LONGITUDINAL IONIZATION COOLING WITHOUT WEDGES*

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Abstract

The emittance of a muon beam must be reduced very rapidly due to the finite lifetime of the muons. The most effective known way to accomplish this is ionization cooling. It is straightforward to reduce transverse emittance through ionization cooling, but the reducing the longitudinal emittance is more challenging. Longitudinal cooling is necessary for a muon collider, and would be helpful for a neutrino factory. The method traditionally proposed for longitudinal cooling is emittance exchange involving wedges of absorber material: the longitudinal emittance is reduced at the cost of increased transverse emittance. The larger transverse emittance can then be reduced straightforwardly. An alternative method is proposed here, which does not require wedges of material but instead makes slight modifications to the standard transverse cooling lattice. We demonstrate a lattice which is a slight modification to a standard "Super FOFO" transverse cooling lattice, which has linear eigenvalues all of which have magnitude less than one.

1 IONIZATION COOLING BASICS

When a beam passes through a material, it loses energy, and therefore its momentum is reduced as well, but its direction (ignoring multiple scattering) doesn't change. If an rf cavity attempts to restore the beam's energy, it will only increase the longitudinal momentum, but leave the transverse momentum unchanged. Thus, a simple cooling channel will reduce transverse momentum (and therefore transverse emittance), leaving the longitudinal emittance unchanged (see Fig. 1). More detailed analysis reveals that an energy dependence in the energy loss in the material will give a small effect on energy spread, which in the energy range typically used for muon cooling gives a slight longitudinal emittance growth.

In addition to the cooling coming from the average energy loss, there is a heating effect coming from multiple scattering. One expects a certain average scattering angle. If the angular spread in the beam is large compared to this average scattering angle, the emittance growth will be correspondingly small. Creating a larger angular spread requires a small beta function at the absorber, and this is an important design criterion for the cooling lattice.

The currently preferred method for achieving a small beta function is the "Super-FOFO" lattice [1] (Fig. 2. The lattice uses solenoids for focusing, with the fields alternating to avoid changing angular momentum (due to absorbers). The beta function as a function of position in the cell is given in Fig. 3, and the beta function at the absorber as a function of momentum is shown in Fig. 4. The Super-FOFO lattice operates in a momentum passband where the beta function at the absorber goes to zero at each end of the passband, causing the beta function to remain small over the passband. Furthermore, the lattice tried to maximize the momentum width of that passband.

The primary method proposed to reduce the longitudinal emittance is "emittance exchange" (Fig. 5) which makes use of wedges of absorber material. One first creates dispersion at the wedges (generally using a bent solenoid). Then the wedge is positioned so that particles with higher energy pass through more material. The result is that the energy spread in the beam can be reduced substantially. In creating the dispersion, however, the transverse size of the beam increased. Since the energy spread has been eliminated, there is no way to bring the transverse beam size down to its original value. Thus some of the large longitudinal emittance is "exchanged" with the smaller transverse

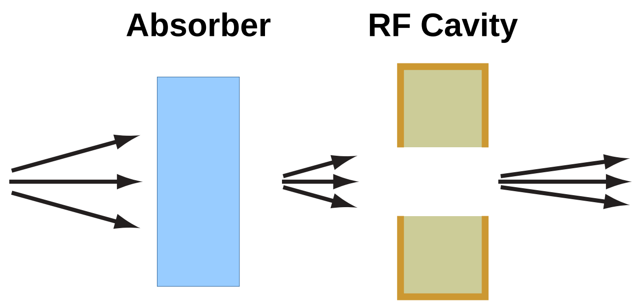

Figure 1: Basic ionization cooling. Momentum is reduced in the absorber, longitudinal momentum restored in the cavity, resulting in transverse momentum reduction.

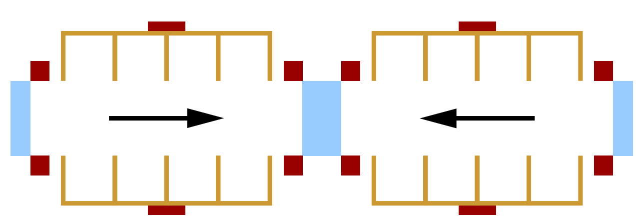

Figure 2: The Super-FOFO lattice, showing absorbers (blue, on axis), cavities, and solenoid coils (red, surrounding absorbers and cavities). The arrows show the direction of the magnetic field.

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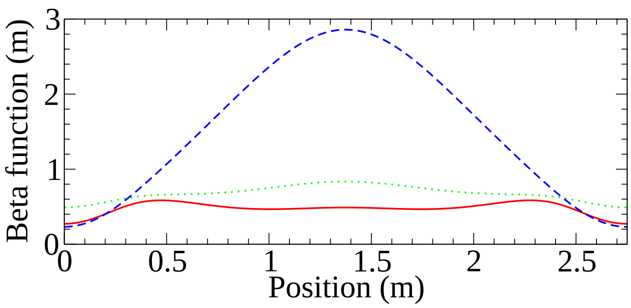

Figure 3: Field as a function of position in the cell. The absorber center is at $s = 0$ and $s = 2.75$ m. The solid red line is at a momentum $p = 140$ MeV/c, the green dotted line is at $p = 200$ MeV/c, and the blue dashed line is at $p = 250$ MeV/c.

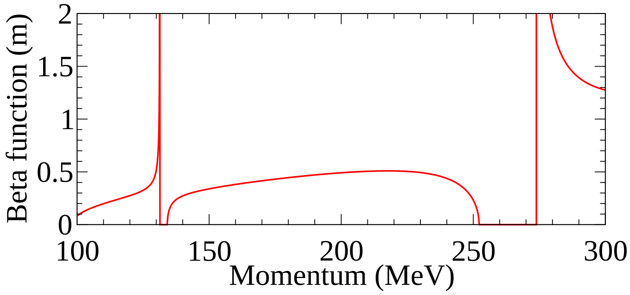

Figure 4: Beta function at the absorber as a function of momentum. The operating region is in the passband extending roughly from 135 MeV/c to 250 MeV/c.

transverse emittance. The longitudinal emittance has been reduced, and the transverse emittance can be reduced by the aforementioned transverse cooling methods.

This method of emittance exchange has its difficulties. One must provide additional focusing at the wedges to reduce the effect of multiple scattering, and this can be difficult since the wedges are within a bent solenoid. Also, the substantial length of the required bent solenoid can lead to bunch lengthening which causes the bunch to extend beyond the rf bucket. While these difficulties are not necessarily insurmountable, it would be useful to have alternative schemes to achieve longitudinal cooling.

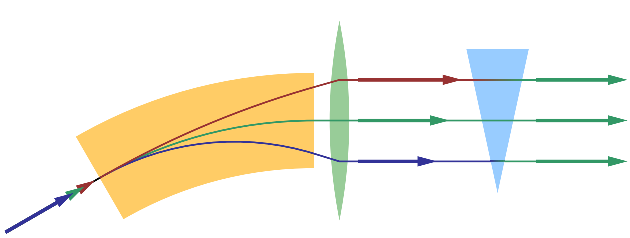

Figure 5: Basic emittance exchange. A bend magnet (left, or bent solenoid) introduces dispersion, and an absorber wedge (right) gives energy loss which depends on transverse position.

2 BASICS OF LINEAR ANALYSIS

We begin by examining the linear map for a single cell of the cooling lattice. Using a single cell in our analysis is valid since the lattice is somewhat repetitive. The linear transformation will at least describe the behavior of the core of the beam; if one cannot achieve longitudinal cooling in a linear analysis, a nonlinear analysis is pointless. The transfer matrix will have six eigenvalues. If there were no absorbers, and the cell corresponded to a stable lattice, the eigenvalues would come in three complex conjugate pairs, each eigenvalue having modulus one. With absorbers, a stable lattice (in most cases) will still have eigenvalues in complex conjugate pairs, but with modulus different from one: less than one for damping, more than one for growth. For a standard transverse cooling cell, four of those eigenvalues will have modulus less than one (transverse cooling), and two will have modulus larger than one (longitudinal heating). The product of the eigenvalues is less than one, indicating that we have net cooling.

Four of the eigenvalues had modulus less than one because the corresponding phase ellipses have nonzero projections in the transverse momentum plane at the absorber. The absorber reduces the transverse momentum components of the ellipses, thus reducing the area of the ellipses and giving cooling. The third ellipse, corresponding to the eigenvalue which is larger than one, has no component in the transverse momentum direction: it corresponded to the synchrotron oscillations. It did have a component in the energy direction, and that component increased slightly, increasing the area of the ellipse.

Thus, if all of the invariant ellipses of a lattice cell had significant projections into the transverse momentum plane, one might expect to make all six eigenvalues have modulus less than one, eliminating the need for a separate emittance exchange. The challenge is to create such a cell. The most straightforward way to create such a cell seems to be to create dispersion inside the rf cavities in the cell. When there is dispersion at the rf cavity (a relationship between transverse position and energy), the symplectic condition also requires that there also be a corresponding relationship between transverse momentum and arrival time. The rf cavity then changes the particle's energy based on its arrival time and therefore its transverse momentum, and the longitudinal and transverse planes have become coupled. If done properly, this coupling will lead to all three invariant ellipses at the absorber having components in the direction of the transverse momentum.

3 DEMONSTRATION LATTICE

To achieve dispersion in the rf cavities, the Super-FOFO lattice is modified by placing bends between the cavities as shown in Fig. 6. Four bends are used in two cells: they extend from the center of each absorber the end of the nearest rf cavity. The signs of those bends are varied in this study. Clearly changing the sign of all the bends will have no effect on the dynamics. There are really four possible com-

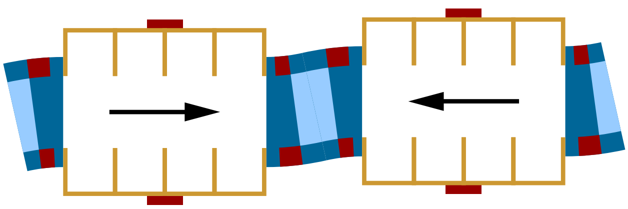

Figure 6: The Super-FOFO lattice with additional bends.

binations for the bending field sign that have a reasonable symmetry: ++++, ++−−, +−+−, +−−+, in the order shown in Fig. 6. Figure 6 corresponds to +−+−.

The lattice used in this example starts with a Super-FOFO lattice cell used for the second neutrino factory design study ("Study II") [2]. Each cell is 2.75 m long, and the with square cross-section solenoid coils:

Position mm	Length mm	Radius mm	Thickness mm	J A/mm^2
175	167	330	175	75.20
1210	330	770	80	98.25
2408	167	330	175	75.20

Here J is the current density. The field on-axis for a straight lattice is computed, and that longitudinal field is used for the on-axis field in the curved coordinate system. The cavity is 1.864 m long with a real-estate gradient (including transit time factor) of 12.39 MeV/m at 200 MHz, 50° off crest. The absorber is infinitely thin, multiplying the transverse momenta by 0.95 and the energy spread by 1.0085. Energy gain and loss are not taken into account.

Figure 7 shows the eigenvalues of the two-cell lattice as a function of the solenoid field. The currents in all the solenoids are varied relative to the values given above. The transverse tunes vary nearly linearly with the coil currents. The longitudinal and transverse eigenvalues retain

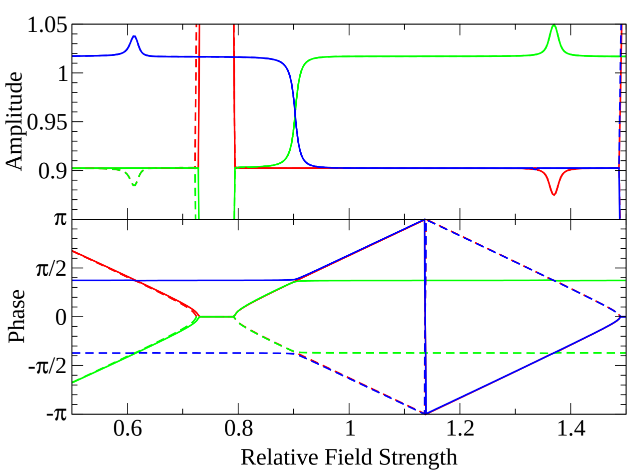

Figure 7: Eigenvalues as a function of solenoid field for the modified Super-FOFO lattice. The bend radius is 5 m with signs +−+−. The horizontal axis is the solenoid field relative to what was described in the text.

their uncoupled values until one approaches a synchro-betatron resonance condition. For one of the resonances (near relative field $k = 0.9$), the longitudinal eigenvalue pair crosses one of the transverse eigenvalue pairs. At that crossing point, all six eigenvalues have a magnitude less than one, and one has cooling in all planes, at least for a region near the origin. For the other resonance (near $k = 1.37$) the eigenvalues get further apart. One of the transverse eigenvalue pairs remains essentially unchanged when passing through the resonance.

This demonstrates cooling in all planes for small amplitudes. A nonlinear analysis is necessary to determine what volume of phase space can be cooled with this lattice. One can make some conclusions from Fig. 7, however. The relative field strength corresponds roughly to the inverse of the momentum. Thus, the energy spread accepted by this lattice will be roughly between the linear resonances (at $k \approx 0.8$ and $k \approx 1.48$), ignoring longitudinal growth.

To achieve damping in all planes, the energy bandwidth will probably be limited to the region where all eigenvalues would be less than 1: in this case a couple percent. One can increase this energy bandwidth by increasing the synchro-betatron coupling strength, which can happen by changing the bend field symmetry or by increasing the bending field. The effect of increasing the bend field is shown in Fig. 8.

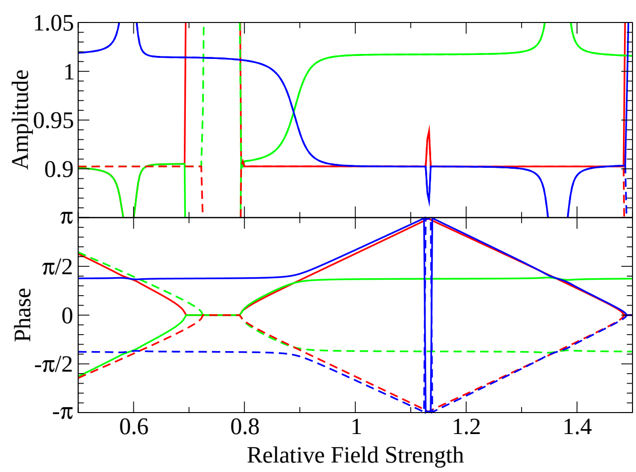

Figure 8: Fig. 7 but with bend radius 2 m.

The range of field strength (and thus momentum) where one has damping clearly increases. A linear resonance also appears, but it seems to be well outside the region where we have damping. Changing the bend symmetry to ++++ also increases the width of the coupling resonance, without starting to drive the linear resonance. Changing the symmetry to ++−− almost completely eliminates the coupling, although the anti-coupling resonance remains.

4 REFERENCES

[1] E.-S. Kim *et al.*, MC Note 36, http://www-mucool.fnal.gov/notes/notes.html.

[2] S. Ozaki *et al.* eds., BNL-52623 (2001).