

# CONFINEMENT OF HIGH-INTENSITY BUNCHED BEAMS IN HIGH-POWER PERIODIC PERMANENT MAGNET FOCUSING KLYSTRONS\*

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## Abstract

Beam confinement in periodic permanent magnet (PPM) focusing klystrons, is a major concern for successful operation of these devices. In an effort to study beam dynamics and confinement for PPM focusing klystrons, we present a model for a relativistic highly bunched beam propagating through a perfectly conducting cylinder in a PPM focusing field. By imposing confinement conditions on the beam, it is shown that the effect of bunching significantly reduces the maximum effective self-field parameter well below the Brillouin density limit for unbunched beams. We compare the self-field parameters of three SLAC PPM klystron experiments, the 50 MW XL-PPM, the 75 MW XP, and the Klystrino, with the theoretical limit. Our analysis shows that these experiments are operating relatively close to the theoretical result. We will discuss the implications of these results in preventing beam losses in these devices.

## 1 INTRODUCTION

The confinement of intense charged-particle beams is an important subject in beam physics [1] and plasma physics [2]. The confinement of intense electron beams is also important to the development of rf-accelerators and high-power microwave (HPM) sources [3], such as klystrons, traveling wave tubes and backward wave oscillators. Recently, beam loss, which is related to beam confinement, has been measured in a number of high-intensity accelerator and high-power microwave experiments. For example, beam power losses have been observed in several periodic permanent magnet (PPM) focusing klystrons [4] at the Stanford Linear Accelerator Center and in other HPM sources elsewhere [5].

## 2 DESCRIPTION OF MODEL

The present model begins with a relativistic Hamiltonian description of a collinear periodic distribution of electron bunches moving in a perfectly conducting cylindrical pipe of radius  $a$ . The electrons are focused transversely by an applied PPM focusing field. Assuming the beam is strongly bunched longitudinally by an rf field, and has a negligibly small transverse size, we approximate the beam bunches by periodic point charges with a periodic spacing of  $L$ . In terms of klystron parameters,  $L$  corresponds to  $v_b/f$ , where  $v_b = \beta_b c$  is the average velocity of the bunches moving parallel to the pipe axis, and  $f$  is the

operating frequency of the klystron. Figure 1 shows a diagram of the system.

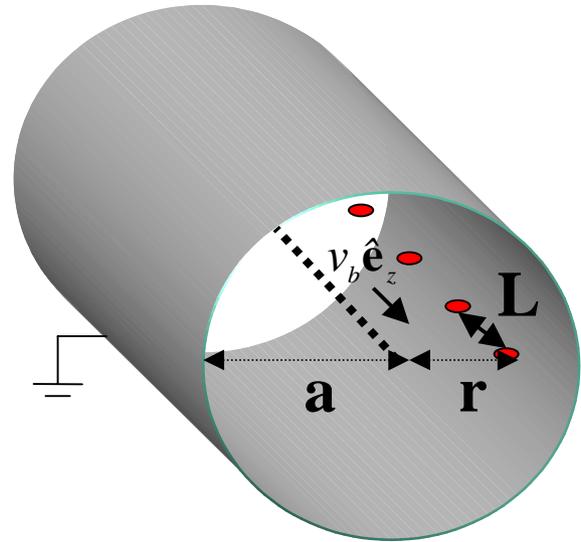


Fig.1 Diagram of a periodic array of charges propagating in a perfectly conducting cylinder with longitudinal velocity  $v_b \hat{e}_z$ .

Since the electron bunches are collinear and periodic, we only need to specify the coordinates of the center of mass of one electron bunch in the Hamiltonian. In the externally applied PPM magnetic field,  $\mathbf{B}^{ext} = \nabla \times \mathbf{A}^{ext}$  with  $\mathbf{A}^{ext} = (rB_0/2)\cos(k_0 z)\hat{e}_\theta$ , and the approximated rf field,  $E \cos(kz - \omega t)\hat{e}_z$  with  $k = 2\pi/L$  and  $\omega = 2\pi f$  being the wave number and angular frequency, the Hamiltonian for this system is given in the laboratory frame by

$$H = \left[ M^2 c^4 + (c\mathbf{P} - Q\mathbf{A})^2 \right]^{1/2} + Q\phi^{self} - \frac{QE}{k} \sin(kz - \omega t) \quad (1)$$

where  $Q = -Ne$  is the total charge of an electron bunch,  $M = Nm_e$  is the total mass of the electron bunch,  $N$  is the number of electrons per bunch,  $-e$  and  $m_e$  are the electron charge and rest mass, respectively,  $\mathbf{P}$  is the canonical momentum of the electron bunch,  $\mathbf{A} = \mathbf{A}^{ext} + \mathbf{A}^{self}$ ,  $\phi^{self}$  and  $\mathbf{A}^{self}$  are the scalar and vector potentials associated with the charge and current on the conductor wall induced by the beam itself, respectively, and  $c$  is the speed of light in vacuum. In expressing Eq. (1), we have implicitly assumed that  $v_b \gg |v_\theta|$  and

$v_b \gg |v_r|$ , which is consistent with the fact that the axial motion remains relativistic, and the usual assumption that the effective Budker parameter is small, or more specifically,  $Ne^2/m_e c^2 L \ll a/L$ . Consequently,  $\mathbf{A}^{self} \equiv A_z^{self} \hat{\mathbf{e}}_z$ . Consistent with the assumptions  $v_b \gg |v_\theta|$  and  $v_b \gg |v_r|$ , it can be shown that  $A_z^{self} \equiv \beta_b \phi^{self}$ .

In order to find the self-field potentials,  $A_z^{self}$  and  $\phi^{self}$ , it is useful to momentarily transform to the rest frame of the beam, using the property that the scalar and vector potentials form 4-vectors,  $(\phi_{rest}^{self}, \mathbf{A}_{rest}^{self})$  and  $(\phi^{self}, \mathbf{A}^{self})$ , in the rest and laboratory frames, respectively. Since there is no longitudinally induced current on the conductor surface in the rest frame,  $\mathbf{A}_{rest}^{self} = 0$ . The beam-wall interaction becomes purely electrostatic in the rest frame, and  $\phi_{rest}^{self}$  may be calculated by solving Poisson's equation. In a previous paper [6], the authors utilized a Green's function approach to compute the electrostatic potential  $\phi_{rest}^{self}$ . The result is given by [6]

$$\phi_{rest}^{self} = \frac{Q}{\gamma_b L} \left[ \ln(\alpha^2 - \hat{r}^2) - 2 \sum_{n=1}^{\infty} \frac{K_0(n\alpha) I_0^2(n\hat{r})}{I_0(n\alpha)} - 4 \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} \frac{K_l(n\alpha) I_l^2(n\hat{r})}{I_l(n\alpha)} \right] \quad (2)$$

Here,  $\gamma_b = (1 - \beta_b^2)^{-1/2}$ ,  $\hat{r} = 2\pi r / \gamma_b L$ ,  $\alpha = 2\pi a / \gamma_b L$ , and  $I_l(x)$  and  $K_l(x)$  are the  $l$ th-order modified Bessel functions of the first and second kind, respectively. Using the Lorentz transformation, we find that  $\phi^{self} = \gamma_b \phi_{rest}^{self}$  and  $\mathbf{A}^{self} = \gamma_b \beta_b \phi_{rest}^{self} \hat{\mathbf{e}}_z = \beta_b \phi^{self} \hat{\mathbf{e}}_z$ .

### 3 CONFINEMENT CRITERION

As is demonstrated in (7), the Hamiltonian may be canonically transformed and decomposed into longitudinal and transverse components, since the longitudinal beam energy is much greater than the corresponding transverse energy in a klystron or an rf-accelerator device. For a deeply trapped electron bunch, the transverse motion occurs on a time scale that is long compared with the beam transit time through one period of the PPM focusing field. The transverse motion of the beam is obtained from the averaged transverse Hamiltonian,

$$\langle H_\perp \rangle = \frac{1}{2\gamma_b M} \left( P_r^2 + \frac{P_\theta^2}{r^2} + \frac{Q^2 B_{rms}^2 r^2}{4c^2} \right) + \frac{Q\phi^{self}}{\gamma_b} \quad (3)$$

where  $B_{rms} = B_0 / \sqrt{2}$  is the rms value of the PPM focusing field, and use has been made of  $P_z = \gamma_b \beta_b M c$ . It follows from Eq. (3) that the radial equations of motion for the deeply trapped electron bunch averaged over one period of the PPM focusing field are

$$\frac{dr}{dt} = \frac{P_r}{\gamma_b M} \quad (4)$$

$$\frac{dP_r}{dt} = \frac{1}{\gamma_b} \left[ \frac{P_\theta^2}{Mr^3} - \frac{Q^2 B_{rms}^2 r}{4Mc^2} - \frac{Q}{\gamma_b} \frac{\partial \phi^{self}}{\partial r} \right] \quad (5)$$

Because  $\langle H_\perp \rangle = const.$ , we have  $P_r^2 = P_{r0}^2 + F(r_0) - F(r)$ , where the subscript zero denotes the initial conditions, and  $F(r) = P_\theta^2 / Mr^2 + Q^2 B_{rms}^2 r^2 / 4Mc^2 + 2MQ\phi_{rest}^{self}$  is an effective radial potential. To determine the condition for radial confinement, we are only interested in orbits near the center of the conductor, i.e. where the beam-wall interaction is weakest. Therefore, by taking the limit of the effective radial potential  $F(r)$  as  $r \rightarrow 0$  ( $P_\theta = 0$ ) and finding the criterion that  $F(r)$  is increasing, we obtain the space charge limit for radially confined orbits,

$$\frac{8c^2 I_b}{\omega_{c,rms}^2 a^2 I_A} \leq \left[ 1 + \sum_{n=1}^{\infty} \frac{n\alpha}{I_0(n\alpha) I_1(n\alpha)} \right]^{-1} \quad (6)$$

where  $I_b = Nef$  is the average current in the klystron (in amperes),  $I_A = \gamma_b \beta_b m_e c^3 / e \equiv \gamma_b \beta_b \times 17 \text{ kA}$  is the electron Alfvén current,  $\omega_{c,rms} = eB_{rms} / m_e c$ , and  $\alpha = 2\pi af / \gamma_b \beta_b c$ . We note that the left-hand side of Eq. (6) is equal to the self-field parameter  $2\omega_p^2 / \omega_{c,rms}^2$  in the rest frame, where  $\omega_p^2 = (4\pi e^2 / m_e) (N / \pi a^2 \gamma_b L)$  is the effective plasma frequency squared. This self-field parameter limit is similar to a limit that the authors computed for a uniform-focusing magnetic field,  $\mathbf{B}^{ext} = B\hat{\mathbf{e}}_z$  [6]. The only difference is that the rms magnetic field on the left-hand side of Eq. (6) should be replaced by  $B$ . Figure 2 shows a plot of the right-hand side of Eq. (6) versus the parameter  $\alpha$ . In the limit where the bunch spacing is small compared to the pipe radius, i.e.  $\alpha \gg 1$ , the system resembles a continuous beam.

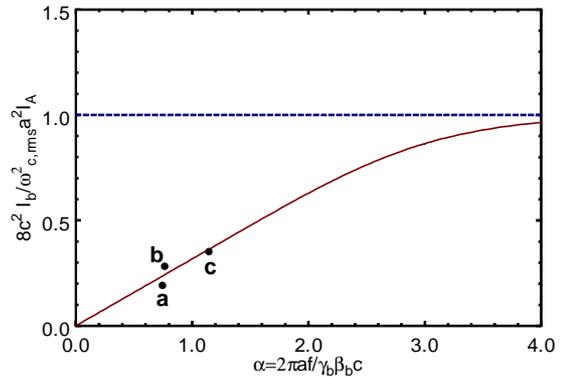


Fig. 2 Plot of the maximum value of the self-field parameter (solid curve),  $8c^2 I_b / \omega_{c,rms}^2 a^2 I_A$ , for bunched beam confinement as a function of the parameter  $\alpha = 2\pi af / \gamma_b \beta_b c$ . Shown in letters are the operating points for three PPM focusing klystrons: a) 50 MW XL-PPM, b) 75 MW XP, and c) Klystrino. The dashed line denotes the Brillouin density limit for an unbunched beam.

Equation (6) approaches the limit of  $2\omega_p^2 / \omega_{c,rms}^2 \leq 1 - 8\pi^3 a^2 \gamma_b^{-2} L^2 e^{-4\pi a / \gamma_b L}$ , and recovers the Brillouin density limit [8] for PPM focusing. However, the more relevant limit for high-power klystrons is when the bunch spacing is much larger than the pipe radius, i.e.,  $\alpha \ll 1$ . Numerical analysis shows that equation (6) becomes  $2\omega_p^2 / \omega_{c,rms}^2 \leq 2a / \gamma_b L$ , which is much less than the Brillouin density limit.

## 4 EXPERIMENTAL APPLICATIONS

We now apply the beam confinement condition in Eq. (6) to three recent PPM focusing klystron experiments at SLAC, namely, the X-band 50 MW XL-PPM and 75 MW XP klystrons [4,9] and the W-band Klystrino [10]. The parameters for all three klystrons are listed in Table 1, and their operating points are marked with letter a, b and c in Fig. 2, respectively. The X-band klystrons were designed and tested for the NLC, whereas the W-band klystrino was designed for sub-millimeter radar applications. As shown in Fig. 2 and Table 1, all three klystrons operate in the regime of  $\alpha \ll 1$  and near the self-field parameter limit. Because the 50 MW klystron operates slightly below the confinement limit, a mild beam loss still occurs in this device [4] through beam halo formation as reported previously [11,12]. The 75-MW XP is operating outside of the confinement limit. This suggests that the 75 MW klystron has greater beam loss than its 50 MW counterpart, which is consistent with more pronounced X-ray emissions measured at the output section of the device [9]. The Klystrino design parameters fall just inside of the theoretical limit, suggesting a marginally stable beam-wall interaction. There are two methods of avoiding the space-charge limitation posed by Eq. (6). One method is to increase the magnetic field by using a hybrid combination of PPM and external solenoidal fields. Another technique is to increase the beam tunnel radius, but this may lead to unwanted mode competition.

## 5 SUMMARY

To summarize, we presented a center-of-mass model for a tightly bunched electron beam in a periodic permanent magnet (PPM) focusing klystron. By analyzing the Hamiltonian dynamics of a train of collinear periodic point charges interacting with a conducting drift tube, an rf field, and an applied PPM focusing field, we derived a space-charge limit for the radial confinement of lightly bunched electron beams, which is significantly below the well-known Brillouin density limit for a continuous beam. We found that several state-of-the-art PPM klystrons developed at SLAC operate close to this limit, thereby shedding some light on the origin of observed beam losses. A further study of PPM confinement, which includes multi-particle simulations in each bunch, is needed to make a more accurate estimate on the amount of beam loss in klystrons.

Table 1. Parameters for SLAC PPM Focusing Klystrons

	50 MW XL-	75 MW XP	KLYSTRINO
	PPM		
$f$ (GHz)	11.4	11.4	95
$I_b$ (A)	190	257	2.4
$\gamma_b$	1.83	1.96	1.22
$B_{rms}$ (T)	0.20	0.16	0.29
$a$ (cm)	0.48	0.54	0.04
$\alpha$	0.75	0.77	1.15
$\frac{8c^2 I_b}{\omega_{c,rms}^2 a^2 I_A} \Big _{exp}$	0.19	0.28	0.35
$\frac{8c^2 I_b}{\omega_{c,rms}^2 a^2 I_A} \Big _{cr}$	0.238	0.244	0.366

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\*This work was supported by the Department of Energy, Grant No. DE-FG02-95ER-40919. The authors wish to thank D. Sprehn and G. Scheitrum for helpful discussions.