

Experimental Measurements of Third-Order Resonance Islands at the Photon Factory Storage Ring

Tsukasa Miyajima*, Yukinori Kobayashi and Kazuhito Ohmi,

KEK, High Energy Accelerator Research Organization, Oho 1-1, Tsukuba, Ibaraki 305-0801, Japan

Abstract

The coherent betatron motions after a vertical kick were examined with various field strengths of the octupole magnets near the third-order resonance line ($3\nu_y = 13$), at the KEK-Photon Factory electron storage ring (PF-ring). At a particular condition, the particles were trapped in the resonance islands. The responses depended on the strength of octupole magnetic fields. As a result, we guess that the dependency of the motion in the resonance islands is generated by a slight difference of the amplitude dependent tune shift due to the octupole magnets.

1 INTRODUCTION

The nonlinear beam dynamics of the transverse betatron motion in circular accelerator has been studied using analytical, numerical and experimental methods [1, 2]. Despite the progress in explanation of nonlinear phenomena, there still is a gap between analytical, numerical prediction and reality. Especially, perturbation theory is limited near a resonance. To reduce this gap, experimental nonlinear-beam-dynamics studies have become increasingly important. Recently, in the PF-ring, transverse phase space monitor system was installed [4]. We have been studied experimentally the phase space topology near resonances using this system.

In the PF-ring, a vertical instability was observed in a multi-bunch mode. To suppress transverse instabilities, four octupole magnets are installed. The octupole magnets can produce the large amplitude dependent tune shift to beam. The effect near a resonance has attracted considerable interest.

This paper concerns the study of phase space topology under various octupole field strengths near the third-order resonance.

2 EXPERIMENTS

2.1 Experimental methods

The experiment was performed in the single-bunch operation mode. The principal parameters of the PF-ring under the low-emittance optics are given in Table 1. The initial stored current was set to be about 5 mA. The closed orbit distortions were corrected by steering dipole magnet. To measure the vertical coherent betatron motions near the third-order resonance line ($3\nu_y = 13$), the initial betatron

Table 1: Principal parameters of the Photon Factory storage ring under the present low-emittance optics.

Parameter	Symbol	Value
Beam energy	E	2.5 GeV
Circumference	C	187 m
Harmonic number	h	312
Horizontal betatron tune	ν_x	9.60
Vertical betatron tune	ν_y	4.28
rf frequency	f_{rf}	500.1 MHz
Revolution period	τ	624 nsec
Emittance	ϵ_x, ϵ_y	36, 0.36 nmrad
Energy spread	σ_ϵ	0.00073
Beam size	σ_x, σ_y	0.58, 0.04 mm

tunes were selected near $(\nu_x, \nu_y) = (9.60, 4.33)$. The phase space monitor system consists of fast kicker magnets and turn-by-turn monitors. The fast kicker magnets provide beam with a large coherent motion. Then the initial vertical amplitude was set to be $y_0 = 3.5$ mm.

To measure the coherent betatron motion in the phase space, two BPM's at a long straight section are used. The beam position y and the beam angle y' are obtained from the data of two positions.

The typical result of the experiment in which the particles was trapped in the third-order resonance island, is shown in Fig.1.

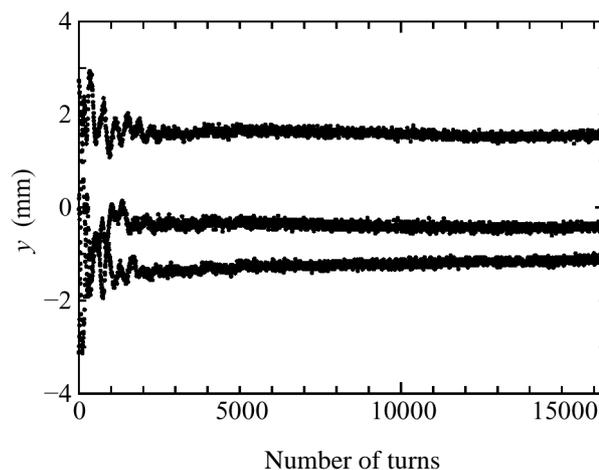


Figure 1: The vertical coherent betatron motion measured at a octupole excitation current of -1.0 A are shown as function of turn.

*Email address: tsukasa@post.kek.jp

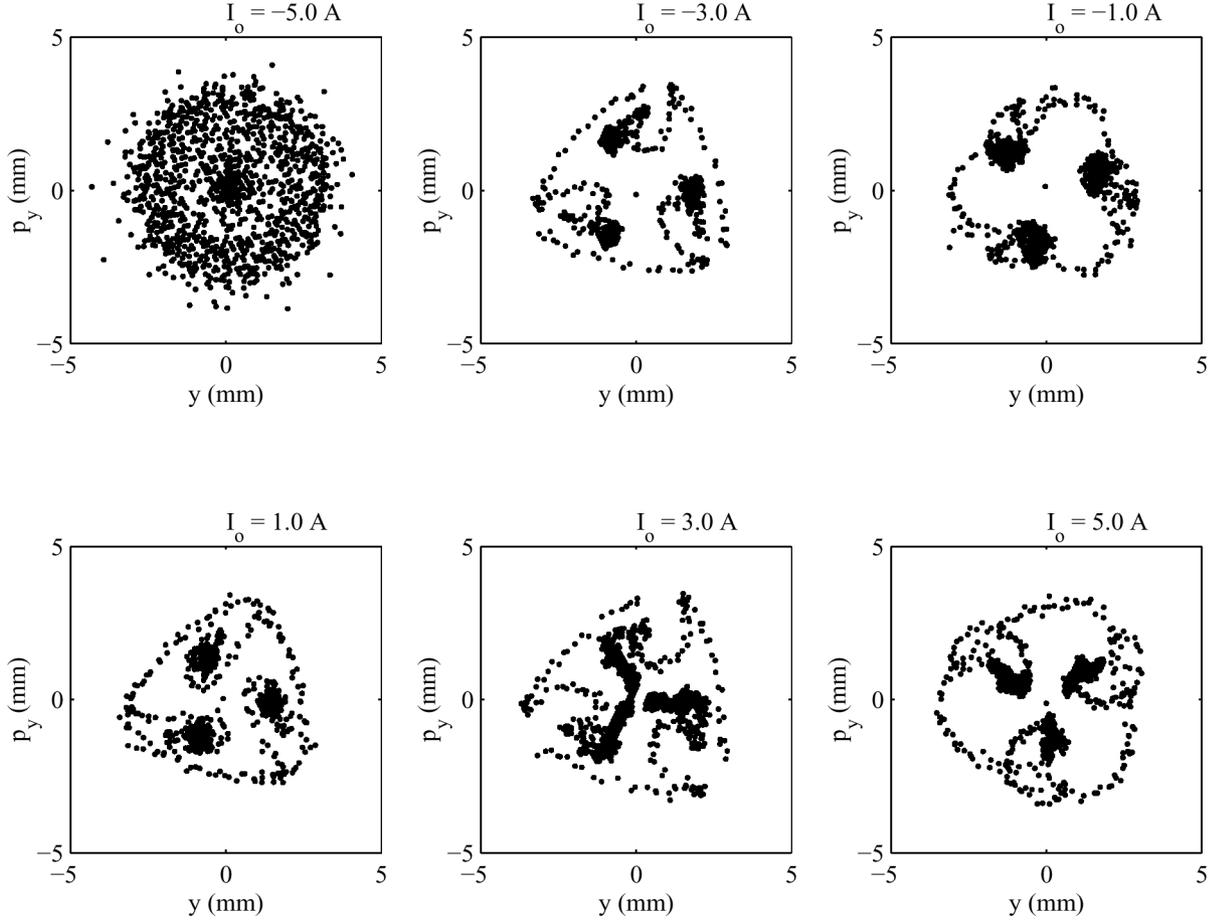


Figure 2: The Poincarè map for motion near the third-order resonance for six different octupole excitation currents: -5.0, -3.0, -1.0, 1.0, 3.0 and 5.0 A.

2.2 Tune shifts due to octupole magnetic field

The octupole magnetic field gradient is given by $d^3B_y/dx^3 = 1.01 \times 10^3 \cdot I_o$ T/m³/A, where I_o is an excitation current of the octupole magnet. The tune shift due to the octupole is given by

$$\Delta\nu_y = a_1 J_x + a_2 J_y. \quad (1)$$

Where J_x and J_y are the horizontal and vertical action, and the coefficients a_1, a_2 are as follows:

$$a_1 = -\frac{1}{16\pi} \int_0^C ds 2K''(s)\beta_x(s)\beta_y(s), \quad (2)$$

$$a_2 = \frac{1}{16\pi} \int_0^C ds K''(s)\beta_y^2(s), \quad (3)$$

$$K''(s) = \frac{1}{B\rho} \left(\frac{\partial^3 B_y}{\partial x^3} \right)_{x=y=0}. \quad (4)$$

In this experiment, the action is $J_x \ll J_y$, because the particles are kicked to only vertical direction. Thus the vertical tune shift is given by

$$\Delta\nu_y \sim a_2 J_y. \quad (5)$$

Using the ring parameters, the vertical tune shift is estimated

$$\Delta\nu_y \sim 1.95 \times 10^3 J_y I_o. \quad (6)$$

3 DATA ANALYSIS

For particle motion in circular accelerator, the deviation from the closed orbit, $y(s)$, satisfies Hill's equation:

$$\frac{d^2 y}{ds^2} + K(s)y = \frac{\Delta B_y}{B\rho}. \quad (7)$$

Where $K(s)$ is a function of the quadrupole strength, $B\rho$ is the magnet rigidity, and s is the longitudinal particle coordinate. When the betatron motion is linear, Hill's equation can be solved using the Floquet transformation [5] to obtain the solution

$$y(s) = \sqrt{2\beta_y J_y} \cos \phi, \quad (8)$$

where J_y and ϕ are action-angle variables, β_y is the vertical betatron amplitude function. Normalized momentum is given by

$$p_y = \alpha_y y + \beta_y y', \quad (9)$$

where $\alpha_y = -1/2 \cdot d\beta_y/dy$. When linear motion is plotted in y - p_y space, it is a circle defined by the equation,

$$p_y^2 + y^2 = 2\beta_y J_y. \quad (10)$$

The values of α_y and β_y , needed to transform the position variables to the normalized momentum, were determined by fitting experimental data. When the motion is linear, the phase space plot (y, y') can be fitted to an ellipse. The Courant-Snyder parameters were determined to be $\alpha_y = 0.186$, $\beta_y = 4.40$ m by fitting experimental data with small amplitude.

4 MEASUREMENT RESULT AND DISCUSSION

In order to search a condition when the particles are trapped in the resonance islands, we changed the initial betatron tunes and the vertical amplitude. Under many experimental conditions, the particles were not trapped in the island, and the coherent motions damped. At a particular condition, the particles were trapped in the island. This condition was very sensitive to the initial betatron tunes and the vertical amplitude.

Fig.2 displays a vertical phase space plot (Poincarè map) for six different octupole excitation currents. When the octupole excitation current was more than -3.0 A, the particles were trapped in the island. The motion in the phase space depended on the octupole excitation current.

The data shown in Fig.1 are plotted in J - ϕ space in Fig.3. $\langle J_y \rangle$ is introduced as the position of the third-order resonance island, which is an average of the action J_y from 9000 turns to 10000 turns. That is reason why the action J_y approached a constant value after 9000 turns. Fig.4 shows the average action $\langle J_y \rangle$ as a function of the octupole excitation current I_o . As shown Fig.4, $\langle J_y \rangle$ was not constant, and was minimum at $I_o = 3.0$ A.

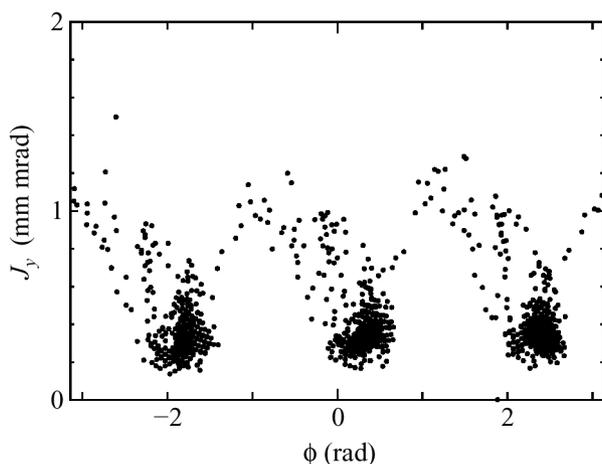


Figure 3: The Poincarè map in action-angle variables for the particles trapped in third-order resonance island at a octupole excitation current of -1.0 A.

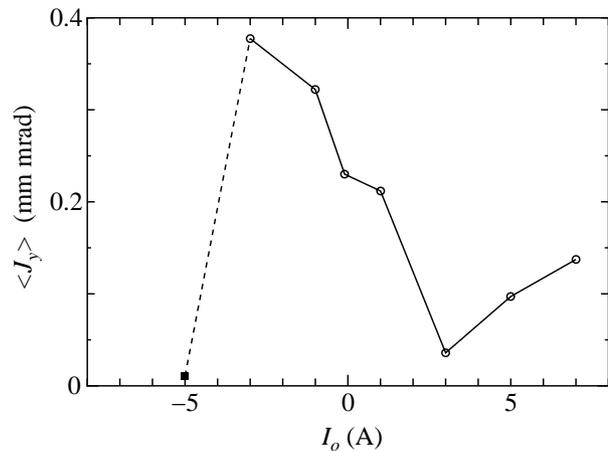


Figure 4: Average action $\langle J_y \rangle$ for the particles trapped in third-order resonance island for eight different octupole excitation currents: -5.0, -3.0, -1.0, -0.1, 1.0, 3.0, 5.0 and 7.0 A. For $I_o = -5.0$ A the particles were not trapped in the island. For $I_o = 7.0$ A slight beam loss was observed.

In this experiment, the coherent motion after a vertical kick was examined with various field strengths of the octupole magnets near the third-order resonance line. We guess that the dependency of the motion in the resonance island is generated by a slight difference of the amplitude dependent tune shift due to the octupole magnets. Now we are going to understand the phenomena through theoretical and numerical approach in detail.

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