TIME DEPENDENCE OF THE PRESSURE PROFILE IN A TUBE WITH AXIALLY-DEPENDENT DEGASSING

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Abstract

We present in this paper analytical results for the pressure profile in an evacuated tube, when subject to two situations common to accelerator vacuum technology. The first case deals with a system presenting two different degassing rates along the tube length. The second part presents results for the time evolution of the pressure profile in a tube where a sudden and localized degassing occurs.

1 INTRODUCTION

The usual geometries of accelerator tubes are straight sections or circle arcs. In terms of vacuum modeling, one usually assumes a straight section with pumping at the extremes, and constant degassing rate. This kind of model is adequate for steady-state situations and where the materials used do not present very different degassing rates. The need to use high thermal conductivity parts, insulating ceramics or bellows, tends to turn this initial hypothesis into a very crude approximation.

In this paper we deal with such problems, presenting analytical results for two basic situations found in accelerator vacuum technology. The first one is a tube with axially-dependent degassing, which may represent situations where the tubes are made of different materials or are subject to different radiation rates. The second one deals with a localized impulsive degassing, which may represent a situation where there is a sudden and intense radiation-induced degassing. We present analytical results for the pressure profile in those situations as well as its time evolution for the second case.

These results are very powerful, since they present means to deal analytically with generic degassing problems, for any extensive degassing rate can be represented by a linear combination of localized impulsive degassing rates.

2 ANALYTICAL SOLUTIONS

Initially we treat the problem of a tube with two different degassing rates, and then present the localized impulsive degassing case.

2.1 Axially-Dependent Degassing

We consider a straight tube section, divided in three parts: the first and third parts are equal in length and have the same degassing rate, while the middle one presents a higher degassing rate and has an arbitrary length. Vacuum pumps with equal effective pumping speeds, S, are placed at each end of the tube. A schematic drawing of the geometry is presented in Fig. 1.

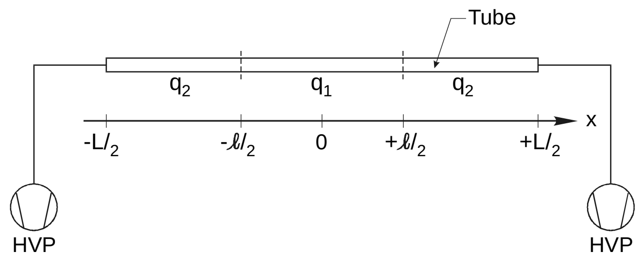

Figure 1: Schematic drawing of the geometry studied. HVP represents a generic high-vacuum pump.

The differential equation [1] is:

$$c\frac{d^2 p(x)}{dx^2} = -q \tag{1}$$

where $p(x)$ is the pressure along the tube axis; c is the specific conductance per unit length; and q is the specific degassing rate per unit length. The solution for $q_2 = q_1$ is the usual parabolic pressure profile. When $q_1 > q_2$, we find two differential equations, analogous to eqn. (1), describing the pressure profile in the regions corresponding to q_1 and q_2. Due to the symmetry of the problem we will treat only the $[0,L/2]$ interval. The following boundary conditions are imposed:

- there is a maximum of the pressure at $x = 0$;
- continuity of the throughput at $x = l/2$,

$$-c\frac{dp_1(x)}{dx}\bigg|_{x=l/2} = -c\frac{dp_2(x)}{dx}\bigg|_{x=l/2}$$

- the pressure at the end of the tube, $x = L/2$, is the total throughput, Q_T, divided by the total pumping speed, $2S$.

The following solution is found:

$$p_1(x) = -\frac{q_1}{2c}x^2 + \frac{q_2\left(L^2 - l^2\right)}{8c} + \frac{(q_2 - q_1)\left(l^2 - lL\right)}{4c} + \frac{q_1 l^2}{8c} + \frac{Q_T}{2S} \tag{2}$$

for x within the $[0,l/2]$ interval; and

$$p_2(x) = -\frac{q_2}{2c}x^2 + \frac{(q_2 - q_1)}{2c}l|x| + \frac{q_2 L^2}{8c} - \frac{(q_2 - q_1)lL}{4c} + \frac{Q_T}{2S} \tag{3}$$

for x within the $]l/2,L/2]$.

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2.2 Localized Impulsive Degassing

We consider a straight tube section with a localized impulsive degassing occurring at the middle (x = 0), superimposed to the uniform background degassing of the tube walls. A schematic drawing representing this setup is shown in Fig. 2.

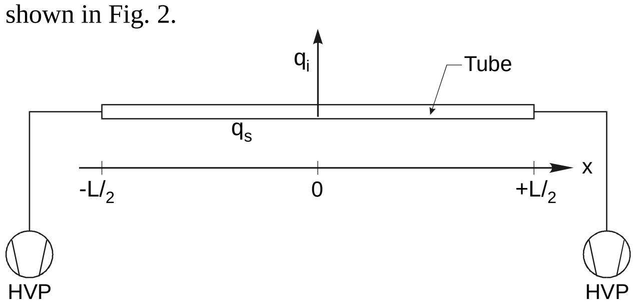

Figure 2: Schematic drawing of the setup, where HVP stands for high-vacuum pump.

The differential equation is:

$$c\frac{\partial^2 p(x,t)}{\partial x^2} = -q(x,t) + v\frac{\partial p(x,t)}{\partial t} \qquad (4)$$

where $q(x,t)$ includes both the steady state degassing from the walls, q_s, and the impulsive source, q_i, and v is the volume per unit length of the tube. A product of two delta functions will represent the impulsive source:

$$q_i(x,t) = q'\delta(x)\delta(t) \qquad (5)$$

where q' is a constant proportional to the amount of gas liberated at x = 0 in t = 0. The solution is:

$$p(x,t) = -\frac{q_s}{2c}x^2 + \frac{Q_T}{2}\left(\frac{1}{S} + \frac{L}{4c}\right) + \\ + \frac{q'}{(4\pi cvt)^{\frac{1}{2}}}\exp\left(-\frac{v}{4ct}x^2\right) \qquad (6)$$

One can see that this solution corresponds to the combination of the usual parabolic steady state result plus a transient solution represented by a Gauss function with a time-varying standard deviation. This result was obtained considering the following boundary conditions:

- there is a maximum of the pressure at x = 0;
- for the steady-state solution, the pressure at the end of the tube, $x = L/2$, is the total throughput, Q_T, divided by the total pumping speed, $2S$.
- for the transient solution, all the gas reaching the pumps is pumped.

3 RESULTS AND DISCUSSION

In order to exemplify the results found, we can consider the following numerical cases.

3.1 Axially-Dependent Degassing

Results for a tube with axially-dependent degassing, divided in three parts, are presented below. The parameters used are the following:

- L = 400 cm
- l = 100 cm
- q_1 = 9.4x10^{-8} mbar.l.s^{-1}.cm^{-1}
- q_2 = 4.7x10^{-8} mbar.l.s^{-1}.cm^{-1}
- c = 324 l.s^{-1}.cm
- Q_T = 2.4x10^{-5} mbar.l.s^{-1}
- S = 100 l.s^{-1}

Taking those values into eqns. (2) and (3), we obtain, for the pressure along the tube, the graph shown in Fig. 3. The plot shows only the pressure in one half of the tube, since the other is symmetrical. One should note that the curve is composed by two parabolas that join smoothly at x = 50 cm. The specific degassing rate per unit length presents a discontinuity at this point. Nevertheless both the pressure functions and their derivatives with respect to x are continuous.

This model can be useful to represent steady-state situations where one has different materials along the vacuum system or a single material presenting distinct degassing rates due to different physical conditions (like temperature, incident radiation).

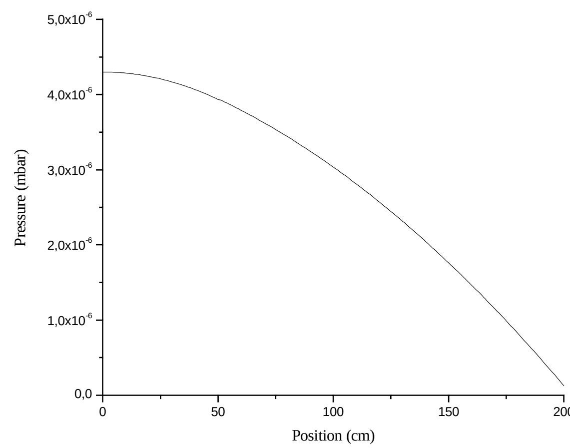

Figure 3: Pressure along the tube for $x \geq 0$ cm.

3.2 Localized Impulsive Degassing

In this case we deal with a tube presenting a constant degassing rate along the whole length, plus a localized impulsive degassing occurring at x = 0 cm and t = 0 s. The parameters are the following:

- L = 400 cm
- q_s = 4.7x10^{-8} mbar.l.s^{-1}.cm^{-1}
- q' = 1.0x10^{-2} mbar.l
- c = 324 l.s^{-1}.cm
- v = 7.1x10^{-3} l.cm^{-1}
- Q_T = 1.9x10^{-5} mbar.l.s^{-1}
- S = 100 l.s^{-1}

Taking those values into eqn. (6) we obtain the results shown in Fig. 4. The plot shows only the pressure in one half of the tube, since the other is symmetrical.

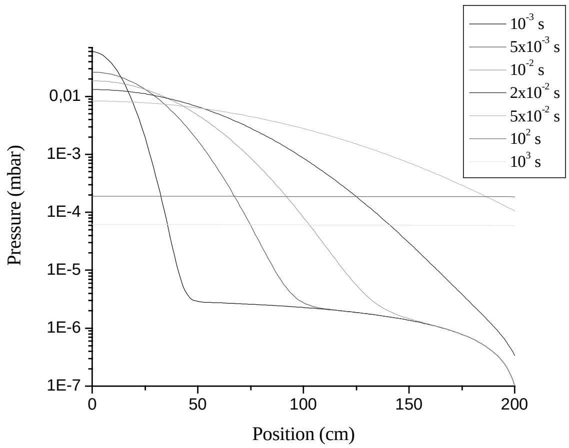

Figure 4: Pressure profile along the tube at different times, for $x \geq 0$ cm.

The reader should be aware that the model assumes that all the gas arriving at the extremities is pumped, which is not a realistic hypothesis depending on the amount of gas (q') and the specific conductance of the tube.

This model can be useful to represent transient-state situations where a very localized and impulsive gas burst happens within the system, as in the case of a particle beam striking the wall [2]. It should be noted that this model presents a wide range of applications, since it can be generalized to conform to whichever shape of degassing burst may happen. Any given function, describing the time or spatial profile of the burst, can be represented by a superposition of delta functions.

On the other hand there is a clear limitation to the model, as presented here. One can see that the pressure along the tube decreases very slowly with time, even though we are assuming that all the gas reaching the tube ends is pumped. This happens because we are not including the natural adsorption that will take place at the tube walls. This effect can be included in the calculation, if a numerical solution is acceptable.

4 REFERENCES

[1] A. Berman, “Vacuum engineering calculations, formulas and solved exercises”, Academic Press, NY, 1997.

[2] J. Gómez-Goñi and A.G. Mathewson, J. Vac. Sci. Technol. A**15**(1997)3093.

5 ACKNOWLEDGMENTS

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