RECYCLER RING BEAM LIFE TIME

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Abstract

We study the Fermilab Recyler Ring beam life time due to various physical processes associated with beam-gas interactions. This includes single coloumb scattering, electronic excitations, nuclear and multiple scattering processes. We compare the measured life time with those obtained from theoretical estimations. The results indicate addtional processes are also contributing to the actual beam life time.

1 RECYCLER RING

The Recycler Ring [1] is located in the Main Injector tunnel at Fermilab is designed to store antiprotons from the Accumulator and the residual Tevatron stores as a part of Run II luminosity upgrade program. The Recycler Ring (RR) is being commissioned using protons. This study estimates the RR beam life time due to the interaction of residual gases with the beam particles. The relevant RR parameters used are listed in Table 1.

Table 1: Recycler Ring Paramaters

Paramater	Value
Acceptance (mm-mr)	40.0π
Average β (m)	40.0
Average beam pipe radius (in m)	0.023
Beam energy (GeV)	8.89
Average beam β	0.998
Average beam γ	9.48
Maximum energy loss (GeV)	0.089

2 BEAM-GAS INTERACTIONS

In this note, we estimate the beam life time due to interaction of beam particles (protons) with the prevalent gases inside the beam pipe via coloumb scattering, electronic excitations, bramstrahlung, nuclear scattering and multiple coloumb scattering. The injected beam is assumed to be Gaussian:

$$f(Z) = \frac{a^2}{2\sigma^2} e^-(a^2/2\sigma^2)Z$$

with $Z = \epsilon/\epsilon_a$ ranging from 0.0 to 1.0. With an initial beam of 10π mm-mr, $\sigma = 11.53 \times 10^{-3}$m-r. Here ϵ, ϵ_a denotes emittance and Recyler acceptance respectively and a is the half aperture equal to the average radius of the beam pipe (0.023 m).

The RR ultra high vacuum is maintained by a regular array of ion and titanium sublimation pumps (TSP). The pressure at a point inside the beam pipe varies significantly depending on the distance from the pump locations. The pressure profile has been computed for each component of the gas present in the beam pipe. The maximum and minimum pressure for each gas are listed in Table 2. For life time estimations, we treat the 'unknown' component as Nitrogen.

Table 2: Vacuum Gas Composition

Gas	Content [%]	Min. Pres. [Torr]	Max. Pres. [Torr]
H_2	67.22	4.9E-11	1.2E-10
H_2O	21.13	1.8E-11	7.6E-11
CO	3.36	1.7E-12	1.3E-11
Ar	0.02	8.9E-12	1.1E-11
CH_4	0.85	2.4E-11	7.9E-11
CO_2	6.53	3.8E-12	2.9E-11
$Unknwon$	0.89	4.5E-13	3.4E-12
Total	100.00	1.1E-10	3.3E-10

2.1 Single Coloumb Scattering

It is possible by a single coloumb scattering off a residual gas nucleus, a proton from the beam can be lost. To estimate the rate of loss by this mechanism, we use the classic Rutherford scattering formula [2].

$$\frac{1}{\tau_{sc}} = \frac{-1}{N}\frac{dN}{dt} = \beta c \sum \sigma_j n_j$$

where the sum is over the species of residual gas molecules with density n_j and σ_j is given by:

$$\sigma_j = \frac{4\pi Z_j^2 r_p^2}{\beta^4 \gamma^2 \theta_{max}^2}$$

with Z_j, the atomic number of jth gas and θ_{max} denoting the maximum angle deflection needed for knocking of the proton. The angle θ_{max} is estimated by:

$$\theta_{max} = \sqrt{\frac{Acceptance}{\pi\beta_{avg}}}$$

Therefore, the beam life time due to single coloumb scattering can be cast as:

$$\frac{1}{\tau_{sc}} = \frac{4\pi r_p^2 c}{\beta^3\gamma^2\theta_{max}^2} \sum Z_j^2 n_j$$

Using Table 2 and the Recyler parameters listed in Table 1, we obtain $\theta_{max} = 0.325$ mr and $\tau_{sc} = 5.29 \times 10^6$ s

(1.47×10^3 hours) for the case of minimum pressure. For the case of maximum pressure, the $\tau_{sc} = 1.79 \times 10^6$ s (4.98×10^2 hours). The contribution from single coloumb scattering for the Recycler beam life is small.

2.2 *Inelastic Scattering:*

There are two important processes in the inelastic scattering: (a) Bremsstrahlung scattering where the proton emits a photon and the nucleus of the gas atom is left unexcited (b) Inelastic scattering (excitation) of the electrons of the atoms from momentum transfer. We examine each case below.

The total cross section for the bremstrahlung can be written for a given gas as [3]:

$$\sigma_{br} = \int_{\epsilon_m}^{E} \{\frac{d\sigma}{d\epsilon}\} d\epsilon$$

where E denotes the energy of the proton, ϵ_m the photon energy and

$$(\frac{d\sigma}{d\epsilon})_{br} = \frac{4\alpha Z^2 r_p^2}{\epsilon}\{[\frac{4}{3}(1-\frac{\epsilon}{E})+\frac{\epsilon^2}{E^2}][\frac{\phi_1(0)}{4}-\frac{1}{3}\mathrm{InZ}]+[\frac{1}{9}(1-\frac{\epsilon}{E})]\}$$

with $\phi_1(0)$ representing the screening function. A similar expression applies to the case of atomic/molecular electron excitations:

$$(\frac{d\sigma}{d\epsilon})_{ee} = \frac{4\alpha Z r_e^2}{\epsilon}\{[\frac{4}{3}(1-\frac{\epsilon}{E})+\frac{\epsilon^2}{E^2}][\frac{\psi_1(0)}{4}-\frac{2}{3}\mathrm{InZ}]+[\frac{1}{9}(1-\frac{\epsilon}{E})]\}$$

Here $\psi_1(0)$ denotes the screening function as $\phi_1(0)$. They can be approximated by $\psi_1(0) \simeq 28.34$ and $\phi_1(0) \simeq 20.836$.

For $\epsilon_m \ll$ E, the above expressions can be evaluated and can be cast as:

$$\sigma_{br} = 4\alpha\{\frac{4}{3}Z^2 r_p^2 \mathrm{In}\frac{183}{Z^{1/3}}[\mathrm{In}(E/\epsilon_m)-(5/8)]+\frac{1}{9}(Z^2 r_p^2)[\mathrm{In}(E/\epsilon_m)-1]\}$$

$$\sigma_{ee} = 4\alpha\{\frac{4}{3}Z r_e^2 \mathrm{In}\frac{1194}{Z^{2/3}}[\mathrm{In}(E/\epsilon_m)-(5/8)]+\frac{1}{9}(Z r_e^2)[\mathrm{In}(E/\epsilon_m)-1]\}$$

The total cross section is then:

$$\sigma_{br+ee} = \sigma_{br} + \sigma_{ee}$$

The σ_{br} is quite negligible compared to σ_{ee} and therefore we drop it from further consideration. The life time due to inelastic scattering τ_{in} becomes as before:

$$\frac{1}{\tau_{in}} = \frac{-1}{N}\frac{dN}{dt} = \beta c \sum \sigma_{eej} n_j$$

Now for ϵ_m = 89 MeV, using the relevant Recycler Ring parameters and Table 2, $\tau_{in} = 1.17 \times 10^7$ s (3.26×10^3 hours) and $\tau_{in} = 3.80 \times 10^6$ s (1.06×10^3 hours) for the minimum and maximum cases of pressure respectively.

2.3 *Multiple Coloumb Scattering:*

Unlike the previous two cases, the mutiple coloumb scattering causes emittance growth of the beam. As a result, protons are lost via diffusion across the boundary of the allowed particle distribution in the beam pipe. Therefore we should approach this problem by solving the diffusion equation [4] for a particle distribution f:

$$\frac{\partial f}{\partial \tau} = \frac{\partial}{\partial Z}(Z\frac{\partial f}{\partial Z})$$

subject to the boundary conditions:

$$f(Z,0) = f_0(Z)$$

$$f(1,\tau) = 0$$

where Z = ϵ/ϵ_a = emittance/acceptance, and τ = tR/ϵ_a with R, the diffusion coefficient. The diffusion coefficient R is given in terms of scattering angle θ by:

$$R = \beta_{avg}\langle\dot{\theta}^2\rangle$$

The general solution of the above equation can be written as:

$$f(Z,\tau) = \sum_n C_n J_0(\lambda_n\sqrt{Z}) e^{-\lambda_n^2\tau/4}$$

with coefficients C_n:

$$C_n = \frac{1}{J_1(\lambda_n)^2}\int_0^1 f_0(Z) J_0(\lambda_n\sqrt{Z}) dZ$$

where λ_n is nth root of the Bessel function $J_0(Z)$. Now we can obtain the total beam particles as a function of time:

$$N(\tau) = \int_0^1 f(Z,\tau) dZ = 2\sum_n \frac{C_n}{\lambda_n} J_1(\lambda_n) e^{-\lambda_n^2\tau/4}$$

The life time due to multiple coloumb scattering can be now computed using the standard expression:

$$\tau_{mc} = -\frac{N(\tau)}{dN(\tau)/d\tau}$$

The beam life time varies with time and normally reaches an asymptotic value:

$$\tau_a = \frac{4\epsilon_a}{\lambda_1^2 R}$$

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To compute $<\dot{\theta}^2>$, we use the small angle limit of the Rutherford scattering cross section, parametrization of atomic and nuclear radii:

$$<\dot{\theta}^2> = \frac{8\pi r_p^2 c}{\gamma^2\beta^3}\sum_j n_j Z_j^2 \mathrm{In}[\frac{38360}{(A_j Z_j)^{1/3}}]$$

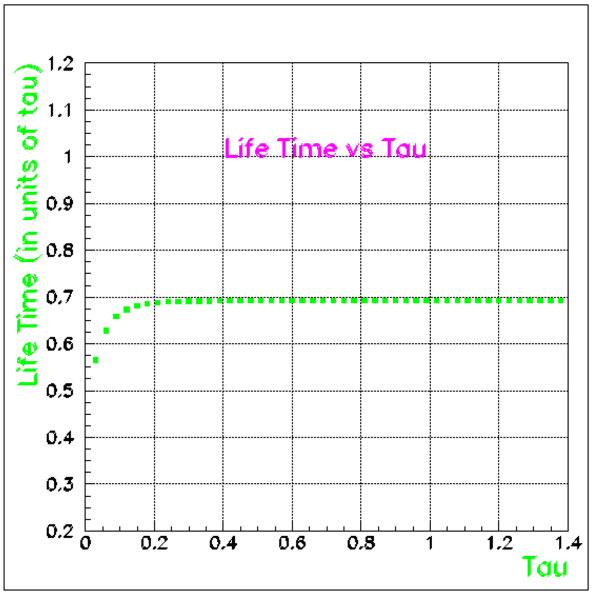

Figure 1: The life time as a function of $\tau = tR/\epsilon_a$ due to multiple coloumb scattering process. The life time reaches an asymptotic value after rising in this case.

with A_j denoting the atomic weight of jth gas component. Using Table 2 and the relevant Recycler Ring parameters, the asymptotic life time due to multiple coloumb scattering is given by $\tau_{mc} = 5.44 \times 10^5$ s (1.51×10^2 hours) for the minimum case. For the maximum, the $\tau_{mc} = 1.84 \times 10^5$ s (5.11×10^1 hours).

2.4 *Nuclear Scattering:*

Here we estimate the beam life time due to the loss of protons from interaction with nucleus of residual gas molecules via strong force. As there is no simple expression for the interaction cross section for the strong force, we use the general formula:

$$\frac{1}{\tau_{nu}} = \frac{-1}{N}\frac{dN}{dt} = \beta c \sum \sigma_j n_j$$

where σ_j denotes the total (elastic + inelastic + quasi-elastic) proton-nucleus cross for each gas in the proton relevant energy range. The total cross sections are [5]: $\sigma_{nc}(H) = 40$mb, $\sigma_{nc}(N) = 387$mb, $\sigma_{nc}(C) = 344$ mb, and $\sigma_{nc}(O) = 429$mb.

Now using the above cross sections and the gas densities in Table 2, we obtain $\tau_{nu} = 2.51 \times 10^7$ s (6.98×10^3 hours) and $\tau_{nu} = 7.12 \times 10^6$ s (1.98×10^3 hours) for the minimum and maximum pressure respectively.

2.5 *Final Beam-Gas Life Time:*

Now we can combine the contributions from various beam-gas interactions to obtain the life time as:

$$\frac{1}{\tau_{bg}} = \frac{1}{\tau_{sc}} + \frac{1}{\tau_{in}} + \frac{1}{\tau_{mc}} + \frac{1}{\tau_{nu}}$$

Using the asymptotic life time for the multiple coloumb scattering, direct evaluation gives the total beam life time due to beam-gas interactions. The results are tabulated in Table 3.

Physical Process	Min. Pressure [hours]	Max. Pressure [hours]
Single Coloumb	1.47×10^3	4.98×10^2
Inelastic Scatt.	3.26×10^3	1.06×10^3
Mult. Coloumb	1.51×10^2	5.11×10^1
Nuclear Scatt.	6.98×10^3	1.98×10^3
Total life time	1.29×10^2	4.64×10^1

Table 3: The total life time summary for maximum and minimum pressure cases considered.

The measured Recycler Ring life time value of 3-10 hours (depending various external situations) indicate that there are other significant factors besides the beam-gas interactions affecting the Recycler life time.

3 REFERENCES

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