

ADAPTIVE ELECTROMAGNETIC FIELD ANALYSIS FOR AXISYMMETRIC STRUCTURE AND UNIFORM WAVEGUIDE USING THE FINITE ELEMENT METHOD

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Abstract

An analysis of the electromagnetic field based on adaptive finite element is presented in this paper. The performance of the adaptive algorithms, based on an element-element h-refinement technique, is assessed. The features of the refinement indicators, adaptation criteria and error estimation parameters are discussed. The strategy of the adaptive mesh refinement method applied to the eigenvalue problem is studied to improve the accuracy of the eigenvector. Numerical results for pill-box cavity and disc-loaded structure are shown.

1 INTRODUCTION

The strategy of FEM is to divide the solution space into a large number of area or volume elements and derive the linear equations based on the physics problem.

Generally in finite element analysis or other mesh based on method, as the mesh is refined, the accuracy of the solution, as well as its cost, goes up. However, whenever refinement is located in areas where the solution has high error, the increase in accuracy is relatively high than the increase in cost. In adaptive mesh generation, error estimates are used to refine the mesh where the error is higher than an acceptable value and to make coarse mesh where the error is lower than an acceptable value. Adaptive meshing is one of the key research topics being investigated to produce more robust and user-friendly finite element analysis environments in many disciplines. This paper studies the adaptive method applied to the RF cavity or wave-guide in accelerators. After giving a brief summary of the FEM (Finite Element Method), we derive a rigorous posteriori bound on the error estimation and adaptive refinement. Examples are also given of the use of adaptive refinement.

ADAPTIVE STRATEGY

The adaptive refinement procedure is based on the use of two key quantities, evaluated on the basis of a tentative solution: the refinement indicator and the convergence parameter. In addition, an estimate of the error of the solution is evaluated.

The usual continuity assumption used in the field based finite element formulations results in a continuous field from element to element, but a discontinuous field gradient. Therefore, the reasonable error norm of the field for each element can be defined as follows

$$\|e\|_e = \left[\int_{\Omega} (\nabla \mathbf{E} - \nabla \hat{\mathbf{E}})^T (\nabla \mathbf{E} - \nabla \hat{\mathbf{E}}) d\Omega \right]^{1/2}, \quad (1)$$

where \mathbf{E} is the exact field, $\hat{\mathbf{E}}$ is the finite element solution. The actual err norm is calculated from the smoothed values of the element nodal gradient by the recovery process instead of the exact field. In this smoothing process, it is assumed that the approximation quantities are interpolated by the same basis function \mathbf{E} and that they fit the original ones in a least square sense. This method is better than the averaging of the element nodal gradient which is used by ANAYSIS[1].

A more practical representation of the error norm in term of a percentage error is

$$\eta_e = \frac{\|e\|_e}{\|q\|_e} \times 100\%, \quad (2)$$

where $\|q\|_e = \left[\int_{\Omega} \nabla E^T \nabla E d\Omega \right]^{1/2}$.

The maximum permissible error for each element can be calculated from the average of $\|q\|_e^2$ over all the elements $\|q\|_{av}^2$, $\|e\|_e^2 \leq \bar{\eta} \|q\|_{av}^2$, here, $\bar{\eta}$ is the specified maximum value. The $\|e\|_e$ values can be used for adaptive mesh refinement. It has been shown by Babuka and Rheinboldt [2] that if $\|e\|_e$ is equal for all elements, then the model using the given number of elements is the most efficient one. This concept is also referred to as "error equilibration".

We define refinement indicator $\xi_e = \|e\|_e / \|\bar{e}\|_e$, if $\xi_e > 1$, the size of element e must be reduced and the mesh will require refinement, otherwise, the size of element must be increased and the mesh will be coarsened. Thus the predicted size of the new element based on an element-element h-refinement technique can be calculated from the current element size as $\bar{h}_e = h_e / \xi_e^{1/P}$, where \bar{h}_e is the predicted element size, h_e is the current element size and P is the order of the shape functions.

The estimate of the error of the solution can be evaluated as

$$\eta = \sqrt{\frac{\sum_e \|e\|_e^2}{\sum_e \|q\|_e^2}} \times 100\%. \quad (3)$$

The summation in above formula is carried on the all elements.

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3 APPLICATIONS

The numerical examples for the application of the adaptive method are shown in this section. The adaptive meshing procedure is dependent on both the geometry and the field. Therefore, the application for different geometry and field will be shown in the following paragraphs.

The pill-box cavity is the typical example which is often used to check the numerical calculation. The adaptive meshing is easy to applied for circular cylindrical coordinate and the accuracy is high. Instead of circular cylindrical coordinate, we use Cartesian coordinate to study the TM01 mode with adaptive method. Quadrant is used to represent the cavity in order to save calculation time. Figure 1 shows the initial mesh and the adaptive mesh. The curve part is metal boundary and symmetric condition is applied on the two radial boundaries. The number of element, estimated error and accuracy of frequency are shown in table 1. The theory frequency for TM01 with radius 10 cm is 1147.42498187125 MHz. The specified maximum error is 1.5×10^{-4} . The estimated error is almost same as the specified maximum error after the first refinement and is two times smaller than the specified maximum error after the second iteration refinement. The estimated error is bigger near the circular boundary, which can be shown from figure 1(c). The accuracy of frequency is about 7.3×10^{-7} with 4112 adaptive refinement elements. However, the frequency accuracy is only 2.75×10^{-5} with 6587 uniform meshed elements. Therefore the adaptive meshing method is very useful to improve the accuracy and save calculation time.

Table 1 refinement results for pill box with radius 10cm

refinement iteration	number of element	estimated error	accuracy of frequency
0(initial mesh)	45	1.358×10^{-2}	2.730×10^{-3}
1	2717	1.594×10^{-4}	3.018×10^{-6}
2	4112	6.361×10^{-5}	7.302×10^{-7}

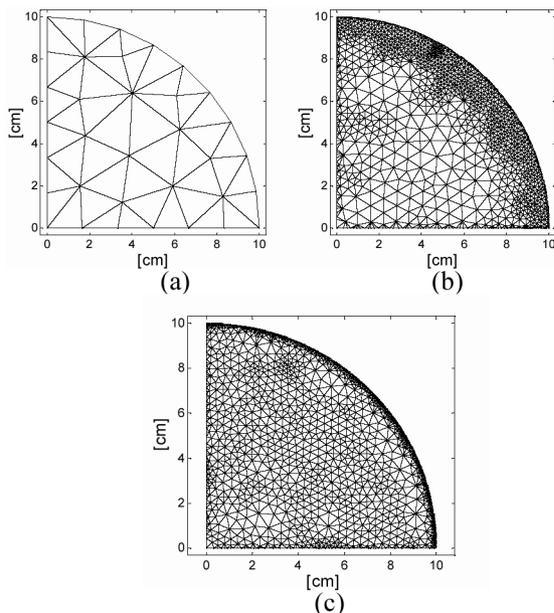


Figure 1 Adaptive mesh for pill box (a: initial mesh; b: first refinement mesh; c: second refinement mesh)

The estimated error by formula (3) is quite different from the field error or the frequency error. An error of 1.5×10^{-4} by formula (3) roughly correspond to an error of 2.25×10^{-8} in the field.

Many accelerator structures and other microwave structures used in accelerator are axisymmetric. The eigenmode equation for the axisymmetric structures is usually written in the cylindrical coordinates R , ϕ and Z . For the disc-loaded structure with round iris, the adaptive mesh is easily to be implemented. Figure 2 shows the initial mesh and the first iteration adaptive mesh which gives the estimated error 4.419×10^{-4} . The structure has parameters: $t=5.842\text{mm}$, $D=34.99\text{mm}$, $b=40.989\text{mm}$, $a=10.363\text{mm}$ and $\rho=2.921\text{mm}$. The specified maximum error is 5×10^{-4} and the frequency of the TM01 mode is 2838.419946 MHz by FEM and 2838.419471 MHz by *LONGTRANSVRS*[3]. The frequency accuracy is about 1.673×10^{-7} .

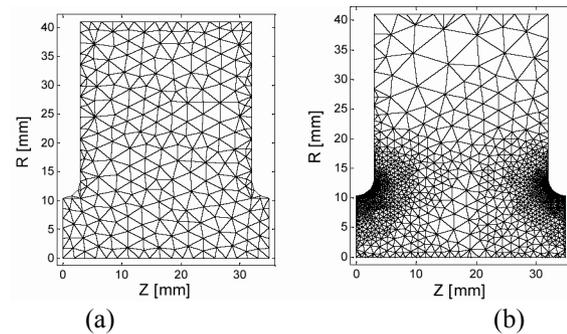


Figure 2 Adaptive mesh for disk-loaded structure (a: initial mesh; b: first refinement mesh)

The efficiency of the adaptive mesh is mainly decided by the iterations. In general, the initial mesh always has small number of elements and the goal of the error can be reached after first refinement. Therefore, our adaptive mesh is efficient.

While adaptive refinement methods are well accepted in the solution of the Poisson equation (electrostatic potential for electrostatic field and partial scalar potential magnetostatic field), little work seems to be done in solving the wave equation. Some significant differences from the Poisson equation exist. For accelerating structure, the discretized globally built finite element matrix equation takes the form

$$A\mathbf{E} = \lambda B\mathbf{E} \quad (4)$$

where λ represents the many eigenvalues of eq.(4), which have directly corresponding relationships to the frequency of the modes.

There is a different eigenvector \mathbf{E} paired with each eigenvalue. The error for eigenvector \mathbf{E} is estimated in eq.(1). Different eigenvector should have different error. Therefore, the adaptive refinement is carried out based on one of the eigenvectors. Figure 3 shows the refinement

mesh based on different eigenvector. The local error for each eigenvector is different. A peculiarity of the cavity analysis is that we are usually not interested in all modes of a cavity, but only in the dominant mode corresponding to the lowest eigenvalue or the second eigenvalue for design reasons. Therefore, we can apply adaptive refinement based on this dominant mode. However, if the high order modes are cared, the adaptive method must be applied on the high order modes. Figure 4 shows the accuracy of the first eight TM modes by refinement based one different eigenvectors (IMOD in the figure). When the refinement is based on the first eigenvector as shown in figure 2, the accuracy is high for the first eigenvalue. However, it is lower for all high order modes (IMOD=1 in figure 4). In general, the field pattern for the lower order modes is simple (uniform), the local error is distributed at small parts of the domain. Therefore, the result based on such error is not good for high order eigenvectors. Figure 4 shows that the refinement based on the 4th eigenvector is best for all eigenvalues.

Certainly, good results can be obtained by applying the adaptive refinement based on one field at one time. However, it will take much time to calculate many fields. In order to get better results for all eigenvectors at the same time, we can apply the adaptive refinement based on all interested eigenvectors. The green circle curve in figure 4 shows the result based on the all first eight eigenvectors. We can see that the accuracy for all eigenvectors is much better than other case. For the second mode, the accuracy reaches 8.0×10^{-8} . The estimated error of all modes is less than the specified maximum error 8×10^{-4} .

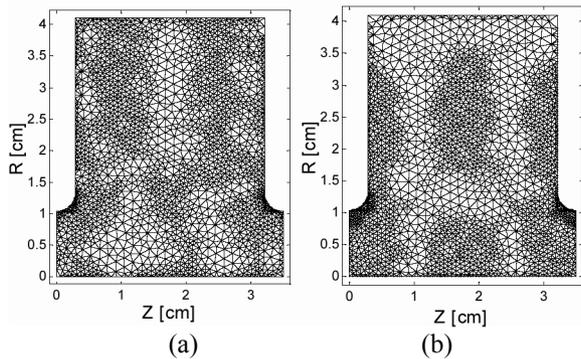


Figure 3 refine mesh based one different eigenvector
 (a) adaptive refinement base on 6th eigenvector
 (b) adaptive refinement base on 8th eigenvector

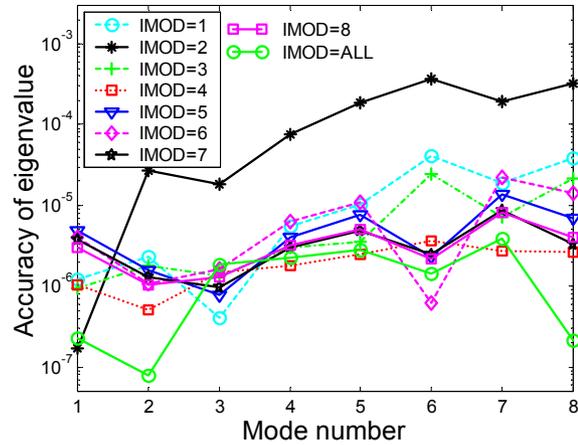


Figure 4 Accuracy of the first 8 TM modes based on different eigenvectors with specified maximum error 8×10^{-4}

4 SUMMARY AND CONCLUSIONS

The adaptive refinement method can be successfully applied to eigenvector problem in the electromagnetic field analysis. The refinement based all interested eigenvectors will greatly help the improvement of all the interested modes.

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