STUDY OF SLOW BEAM EXTRACTION THROUGH THE THIRD ORDER RESONANCE WITH TRANSVERSE PHASE SPACE MANIPULATION BY A MONO-FREQUENCY RFKO

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Abstract

An electron pulse stretcher ring (STB ring) [1] at Laboratory of Nuclear Science (LNS) in Tohoku University provides quasi-cw electron beam for the study of nuclear physics. The extracted beam from the ring has certain spread in time and space resulted from the emittance of injected beam from linac even if the injected beam is perfectly matched to the ring optics. However, the extracted beam emittance can be reduced by applying a phase space manipulation. Under the effect of perturbation using an RF shaker driven by a mono-frequency, the betatron amplitude of circulating beam can be controlled in order to reduce the extracted beam emittance. We have demonstrated the reduction of emittance experimentally on the STB ring.

PULSE STRETCHER RING

The STB ring works as a pulse stretcher that converts the pulse beam generated by RF linac into a quasi-continuous beam. The parameters related to the stretcher operation of the STB ring are shown in Table 1.

Table 1: Parameters for stretcher mode of the STB ring

Lattice type	Chasman-Green
Superperiodicity	4
Circumference	49.75 m
Beam energy	200 MeV (nominal)
Betatron tune	(3.31, ~1.20)
Natural chromaticity	(-5.78, -4.97)
Energy loss / turn	46 eV @ 200 MeV
Repetition period (rate)	3.33 ms (300 Hz)
Relative energy loss / period	0.46 % @ 200 MeV
Horizontal tune shift / period	0.0267 @ 200 MeV
Number of harmonic sextupole	1

Because there is no accelerating field in the ring, the beam approaches a transverse resonance condition due to synchrotron radiation loss and finite chromaticity. The beam is extracted by the third order resonance excited by a sextupole magnet. Since there is no sextupole for correction of the chromaticity in the ring, a rate of the tune shift is dominated by the natural chromaticity and the radiation loss. Accordingly, total energy width of the injected beam should be equal to a total energy loss integrated over one repetition period of the beam injection to achieve the continuous beam. Fig. 1 shows the tune spread of the injected beam on tune diagram.

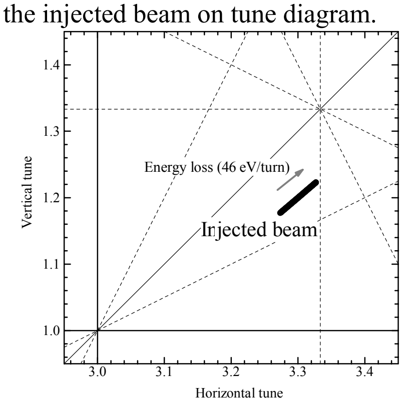

Figure 1: The tune spread of the injected beam on diagram. The energy width of the injected beam causes tune spread depending on the horizontal natural chromaticity.

BEAM EXTRACTION FROM STB RING

Choice of Resonating Condition

Defining the betatron amplitude J using Courant-Snyder invariant as

$$J = \frac{1}{2\beta}\left[x^2 + \left(\beta x' + ax\right)^2\right], \qquad (1)$$

then reduced Hamiltonian including the sextupole potential is approximately written as

$$H \approx \delta J + GJ^{2/3}\cos\left(3\varphi - l\theta + \xi\right), \qquad (2)$$

where δ is the tune deviation of fractional part from the resonance $\delta = \nu - l / 3$, φ is the betatron phase, and the orbiting angle θ is s / R. Fourier amplitude of the sextupole moment G is averaged over the ring circumference, and the phase ξ is the phase defined in Ref. [2].

From the Hamiltonian Eq. 2, betatron amplitude at an unstable fixed point (ufp) of $\varphi = \pi$ is found to be

$$J_{ufp}^{1/2} = \left|\frac{2\delta}{3G}\right|. \qquad (3)$$

It is obvious that the resonating condition is defined by the tune deviation δ and the Fourier strength G. In the STB ring, the wire septum is located on −32 mm from the central orbit just downstream of the sextupole (ξ = 0),

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therefore the betatron amplitude in the real space should be less than the distance to the wire septum. In addition, a significant parameter for optimization of the resonating condition is the step of the increase of the betatron amplitude at every 3 turn [3]. The 3-turn separation should be of course larger than the width of the septum wire.

In STB ring, the betatron amplitude of circulating beam is almost preserved until extraction, because there is no accelerating field. Therefore, the time until the injected beam is extracted and the betatron tune of extracted beam depend on the betatron amplitude at injection. This suggests that the extracted beam from the ring has some spread in time and space corresponding to the finite emittance of injected beam from linac even if the Twiss parameter of the injected beam is perfectly matched to the ring optics.

Beam Extraction with Mono-Frequency RFKO

In order to suppress dependence on the beam injection, the phase space manipulation using the transverse RF kick, RF knock out (RFKO), is employed. An analytical solution that describes the response of the betatron amplitude with mono-frequency RF kick is derived in this research.

Hill's equation with mono-frequency RF kick is written as

$$\frac{d^2\eta}{d\phi^2} + \nu(t)^2 \eta = \nu_0^2 \beta^{3/2} f(t), \tag{4}$$

where η and ϕ are the transverse coordinate and the betatron phase, which are applied by the Floquet transformation. The betatron tune ν and perturbation function f by RF kick are written as

$$\begin{aligned} \nu(t) &= \nu_0 - at, \\ f(t) &= \theta_a \sin(\omega_m t + \varphi) \sum_{n=-\infty}^{\infty} \delta(s - nC). \end{aligned} \tag{5}$$

Because the betatron tune has linear dependence on time in a pulse stretcher ring, ν is defined as the function of time $t = s / \beta c$. The constant a is tune shift in each unit time, ν_0 is initial tune, θ_a, ω_m and φ are the kick angle, the angular frequency and the initial phase of the RF kick respectively. The analytical solution derived from Eq. 4 is

$$\begin{aligned} x = &-i \frac{\nu_0^2 \beta^2 \theta_a}{R} \frac{1}{\sqrt{\nu(\phi)}} e^{i\left[\nu_0 \phi - \frac{a'}{2}\phi^2\right]} \\ &\times \sum_{m=0}^{k} \frac{1}{\sqrt{\nu(2\pi m)}} e^{-i\left[\nu_0 2\pi m - \frac{a'}{2}(2\pi m)^2\right]} \sin(\nu_m 2\pi m + \varphi) \end{aligned} \tag{6}$$

where x is the transverse coordinate in real space, R is the averaged radius, k is revolutions of particle from injection and $\nu_m = \omega_m / \omega_0$ is the modulation tune. ϕ is the phase function which increases by 2π in one revolution, therefore the betatron oscillation ν is converted to the function of the phase ϕ, and a' is the tune shift for each turn.

However, this solution does not take into account the sextupole potential. In the estimation of the characteristics of the extracted beam, the relation between the betatron amplitude from this solution and separatrix by the third order resonance should be considered. Fig. 2 shows the principle in order to attain the lower emittance beam with the phase space manipulation, where the response of the betatron amplitude by RFKO derived from Eq. 6. The separatrix by the third order resonance are also shown.

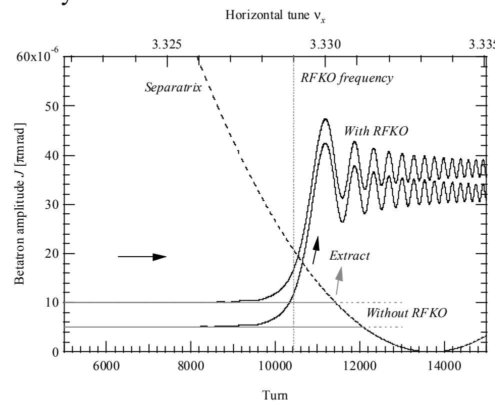

Figure 2: The schematic draw of extraction with phase space manipulation. The difference of betatron amplitudes when the electrons reached separatrix with RFKO decreases in the particular condition by comparison to the case without RFKO.

In the case of no RF kick, the electrons, which have different initial betatron amplitudes, are extracted from the ring having different tune deviations when the electrons reach separatrix. Since the δ decides the size of separatrix for the extracted particle, the difference in δ causes an emittance growth. According to the results of the analytical calculation, however, it is predicted that the difference in δ can be reduced by this phase space manipulation using RF kick in a particular condition. As a result, the lower emittance for the extracted beam can be realized.

Emittance of the Extracted Beam with RFKO

The betatron amplitude J when an electron reaches separatrix is affected by the initial phase and the kick angle (see Fig. 3). The emittance of extracted beam is determined by the spread of J over all initial phases because there is no synchronous system between RFKO and beam injection. In this case, the beam emittance is expected to be the minimum value when the kick of about 3 μrad is provided.

The achievement of the emittance improvement was also confirmed by the particle tracking simulation that employs the sextupole field and the same condition of RF kick as analytical calculation. Fig. 4 shows the phase space distribution of the extracted beam and the circulating beam before resonating at the location of the wire septum by the tracking simulation. The emittance with RF kick of 3 μrad (b) which is denoted by the area occupied by the extracted particles is found to be smaller than without RFKO (a).

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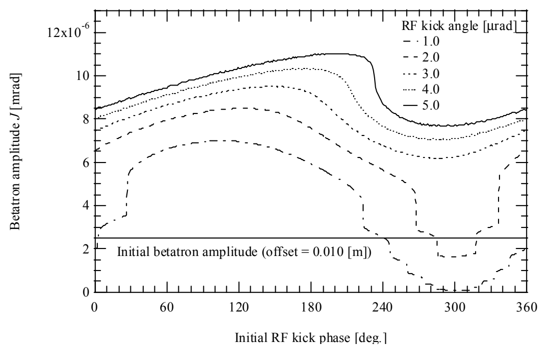

Figure 3: The betatron amplitude when an electron reaches separatrix. The emittance of extracted beam is affected by the kick angle and the initial phase of RF kick.

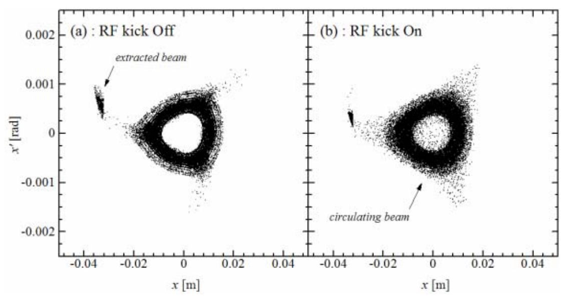

Figure 4: The phase space distribution of the extracted beam and the circulating beam before resonating at the location of the wire septum by the tracking simulation: without RFKO (a), with RFKO (b).

EXPERIMENTAL RESULTS AND DISCUSSIONS

Measurement of Extracted Beam Size

The measurement of the extracted beam size was performed by using wire scanner (1.0 mm ϕ) at location in front of extraction septum magnet and plastic scintillator to detect gamma ray radiated by electrons that collide with the wire. The RF kicker that has 4 strip-line type electrodes was installed at straight section of STB ring. Fig. 5 shows the measured distribution of the density of the extracted electron for various RF kick angles at one initial amplitude.

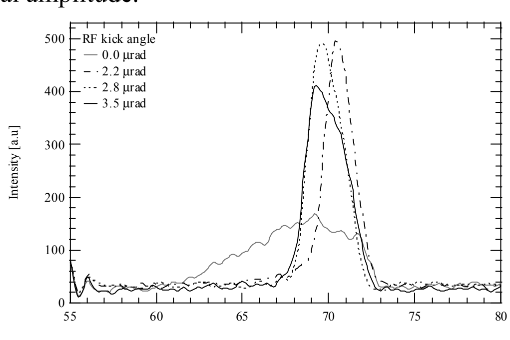

Figure 5: The measured distribution of the density of the extracted electron for various RF kick angles.

The experimental results were almost consistent with the expectations by the tracking simulation for various initial amplitudes not only in a case shown in Fig. 5 but also in other conditions.

Emittance of the Extracted Beam

The emittance is not given by the information of the beam size only, therefore, the emittance of the extracted beam was expected from the results by the tracking simulation. Fig. 6 shows the emittance of the extracted beam plotted as functions of an RF kick angle for various initial betatron amplitudes.

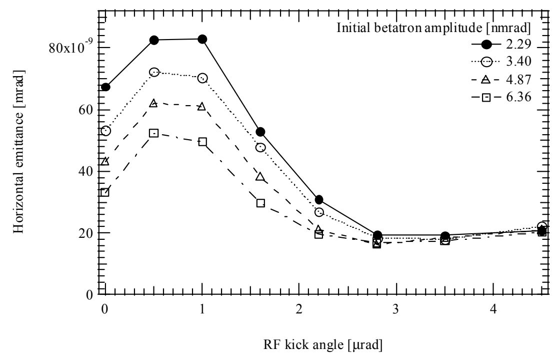

Figure 6: The extracted beam emittance estimated by tracking simulation for various RF kick angles and initial betatron amplitudes.

Since the small initial betatron amplitude of injected beam corresponds to the large distance to septum wire, it leads to the large turn separation. Consequently, this causes the emittance growth of extracted beam. However, the emittance can be minimized by the phase space manipulation with kick angle of about 3 μrad, and also this make it possible to be almost independent on the initial amplitude.

CONCLUSION

The suppressing the difference of betatron amplitude at the extraction from the pulse stretcher is experimentally demonstrated by the phase space manipulation using RF kick. The analytical formula that describes the motion of particle with RFKO brings the expectation of the certain characteristics for the extracted beam with the phase space manipulation. The reduction of extracted beam emittance was proved experimentally. The decrease of the amplitude deviation, which depends on an optical mismatch between the ring and the transport, was also proved. Further investigation of the beam dynamics in the stretcher ring may surely improve the quality of extracted beam.

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