A LOW-β STRIPPING INSERTION IN THE CERN PS TO SPS TRANSFER LINE FOR THE LHC ION PROGRAMME

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Abstract

In the framework of the LHC ion programme the charge state will be changed by passing through a thin metal stripping foil installed in the transfer line between the CERN PS and SPS machines. The tight transverse emittance budget of the LHC beam allows only a minor blow-up. Hence, to minimise the emittance growth due to multiple Coulomb scattering (a non-symplectic process leading also to Twiss parameter modification) the stripping foil has to be located in a low-β region. Furthermore, any change of optics has to be compliant with the simultaneous operation of the transfer line with proton and ion beams. This paper describes a possible implementation of a low-β insertion with the effect of scattering included. The impact of global constraints on the effective degrees of freedom of the matching problem has been studied in a systematic way and its outcome is reported.

1 INTRODUCTION

For the operation of the PS complex as an ion injector for the LHC, Pb^{54+} ion beams will be accelerated in the PS machine to 4.25 GeV/u and then ejected and fully stripped to Pb^{82+} in the TT2/TT10 transport channel between PS and SPS.

The transverse emittance budget of the ion beams required to obtain the specified luminosity for the lead ion programme in the LHC is listed in Table 1 together with the emittances of the present lead ion beam for the SPS fixed target programme. The quoted PS normalised RMS emittance $\varepsilon^*_{h,v} = 1.0$ μm refers to the value at the end of the TT2 transfer line after the stripping process. Hence, all along the chain, small emittances (about 1/4 of the present lead ion beam emittances) are required. Thus emittance preservation is of great importance.

The stripping process, however, generates transverse emittance blow-up due to the multiple Coulomb scattering in the stripping foil. In the presence of non-zero dispersion at the foil, the stripping may also lead to additional emittance increase due to coherent energy loss and straggling of the ions traversing the stripper. Motivated by the tight emittance budget for the LHC injector chain, a careful analysis of these different effects was carried out. The performance of a new low-β stripping insertion is compared to the present situation where there are relatively large β values at the stripper location.

MACHINE (top energy)	Ions for LHC $\varepsilon^*_{h,v}$ [μm]	Ions for SPS fixed target $\varepsilon^*_{h,v}$ [μm]
LHC	1.5	–
SPS	1.2	4.0–4.5
PS	1.0	3.8
LEIR	0.7	–
BOOSTER	–	3.0

Table 1: Emittance budget (normalised RMS) for the LHC ion programme [1]. The normalised emittance is related to the physical by $\varepsilon^* = (\beta\gamma)\varepsilon$.

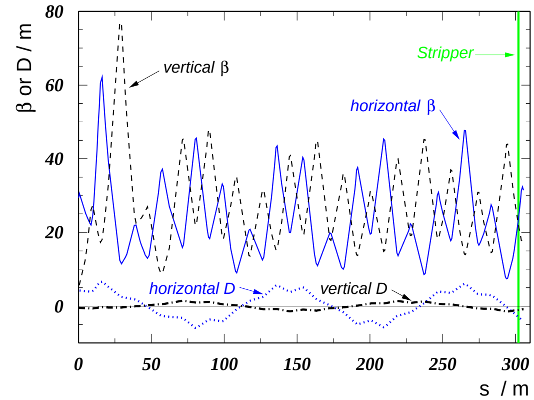

Figure 1: Betatron and dispersion functions of the present lead ion optics in the TT2 transfer line. The solid line at around 302 m denotes the current position of the stripper. The line is composed of a matching sector and a FODO string.

In the fixed-target lead ion runs performed up to now the charge state of the ions is changed from 53^+ to 82^+ in TT2 using a 0.8 mm thick aluminium stripping foil installed about 302 m from the entrance of TT2. Optics parameters at the present stripper location as calculated with MAD [2] are listed in Table 2. The horizontal and vertical betatron and dispersion functions of the current lead ion optics in the TT2 transfer line are shown in Fig. 1.

2 EMITTANCE PRESERVATION

2.1 Scattering and Energy Straggling

The main mechanism leading to emittance blow-up is multiple Coulomb scattering, which leads to an increase in angle and therefore in emittance. The scattering process can be described in the covariance matrix formalism by a convolution of the matrix representing the beam particle distribution and a matrix representing a scatterer of arbitrary length L [3]:

$$\Delta C_L = \begin{pmatrix} \frac{L^2}{3}\langle\theta^2\rangle & \frac{L}{2}\langle\theta^2\rangle \\ \frac{L}{2}\langle\theta^2\rangle & \langle\theta^2\rangle \end{pmatrix} \qquad (1)$$

where the RMS scattering angle is given by [4]

$$\sqrt{\langle\,\theta^2\rangle} = 13.6\,z\,\frac{1}{\beta p}\sqrt{\frac{x}{X_0}}\left\{1 + 0.038\,\ln\frac{x}{X_0}\right\}. \qquad (2)$$

In this equation, X_0 is the radiation length, z, p and β are the charge state, total momentum (in MeV/c) and velocity of the incident ion relative to the speed of light. The covariance matrix representing the beam particle distribution can also be expressed in terms of Twiss parameters and beam emittance:

$$C = \begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix} = \begin{pmatrix} \varepsilon_0\beta_0 & -\varepsilon_0\alpha_0 \\ -\varepsilon_0\alpha_0 & \varepsilon_0\gamma_0 \end{pmatrix} \quad (3)$$

with $C_{ij} = \langle ij\rangle - \langle i\rangle\langle j\rangle$. The covariance matrix of the convoluted distribution C' is given by the sum of the initial beam's covariance matrix C and the covariance matrix of the interaction ΔC_L: $C' = C + \Delta C_L$. The emittance can be defined from the determinant of the resulting matrix. The increase in emittance in this case is found to be

$$\Delta\varepsilon = \frac{1}{2}\langle\theta^2\rangle\left[\beta_0 + L\alpha_0 + \frac{L^2}{3}\gamma_0\right] \quad (4)$$

An additional contribution to the emittance increase is caused by coherent and incoherent energy loss in the stripping foil. The coherent (average) loss in kinetic energy of the lead ions, however, does not lead to blow-up if the optics downstream of the stripper is adjusted for the lower reference momentum. The incoherent energy loss ("energy straggling") is caused by the variance of the energy loss distribution. For a thick stripping foil the energy loss distribution can be treated in the Gaussian limit. The energy spread introduced by the straggling process is $\sigma_T \approx 42$ MeV yielding a relative momentum spread $\sigma_p/p \approx 4\times10^{-5}$ for a 0.8 mm aluminum foil. The covariance matrix of a process introducing momentum spread can be expressed as

$$\Delta C_{\text{mom.}} = \left(\frac{\sigma_p}{p_0}\right)^2 \begin{pmatrix} D^2 & DD' \\ DD' & D'^2 \end{pmatrix} \quad (5)$$

where D and D' are the dispersion and its derivative at the stripper. The emittance blow-up resulting from the determinant of the matrix sum $C' = C + \Delta C_{\text{mom.}}$ is

$$\Delta\varepsilon = \frac{1}{2}\left[\beta_0 D'^2 + \gamma_0 D^2 + 2\alpha_0 DD'\right]\left(\frac{\sigma_p}{p_0}\right)^2. \quad (6)$$

Measurements of the emittance blow-up due to stripping were performed in 1995 [5]. The experimental results for a 0.8 mm aluminium stripper are given in [5] as $\Delta\varepsilon_h^* \approx 0.77$ μm and $\Delta\varepsilon_v^* \approx 0.58$ μm.

2.2 Impact on Twiss Parameters

From Eq.(3) it is clear that a change in beam emittance, or in the determinant of C, will also affect the Twiss parameters. The covariance matrix of a process involving a scatterer yields immediately the new Twiss parameters, altered by multiple scattering:

$$\alpha = \frac{\varepsilon_0\alpha_0 - \frac{L}{2}\langle\theta^2\rangle}{\varepsilon_0 + \Delta\varepsilon},\ \beta = \frac{\varepsilon_0\beta_0 + \frac{L^2}{3}\langle\theta^2\rangle}{\varepsilon_0 + \Delta\varepsilon},\ \gamma = \frac{\varepsilon_0\gamma_0 + \langle\theta^2\rangle}{\varepsilon_0 + \Delta\varepsilon}.$$

For the case of momentum change we get

$$\alpha = \frac{\varepsilon_0\alpha_0 - DD'V_\delta}{\varepsilon_0 + \Delta\varepsilon},\ \beta = \frac{\varepsilon_0\beta_0 + D^2V_\delta}{\varepsilon_0 + \Delta\varepsilon},\ \gamma = \frac{\varepsilon_0\gamma_0 + D'^2V_\delta}{\varepsilon_0 + \Delta\varepsilon}$$

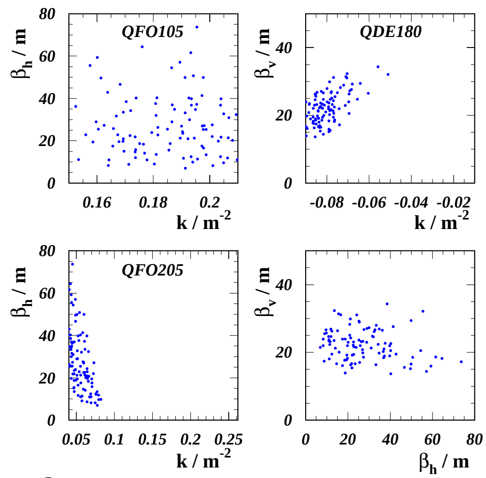

Figure 2: Results of Monte Carlo simulations for TT2 at the new stripper location. Each point corresponds to a given combination of quadrupole strengths. The full range of the variation is indicated by the plot range. Top left, right and bottom left: β as function of gradient for different quadrupoles. Varying degrees of correlation are visible. The limits are due to global constraints. Bottom right: Vertical β as function of horizontal β. No solution with sufficiently small β in both planes exists.

with $V_\delta = (\sigma_p/p_0)^2$. For both cases the actual relative emittance blow-up due to the resulting injection mismatch into the SPS machine is small, of order 10^{-3}.

3 LOW-β STRIPPING INSERTION

According to Eq.(4), the minimisation of the emittance blow-up from multiple Coulomb scattering requires betatron functions as small as possible at the stripper, and a negligible α. Therefore, a low-β insertion is proposed at about 70 m from the beginning of TT2. To design a feasible insertion, it is necessary to match Twiss parameters and dispersion function in the horizontal and the vertical planes, and at the same time respect the "geometrical" constraints imposed by the aperture of the vacuum chamber. These global constraints reduce the effective number of degrees of freedom of the system. To quantify this, the method outlined in the following section was used.

3.1 Effective Number of Degrees of Freedom for the Matching Problem

The eight optical parameters ("local constraints") at the stripper depend on all $n = 7$ normalised quadrupole gradients and constitute a vector of dimension $m = 8$: $\vec{f}(k_1,\ldots,k_n) = \{\alpha_h, \alpha_v, \beta_h, \beta_v, D_h, D_v, D'_h, D'_v\}$. The change of the parameters with respect to the degrees of freedom (i.e. the gradients) can be described as $\Delta\vec{f} = A\,\Delta\vec{k}$, where the elements of the response matrix A are $A_{ij} = \frac{\partial f_i}{\partial k_j}$. An evaluation of A is done by Monte Carlo: all free parameters (k) are varied and MAD is used to calculate f_i at the point of interest. To account for global constraints, only solutions with $\beta < \beta_{\max}$ (acceptance of the line) are permitted. Figure 2 shows some

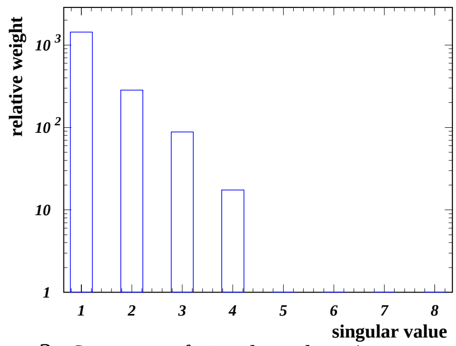

Figure 3: Spectrum of singular values (components of the diagonal matrix) for the response matrix of the TT2 line.

results of these simulations with weak and strong correlations. The correlations are given by $\rho_{ij} = \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$ with $C_{ij} = \langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle$ and $\vec{v} = \{k_1, \ldots, k_n, f_1, \ldots, f_m\}$. The correlation matrix is related to A by $\rho_{ij} = \frac{\partial f_i}{\partial k_j}\sqrt{\frac{C_{jj}}{C_{ii}}}$. To extract the effective number of degrees of freedom of the problem, a singular value decomposition (SVD) is applied to the response matrix A. Figure 3 shows clearly that although there are seven quadrupoles present, the problem possesses only four effective degrees of freedom instead of the eight required for a perfect matching. As a consequence, additional quadrupoles have to be introduced.

3.2 Final Low-β Optics

To create the insertion, four quadrupoles had to be added and in addition the first two quadrupoles of the string needed individual supplies. Figure 4 shows the horizontal and vertical betatron and dispersion functions of the proposed lead ion optics in the TT2 transfer line [1]. The optical parameters at the new stripper location are listed in Table 2. The calculated emittance increase is found in Table 3 for the present and the new low-β optics assuming fully stripped lead ions (charge state z=82).

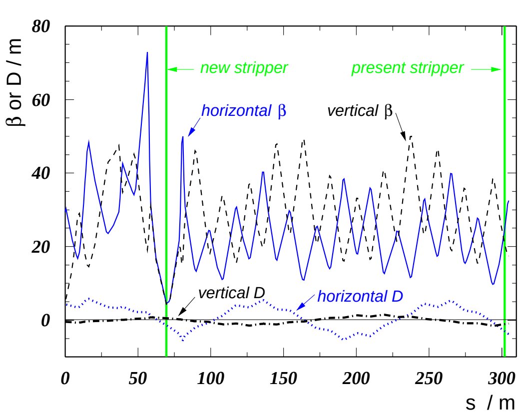

Figure 4: Betatron and dispersion functions of a new lead ion optics in the TT2 transfer line with a low-β insertion. The solid line at around 70 m denotes the new stripper position, the line at around 302 m the current position of the stripper. This optics is compliant with simultaneous proton and ion beam operation.

	β_h [m]	α_h	D_h [m]	D'_h
present	23.56	-1.70	-2.94	-0.34
low-β	4.56	$7{\cdot}10^{-2}$	-1.41	-0.24
	β_v [m]	α_v	D_v [m]	D'_v
present	22.05	1.12	-1.03	0.06
low-β	4.33	$8{\cdot}10^{-3}$	0.51	-0.02

Table 2: Optical parameters at the present and the new stripper position ("low-β").

Optics	$\Delta\varepsilon^*_h$ [μm]		$\Delta\varepsilon^*_v$ [μm]	
	Angle	Energy	Angle	Energy
Calc. present	0.447	0.003	0.418	0.0002
Calc. low-β	0.086	0.003	0.082	0.0003
	Combined		Combined	
Meas. 1995	0.766		0.584	
"Scaled" low-β	0.175		0.158	

Table 3: Increase in normalised r.m.s. emittance due to multiple scattering and energy straggling for the calculated present and proposed low-β optics and for the measured (1995) and "scaled" low-β optics (Measured emittances include the multiple scattering and the energy straggling effects).

4 SUMMARY

A detailed study of possibilities to reduce the emittance blow-up due to the final stripping of the lead ion beam has been performed. It leads to the proposed insertion with four extra quadrupoles in TT2. A change in Twiss parameters is associated with a change in emittance and has to be taken into account for the matching. A method to estimate the effective number of degrees of freedom has been described and applied to determine the number of effective quadrupoles needed to implement the low-β insertion in TT2. It was shown that the proposed low-β optics would keep the emittance within the tight budget of the LHC ion programme.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

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