu_\beta + \nu_s$, which shifts to higher frequency for increasing cloud density. This behavior is consistent with experiment, but we can not say that the signals $\nu_\beta + \nu_s$ is dominant compare than other modes. Mode with $\nu_\beta - \nu_s$ always seems to dominate.

## SUMMARY

We have try to reproduce the measured bunch oscillation spectrum which arise from electron cloud induced head-tail instability. The simulation gave a sideband peak near $\nu_\beta + \nu_s$ and a shift of the peak for increasing cloud density. However the peak with $\nu_\beta + \nu_s$ was not dominant compare than other mode with $\nu_\beta - \nu_s$. The trial has succeeded partially, but has been far from perfect yet.

There are ambiguities to obtain the spectra in the simulation: e.g., choice of feedback gain, chromaticity and others. Since the phenomenon is strongly nonlinear, the spectrum depends on the amplitude $y(z)$. The number of data is limited when the instability growth is fast. It is difficult to get

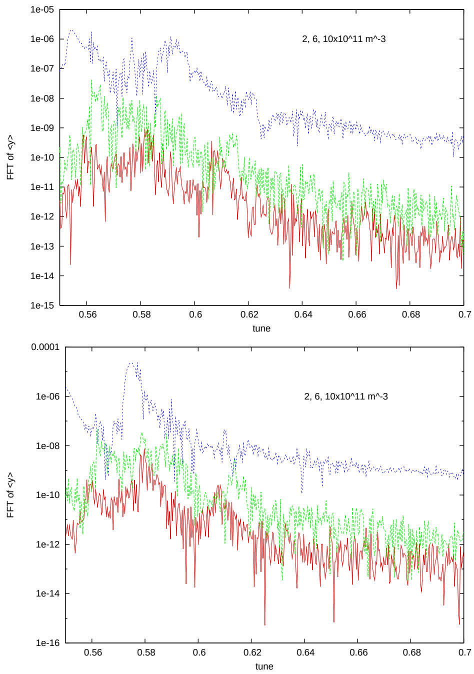


Figure 4: Frequency spectra for vertical averaged position ($\langle y \rangle$) obtained by a simulation. Top and bottom pictures depict spectra for damping time of 50 and 200 turns, respectively. Three lines, red, green and blue, corresponds electron cloud densities, 2, 4 and 6 $\times 10^{11}$ m$^{-3}$, respectively.

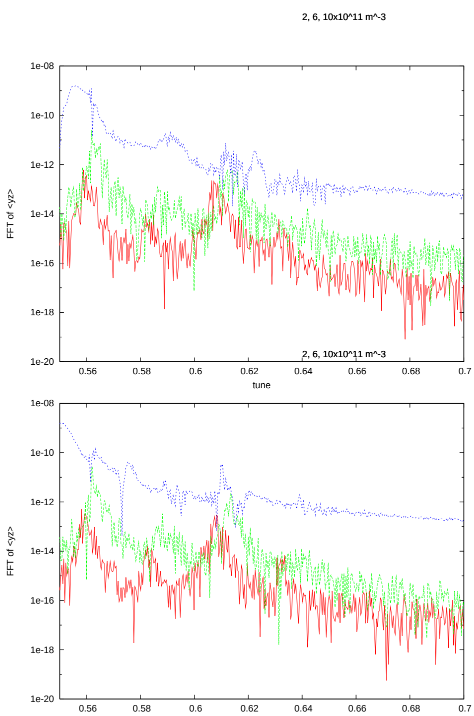


Figure 5: Frequency spectra for the correlation function ($\langle yz \rangle$) obtained by a simulation. Top and bottom pictures depict spectra for damping time of 50 and 200 turns, respectively. Three lines, red, green and blue, corresponds electron cloud densities, 2, 4 and 6 $\times 10^{11}$ m$^{-3}$, respectively.

clear result as seen in the ordinary wake at Figure 2.

While the sideband signal obtained by experiments is very clear. Therefore we hope it has to be reproduced by simulations more clearly. If we can not reproduce it, there may remain unknown phenomenon in this subject.

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