

QUENCH SIMULATION PROGRAM FOR SUPERCONDUCTING ACCELERATOR MAGNETS

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Abstract

In the design of superconducting magnets for accelerator and the quench protection systems, it is necessary to calculate the current, voltage and temperature during quench. The quench integral value (MIITs) is used to get a rough idea about the quench, but we need numerical calculations to obtain more detailed picture of the quench. A simulation program named "KUENCH", which is not based on the MIITs calculation, was developed to calculate voltage, current and temperature of accelerator magnets during quenches. The software and calculation examples are introduced. The example also gives some important information about effects of copper content in the coil and quench protection heaters.

1 INTRODUCTION

The program "KUENCH" simulates quench processes in long magnets such as dipole magnet, quadrupole magnet, race-track magnet and so on. During the simulation, current, temperatures, resistive voltage, quench integral are calculated and displayed.

This paper introduces the program "KUENCH" and shows some useful results calculated by using "KUENCH 1.6."

2 SIMULATION MODEL

As shown in Figure 1, the magnet is considered as a long cable. This long cable is divided by elements. Heat balance equation for each element is used to calculate voltage and temperature of the element, and the circuit equation is used to calculate the current. Cooling and the thermal contact between turns and layers are considered also. To mimic the transverse heat transition, thermal contacts between elements with a certain distance (length of one turn) are taken in account.

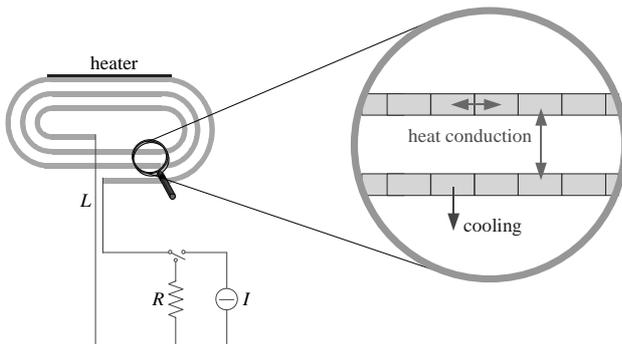
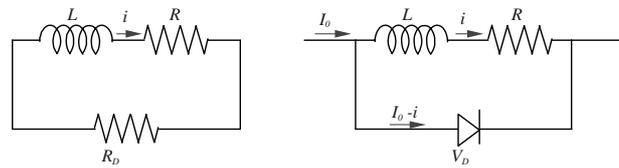


Figure 1 : Calculation model.

3 CURRENT CALCULATION

In a normal operation condition, current of the magnet is constant. When the magnet quenches the current decays and we need to calculate the current. All of the elements have the same current, because all the elements are in series. Therefore, current of the magnet is calculated from inductance and resistance of the magnet and the quench protection circuit.



Dump resistor circuit

Bypass diode circuit

Figure 2 : Typical circuits of quench protection systems.

In this figure L and R are inductance and resistance of the magnet, R_D is resistance of the dump resistor, V_D is voltage across the bypass diode, i is current of the magnet and I_0 is total current which is constant.

In the case of dump resistor protection, the circuit equation is as follows,

$$L \frac{di}{dt} + Ri + R_D i = 0 \quad (1)$$

In the case of bypass diode protection, the circuit equation is as follows,

$$L \frac{di}{dt} + Ri = V_D \quad (2)$$

4 TEMPERATURE CALCULATION

Relationship between heat energy increment ΔQ and temperature increment ΔT is given by the following equation.

$$\Delta Q = C_p(T) \cdot (\text{volume}) \cdot \Delta T \quad (3)$$

where, $C_p(T)$ is volumetric specific heat, which will be described in the following section.

From the above relationship temperature of each element T_i for the next time step can be calculated as follows.

$$T_{i, \text{next step}} = T_i + \frac{\Delta Q_i}{C_p(T_i) \cdot A_e \cdot l_e} \quad (4)$$

where A_e and l_e are cross-section area and length of an element.

Heat energy increment in one element during one time step is given by the following equation.

$$\begin{aligned} \Delta Q_i = & (\text{heat generation}) - (\text{Cooling}) \\ & + (\text{conduction heat along the cable}) \\ & + (\text{conduction heat between turns}) \\ & + (\text{conduction heat between layers}) \end{aligned} \quad (5)$$

Heat generation and cooling for an element during one time step is given by the following equations.

$$(\text{heat generation}) = \rho(T_i) \frac{l_e}{A_e} \cdot i^2 \cdot \Delta t \quad (6)$$

$$(\text{cooling}) = h \cdot P_e \cdot l_e \cdot (T_i - T_{He}) \cdot \Delta t \quad (7)$$

where, h is cooling coefficient, P_e is perimeter of one element and T_{He} is helium temperature which is 4.2K in this software.

Conduction heat between elements i and $i+1$ is calculated by the following equation.

$$(\text{conduction heat along cable}) = K \left(\frac{T_i + T_{i+1}}{2} \right) \cdot \frac{A_e}{l_e} \cdot (T_i - T_{i+1}) \cdot \Delta t \quad (8)$$

where $K(T)$ is thermal conductivity function of temperature, which will be discussed in the following section

Unfortunately, the thermal conductivity from turn to turn is not known. We do not have a measured value or an analytic equation to calculate the value. Hence, in this software, constants are used for the thermal conductivity between cable turns and cable layers.

Conduction heat between adjacent turns is calculated by the following equation, and conduction heat between inner and outer layers of the magnet is calculated in the same manner.

$$(\text{conduction heat between turns}) = K_t \cdot (\text{contact area}) \cdot (T_i - T_j) \cdot \Delta t \quad (9)$$

where K_t is the heat transfer coefficient between turns, which is constant.

5 MATERIAL PROPERTIES

The critical current density j_c of an element is given by a function of magnetic field B and temperature T , as shown in the following equation. If the current density of the element is less than the value given by this equation, we can see the element is in superconducting state.

$$j_c(B, T) = j_{c0} \left(1 - \frac{4 \frac{B}{B_{c0}}}{4 - 3 \left(\frac{T}{T_{c0}} \right)^2 - \frac{T}{T_{c0}}} \right) \left(1 - \frac{T}{T_{c0}} \right) \quad (10)$$

where j_{c0} is critical current at 0 K, 0 A condition. B_{c0} and T_{c0} are defined in the same manner. These three values are critical values on the critical surface.

Figure 3 shows approximated specific heats of materials used in this simulation.

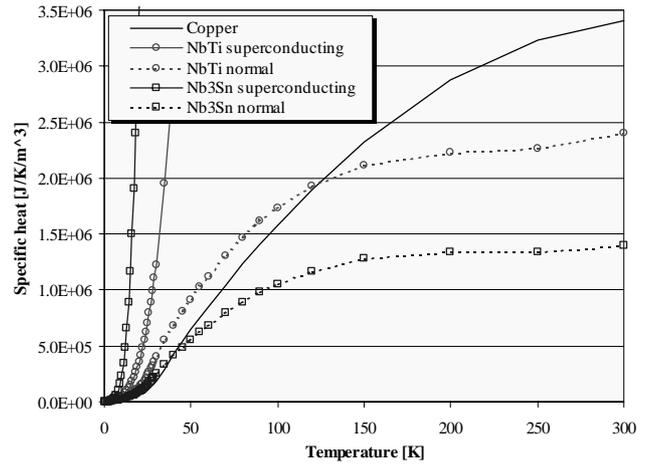


Figure 3 : The volumetric specific heat for copper and superconductors.

Resistivity of copper is given by the following equation. RRR stands for Residual resistance ratio.

$$\rho(T, B) = \text{Max} \left(\frac{1.67 \times 10^{-8}}{RRR} + 0.5 \times 10^{-10} B, 5.9 \times 10^{-11} T - 1.0 \times 10^{-9} \right) [\Omega \cdot \text{m}] \quad (11)$$

Resistivity of a superconductor is given by the well-known equation.

$$\rho(j) = \rho_0 \left(\frac{j}{j_c} \right)^n [\Omega \cdot \text{m}] \quad (12)$$

In normal state NbTi has resistivity of $5.6 \times 10^{-7} \Omega \cdot \text{m}$. Resistivity of Nb₃Sn in normal state is given by the following equation.

$$\rho_{\text{Nb}_3\text{Sn}}(T) = \text{Min} \left(7.65 \times 10^{-10} \times T + 2.3 \times 10^{-7}, 2.6 \times 10^{-10} \times T + 3.32 \times 10^{-7} \right) [\Omega \cdot \text{m}] \quad (13)$$

Thermal conductivity of copper is presented by the Wiedemann-Franz-Lorentz law,

$$K(T) = \frac{2.45 \times 10^{-8} T}{\rho(T)} [\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}] \quad (14)$$

Thermal conductivities of superconductors are given by measured data.

6 EXAMPLE 1 : DUMPING AND HEATING

As the first example the effect of dumping and heating to the maximum temperature is calculated. Table 1 shows input parameters for this calculation and Figure 4 shows the summary of the results.

This result shows that a dump resistor has more effect for a short magnet, and heating affects a long magnet more. This suggests that even if a heater works fine for a short prototype magnet, the same heater can be not enough for a long real magnet.

We can use a bigger dump resistor for a longer magnet to suppress the temperature, but we need to be more careful for the higher voltage

Table 1 Input parameters for example 1

Superconductor	Nb ₃ Sn
Number of turns	25(inner 11, outer 14)
Inductance of the magnet	1.42 mH per meter
Cable cross-section	20.00 mm × 1.867 mm
Cu/super volume ratio	1.6
Quench detection	100 mV of total voltage
Dump resistor	20 mΩ
Time delay of dump resistor	2 ms
Heater for magnet protection	55W/cm ² on outer 14 turns
Heater delay	50 ms

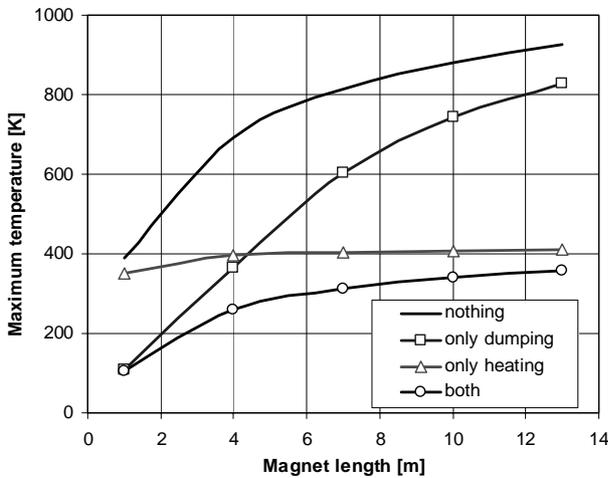


Figure 4 : Effect of energy extraction and heating on the maximum temperature.

7 EXAMPLE 2 : AMOUNT OF COPPER

Second example is magnets of three strands with different copper and superconductor amount. Table 2 shows the three different conductors. Parameters not shown in this table are the same as Table 1. Values in Table 2 except Cu/SC ratio are normalized by the values of the big cable which was used in example 1.

Big cable and small cable have the same amount of superconductor, but different amount of copper. High density cable has the same total volume and copper ratio as the big cable. This means the high density cable has more copper than the small cable, but less than the big cable.

Figure 5 shows maximum temperatures of these three cables. In comparison with big and small cables, we can see that the half amount of copper in the small cable makes the temperature more than twice. The result of high density cable suggests that the maximum temperature is

controlled by the amount of copper, not by the copper ratio.

A long real magnet has bigger inductance and hence a slower current decay than a short prototype magnet. Therefore a longer magnet should have more copper for a safe operation.

Table 2 Three different wires for example 2

Strand in the cable			
	Big cable	Small cable	High density cable
Cross-sectional area	1	0.69	0.69
Copper/Super ratio	1.6	0.8	1.6
Amount of copper	1	0.5	0.69
Critical current density (Jc)	1	1	1.45
Cable critical current (Ic)	1	1	1

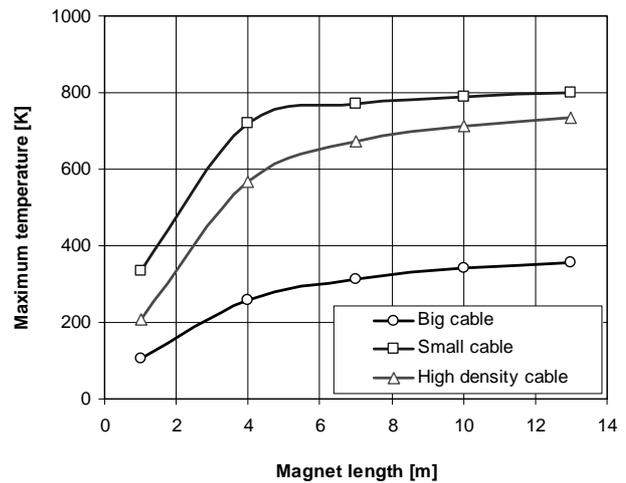


Figure 5 Cables with different amount of copper and superconductor.

8 REFERENCES

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