NONLINEAR CHARGE AND CURRENT NEUTRALIZATION FOR AN ION BEAM PULSE PROPAGATING THROUGH A BACKGROUND PLASMA

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Abstract

This paper demonstrates the possibility of quasi-steady-state transport of a finite-length ion beam through a chamber filled with plasma of arbitrary density. We conclude that, in principle, partially-current-neutralized equilibria exist in the reference frame moving with the ion charge bunch for arbitrary ratio of beam density to plasma density. The electric and magnetic fields generated by the ion beam are studied analytically for the nonlinear case where the plasma density is comparable in size with the beam density. Particle-in-cell simulations of current and charge neutralization agree well with analytical results. An important conclusion is that for long, nonrelativistic ion beams, with length much longer than the beam radius and the plasma neutralization length, which is the ratio of the beam velocity to the electron plasma frequency, the charge neutralization is, for all practical purposes, complete even for very tenuous background plasmas. Current neutralization is usually much weaker than charge neutralization. As a result, the magnetic pinching force dominates the electric defocusing force, and the beam ions are always pinched during quasi-steady-state beam propagation through the background plasma.

1 INTRODUCTION

Neutralization of the beam charge and current is an important issue for many applications. Heavy ion fusion and high energy physics applications involve the transport of positive charges in plasma: partially-stripped heavy elements for heavy ion fusion [1]; positrons for electron-positrons colliders [2]; and high-density laser-produced proton beams for the fast ignition of inertial confinement fusion targets [3]. Beam focusing schemes rely on complete charge neutralization and partial current neutralization for magnetic focusing in plasma lenses [2], and for ballistic ion focusing in heavy ion fusion [1]. In these applications, the plasma is pre-formed by an external plasma source and is independent of the beam characteristics.

The goals of the present calculation are: (a) to derive a system of reduced equations for the electric and magnetic fields generated by an ion beam propagating through background plasma, and (b) to develop a semi-analytical method for robust and easy assessment of the effects of these fields on the beam transport. The case where the beam propagates through a cold unmagnetized plasma, with plasma density large compared with the beam density, can be studied by use of linear perturbation theory [4]. Here, we focus on the nonlinear case where the plasma density has an arbitrary value compared with the beam density, and correspondingly the degrees of current and charge neutralization are arbitrary.

The transport of stripped, pinched ion beams has also been discussed in Ref. 5, where the assumptions of current and charge neutrality were made to determine self-consistent solutions for the electric and magnetic fields. Rosenbluth, *et al.* have considered the equilibrium of an isolated, charge-neutralized, self-pinched ion beam pulse in the absence of background plasma [6]. In contrast, we consider here the case where "fresh" plasma is always available in front of the beam, and there are no electrons co-moving with the beam.

2 BASIC EQUATIONS FOR DESCRIPTION OF ION BEAM PULSE PROPAGATION IN A PLASMA

The electron fluid equations together with Maxwell's equations comprise a complete system of equations describing the electron response to a propagating ion beam pulse. The plasma electrons are assumed to be cold, and electron thermal effects are neglected. The electron fluid equations consist of the continuity equation,

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \boldsymbol{V}_e) = 0 , \qquad (1)$$

and the force balance equation,

$$\frac{\partial \boldsymbol{p}_e}{\partial t} + (\boldsymbol{V}_e \cdot \nabla) \boldsymbol{p}_e = -e(\boldsymbol{E} + \frac{1}{c} \boldsymbol{V}_e \times \boldsymbol{B}) , \qquad (2)$$

where *-e* is the electron charge, $\boldsymbol{V_e}$ is the electron flow velocity, $\boldsymbol{p}_e = \gamma_e m_e \boldsymbol{V}_e$ is the electron momentum, m_e is the electron rest mass, and γ_e is the relativistic mass factor.

Maxwell's equations for the self-generated electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$, are given by

$$\nabla \times \boldsymbol{B} = \frac{4\pi e}{c} \left(Z_b n_b \boldsymbol{V}_b - e n_e \boldsymbol{V}_e \right) + \frac{1}{c} \frac{\partial \boldsymbol{E}}{\partial t} , \qquad (3)$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t} , \qquad (4)$$

where $\boldsymbol{V_b}$ is the ion beam velocity, n_e and n_b are the number densities of the plasma electrons and beam ions, respectively, and Z_b is the ion beam charge state. The plasma ions are assumed to be stationary with $\boldsymbol{V_i}$=0.

Considerable simplification can be achieved by applying the conservation of generalized vorticity Ω [7]. If Ω is initially equal to zero ahead of the beam, and all streamlines inside of the beam originate from the region ahead of the beam, then Ω remains equal to zero everywhere, i.e.,

$$\Omega \equiv \nabla \times \boldsymbol{p}_e - \frac{e}{c}\boldsymbol{B} = 0 \tag{5}$$

Substututing Eq.(5) into Eq.(2) yields

$$\frac{\partial \boldsymbol{p}_e}{\partial t} + \nabla K_e = -e\boldsymbol{E} \ , \tag{6}$$

where $K_e = \gamma_e m_e c^2$ is the electron energy.

3 APPROXIMATE SYSTEM OF EQUATIONS FOR LONG DENSE BEAMS

In this section, an approximate set of equations is derived for a long ($r_b << l_b$), cylindrically symmetric ion charge bunch satisfying

$$V_b / \omega_p << l_b \ . \tag{7}$$

Here r_b and l_b are the beam radius and length, respectively, and $\omega_p = \left(4\pi n_e e^2 / m_e\right)^{1/2}$ is the electron plasma frequency. For long beams ($r_b << l_b$), radial derivatives are much larger than longitudinal derivatives. Therefore, for cylindrically symmetric beams, the azimuthal magnetic field is determined in terms of the longitudinal flow velocity from Eq.(5), which gives

$$B = -(e/c)\partial p_{ez} / \partial r \ . \tag{8}$$

The displacement current, the last term on the right-hand side of Eq.(3), can be neglected under the condition of Eq.(7). Thus, Eq.(3) simplifies to become

$$-\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} p_{ez} = \frac{4\pi e}{c}\left(Z_b n_b V_{bz} - n_e V_{ez}\right). \tag{9}$$

Equation (9) describes current neutralization of the beam. Under the condition in Eq.(7), the degree of charge neutralization is very close to unity [7], and the quasineutrality condition holds with

$$n_e = Z_b n_b + n_p \ , \tag{10}$$

where n_p is the background plasma ion density. The radial electron flow velocity can be obtained from the beam current ($\boldsymbol{j_b}$) and the longitudinal electron flow velocity by making use of

$$\nabla \cdot (\boldsymbol{j}_b + \boldsymbol{j}_e) = 0 \ . \tag{11}$$

The electric field is then obtained from Eq.(6). Small departure from charge neutrality can be estimated by making use of Poisson's equation

$$\nabla \cdot \boldsymbol{E} = 4\pi e(Z_b n_b + n_p - n_e) \ . \tag{12}$$

It can be readily shown [7] that the maximum deviation from quasineutrality occurs when $r_b \sim c/\omega_p$ and $(Z_b n_b + n_p - n_e)/(Z_b n_b) < 0.25\beta_b^2$, where $\beta_b = V_b / c$. Therefore, for nonrelativistic, long ion pulses, there is almost complete charge neutralization.

The focusing force acting on the beam ions is [5, 7]

$$F_r = -e\frac{\partial}{\partial r}\left(K_e - V_{bz} p_{ez}\right). \tag{13}$$

Thus, Eqs. (6)-(12) describe the self-consistent electron motion driven by long dense beams. Examples of calculations and comparisons with the results of electromagnetic particle-in-cell simulations can be found in Ref. 7.

4 APPROXIMATE SYSTEM OF EQUATIONS FOR LONG BEAMS

We have seen in Sec.3 that under the condition in Eq.(7) charge neutralization is complete. It is also important to consider the transition to the regime of low plasma density and determine the condition under which the beam charge is not neutralized. We use the assumption of a long beam ($r_b << l_b$), but relax the assumption in Eq.(7). The typical longitudinal scale of electron density perturbations is V_b / ω_p. If $V_b / \omega_p >> r_b$, the main dynamics is in the radial direction, and longitudinal derivatives are neglected in comparison with radial derivatives. This gives

$$\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r n_e V_{er}\right) = 0 \ , \tag{14}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_r) = 4\pi e(Z_b n_b + n_p - n_e) \ , \tag{15}$$

$$\frac{\partial p_{er}}{\partial t} + \frac{\partial K_e}{\partial r} = -eE_r \ . \tag{16}$$

The longitudinal electron flow velocity can be still determined from Eq.(9). In Eq.(9) we neglected the displacement current. In contrast with the results of Sec.3, the displacement current can be now comparable with the electron current if $V_b / \omega_p \sim l_b$. However, in this case both the displacement current and the electron current are small compared with the other terms in Eq.(9). Indeed, the skin depth is larger than beam radius $c/\omega_p = V_b /(\omega_p \beta_b) >> r_b$ provided $V_b / \omega_p >> r_b$.

The results of numerical solutions of the system of equations (9) and (14)-(16) are presented in Fig.1. It can be seen that the beam charge is well neutralized under the condition in Eq.(7) [compare Fig.1(a) and Fig.1(d)] and is not neutralized in the opposite limit [Figs.1(b) and 1(c)]. The results in Fig.1 agree well with the results of two-dimensional electromagnetic particle-in-cell simulations described in Ref. 7.

Note that in the linear case the equation for plasma oscillations [7] is

$$\frac{\partial^2 n_e}{\partial t^2} + \omega_p^2(n_e - Z_b n_b - n_b) = 0 \ . \tag{17}$$

Equation (17) is readily recovered from the linearized version of Eqs.(1), (2) and Poisson's equation (12). It can be also derived from Eqs. (14)-(16) when longitudinal derivatives were neglected.

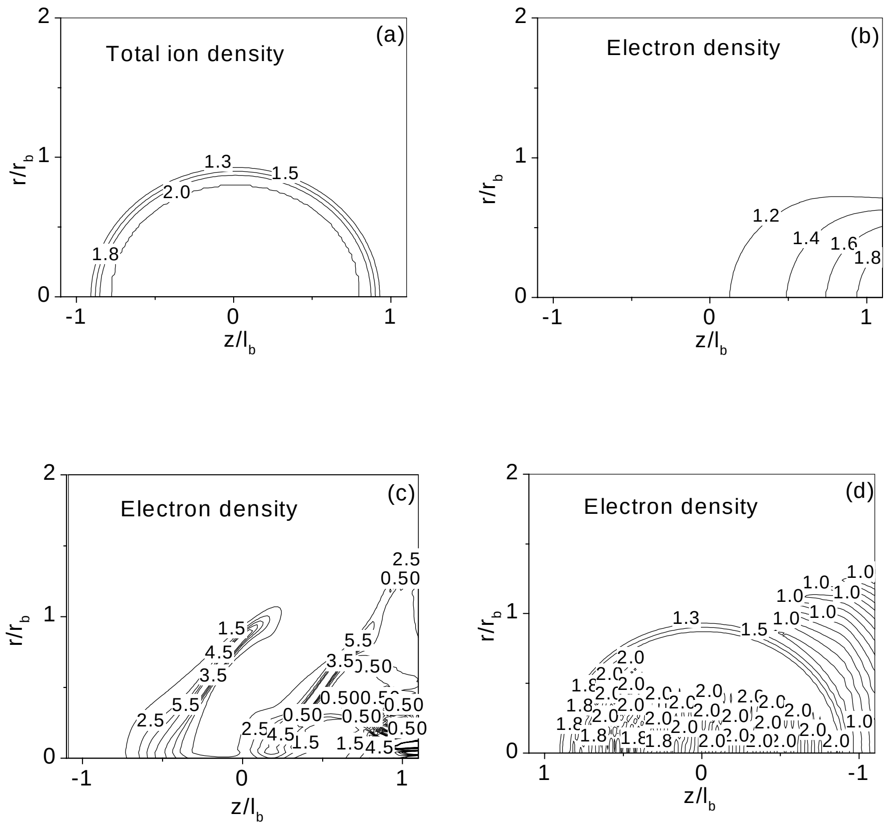

Fig.1 Constant density contours in $(r/r_b, z/l_b)$ space in the beam frame obtained in numerical simulations Eq. (9) and Eqs.(14)-(16). The analysis assumes $r_b = l_b/10$, $\beta_b = 0.5$, $Z_b = 1$ and $n_b = n_p$ (beam density, for an ellipsoidal beam pulse, is equal to the background plasma ion density). The beam pulse moves from right to left, with z=0 corresponding to the center of the beam pulse (in the beam frame). The constant-density contours correspond to: (a) total ion density, (sum of background ion density and beam density), and the electron density contours for (b) $l_b = 0.3c/\omega_p$, (c) $l_b = 3c/\omega_p$, and (d) $l_b = 30c/\omega_p$. The contour "numbers" in Fig.1 correspond to values of density normalized to the (constant) background plasma ion density.

5 DISCUSSION

The propagation of a finite-length ion beam pulse through a background plasma has been studied. The analytical solutions for the electric and magnetic fields generated by the ion beam pulse have been determined in the nonlinear case for arbitrary values of beam and plasma densities, under the assumption of a long beam, where the beam length is much longer than the beam radius. Under these conditions, a reduction in the dimensionality of the problem is possible. Assuming an axisymmetric beam pulse, the longitudinal electron flow velocity is determined for one-dimensional variations in the radial direction for each axial slice of the beam [Eq.(9)]. The electric and magnetic fields are then readily calculated from the longitudinal electron flow velocity from Eqs. (6) and (8), respectively.

The approach used here can be generalized to the case of nonuniform, nonstationary plasma density and beam density profiles, and forms the basis for a hybrid semi-analytical approach to be used for calculations of beam propagation in the target chamber. This work is now underway. The analytical formulas derived in this paper can provide an important benchmark for numerical codes and provide scaling laws for different beam and plasma parameters. The charge neutralization depends crucially on the beam length, and is determined by the ratio of the beam length to the plasma neutralization length, $l_b/(V_b/\omega_p)$. If $l_b/(V_b/\omega_p) >> 1$, the degree of charge neutralization is very close to unity. The degree of current neutralization is determined by the ratio of the beam radius to the skin depth, $r_b/(c/\omega_p)$.

Acknowledgment: This research was supported by the U.S. Department of Energy Office of Fusion Energy Sciences and the Division of High Energy Physics. It is a pleasure to acknowledge the benefit of stimulating discussions with Ed Lee and Marshall Rosenbluth.

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