

SOLITON-LIKE LONGITUDINAL HOLES IN DEBUNCHED BEAMS PERPETUATED BY SPACE-CHARGE FORCES

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Abstract

Because the longitudinal space-charge forces within a local rarefaction of the particle density in phase space are focusing toward the centre of the perturbation, holes in beams might be expected to be self stabilized below transition energy. We shall report measurements[7, 8, 9] made at the CERN PS Booster (PSB) and computer simulations that demonstrate the surprising longevity of such perturbations on an unbunched charged-particle beam. The negative mass instability is not an explanation for this persistent beam structure and the usual wave theory for perturbations on unbunched beams (the Keil-Schnell criterion) cannot explain their stability. The holes are solutions of the non-linear Vlasov-Poisson equation for which we have found self-consistency and stationarity conditions; these have scaling-laws similar to the Keil-Schnell criterion.

1 INTRODUCTION

Localized longitudinal density perturbations on an unbunched charged-particle beam, have recently aroused interest. Colestock[1] and Spentzouris[2] have reported experimental observations and Schamel[4, 5, 6] has made theoretical studies and claims that the perturbations are soliton-like and cannot be predicted by the linearized Vlasov treatment used by Keil[3] and Schnell; we agree with this claim.

1.1 CERN PS Booster Measurements

The PS Booster is a proton synchrotron cycling between 50 MeV and 1.4 GeV kinetic energy. The machine operates with a harmonic $h = 1$ fundamental rf system and in addition there is an $h = 2$ rf system and a high harmonic cavity capable of generating harmonics from $h = 12$ to $h = 26$ at the injection energy. The Booster may also be operated in storage ring mode at 50 MeV. Beam is injected over a few turns (from 0.1 to 12) from a 200 MHz linac.

A wide-band beam pick-up and a fast digital oscilloscope with deep memory are available for recording bunch shapes turn-by-turn. The data may be graphed as a waterfall display: a "bird's-eye" view of the conventional mountain-range display. A grey scale, from black to white, denotes relative particle density from most to least dense. The instrument and software is called the "tomoscope".

Linac bubbles The tomoscope was used to observe the debunched beam after injection. Beam structure which we refer to as "bubbles", may survive for at least 50 ms. This

longevity cannot be ascribed to narrow momentum width: no appreciable debunching of the bubbles implies an energy spread less than 5 eV, which is so small that the bubbles would be undetectable. The bubbles become more prominent at higher intensity (figure 1).

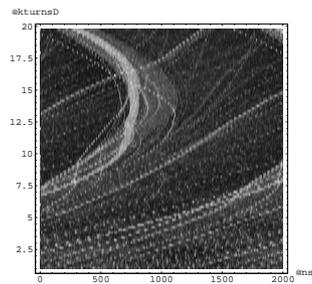


Figure 1: 3.6×10^{12} protons, 1-trace each 200-turns, vertical span 31.7 ms.

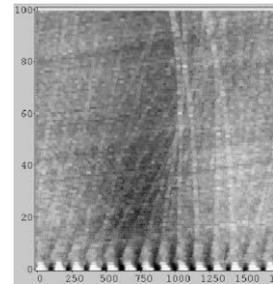


Figure 2: 2.4×10^{12} protons, 1-trace/20-turns, span 3.3 ms; abscissa ns.

1.2 Periodic high-harmonic holes

With the fundamental rf turned off, the high harmonic cavity can be used to introduce periodic holes with known properties. For example, $h = 13$ holes having an energy spread of some 190 keV were introduced into a beam of 3×10^{12} protons. With the $h = 13$ cavity switched off once the empty buckets were introduced, the top and bottom of the resulting holes should shear by a machine circumference each milli-second. No such shearing was detected.

1.3 Momentum steering of holes

Because the longitudinal space-charge force is proportional to the derivative of the line-current density, there is no force between holes until they contact. However, if there are differences in their central momenta, then holes will move relative to one another. To test this idea, one impresses a $h = 1$ coherent energy modulation on the holes (using small voltage of short duration). Figure 2 shows a waterfall display after ± 60 keV modulation at $h = 1$; the holes' tracks converge in response to the modulation; and though some holes collide none coalesce.

1.4 Momentum spread versus beam current

A simple theoretical model (section 1.5) predicts a definite relation between the momentum spread of the holes and the beam current, because of a fine balance between shearing and focusing. The Booster experiments were

adapted to test the theory using $h = 18$ holes. In figure 3, the tendency for holes to shear due to their momentum spread is well balanced against the focusing provided by space-charge forces and the tracks stay sharply defined. In figure 4, the momentum spread of the holes has been enlarged and this enhances the shearing effect. Comparison of figures 4, 5 and 6, where the beam current is progressively increased, shows that space-charge focusing on holes can overcome the shearing due to momentum spread, leading to narrower and more self-sustained holes.

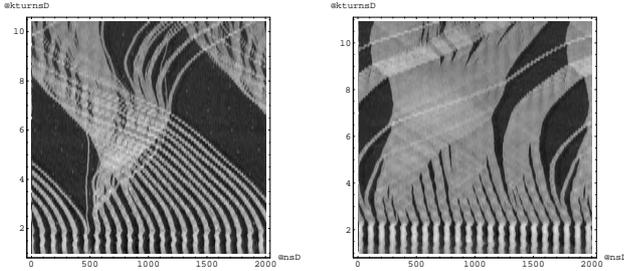


Figure 3: 1.8×10^{12} protons, 1-trace each 100-turns, span 16.7 ms, $V_{rf} = 0.3$ kV.

Figure 4: 1.8×10^{12} protons, 1-trace each 100-turns, span 16.7 ms, $V_{rf} = 0.6$ kV.

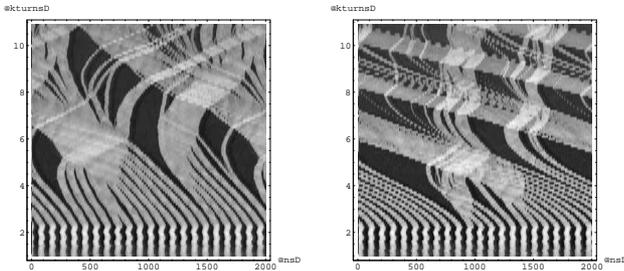


Figure 5: 2.4×10^{12} protons, 1-trace each 100-turns, span 16.7 ms, $V_{rf} = 0.55$ kV.

Figure 6: 3.2×10^{12} protons, 1-trace each 100-turns, span 16.7 ms, $V_{rf} = 0.6$ kV.

1.5 Simple theoretical model

Suppose that space charge is the dominant longitudinal source of impedance. Let g_0 be the geometrical coupling parameter which depends on the ratio of beam and vacuum pipe transverse radii. Let p and q be canonical momentum and position. The Hamiltonian is of the form $H = p^2/2 + Z\lambda(q)$ where $Z > 0$ is the space-charge coupling constant and λ is the line density. Self-consistency of a rectangular void of momentum width $2\hat{p}$ and length $2b$ radians, leads to the equilibrium condition:

$$\left(\frac{\hat{p}}{P_s}\right)^2 = \frac{g_0}{2\pi\epsilon_0} \left(\frac{e}{\beta_s}\right)^2 \frac{1}{E_s} \frac{N}{R_s} \frac{1}{[(\gamma_s/\gamma_t)^2 - 1]} \times \frac{f_{\text{empty}}}{f_{\text{filled}}} \frac{1}{2b}. \quad (1)$$

Here E_s, P_s are energy and momentum, $\beta_s c =$ speed, $R_s =$ ring radius, $N =$ number of particles, f_{empty} and f_{filled} are the fractions of empty and filled phase space.

2 COMPUTER SIMULATIONS

Computer simulations were made using the LONGID tracking programme[10]. Two types of system were stud-

ied: (i) a periodic array of holes as the result of depositing $h = 15$ empty rf buckets; and (ii) the behaviour of one or two rectangular holes. Example GIF animations may be found on the world-wide-web[11].

2.1 Periodic holes

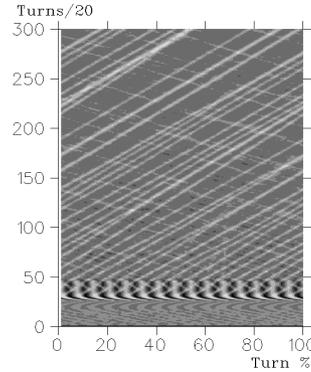


Figure 7: 4×10^{12} protons, 10^6 macro-particles.

Simulations were made in which the beam was increased from zero to 4×10^{12} protons. The zero intensity case shows the holes to debunch in ≤ 0.5 ms. For 2×10^{12} protons, the holes leave obvious tracks, though space-charge is not strong enough to prevent the holes from widening. For 4×10^{12} protons (figure 7) the higher space-charge forces provide better focusing and the holes do not widen.

2.2 Single rectangular holes

Simulations were made for the cases of beam current one half and double the value required for stationarity. In the former, the hole debunches because the shearing due to momentum spread overcomes the space-charge focusing. In the latter (figure 8) the hole quickly breaks up into two smaller holes which thereafter appear stationary; the momentum spread is unaltered, but the charge in each is halved and so the stationarity condition is restored.

2.3 Collision of two holes

The case of two initially matched holes prepared with slightly different central momentum and initially separated in azimuth is shown in figure 9. The holes encounter and slide past one another. They do not coalesce.

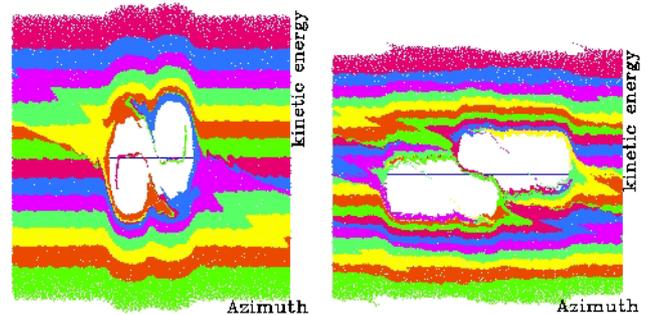


Figure 8: 10^{13} protons, 2.8×10^5 macro-particles.

Figure 9: 5×10^{12} protons, 2.2×10^5 macro-particles.

3 VLASOV EQUATION

We now examine the question of whether stationarity conditions such as (1) can be obtained from solution of the time-independent Vlasov equation. Let ψ_0 be a steady state density function and ψ_1 some perturbation.

3.1 Keil-Schnell type condition

Let Z represent the space-charge coupling to $\partial\lambda/\partial q$. The Vlasov equation is linearized and the time derivative set to zero, leading to

$$\dot{p}(\psi_1)\frac{\partial\psi_0}{\partial p} + p\frac{\partial\psi_1}{\partial q} = 0, \quad \frac{dp}{dt} = -Z\frac{\partial}{\partial q}\int_{-\infty}^{+\infty}\psi_1(p,q)dp. \quad (2)$$

Hence the Keil-Schnell-type dispersion relation (for zero modulation frequency) is:

$$Z\int_{-\infty}^{+\infty}\frac{1}{p}\frac{\partial\psi_0}{\partial p}dp = +1. \quad (3)$$

The PV integral is negative/positive if ψ_0 is locally a maximum/minimum. Thus for capacitive impedance ($Z > 0$), an inverted bell ψ_0 is required to perpetuate a perturbation below transition energy. Despite this prediction, the observed stable holes in the PSB debunched beam occur for a quasi-parabolic distribution with $\int(\partial\psi_0/\partial p)dp/p < 0$.

3.2 Whether or not to linearize

When linearizing the Vlasov equation, it is customary to state that the perturbation ψ_1 is small compared with the steady state function ψ_0 and that the product $\dot{p}\psi_1$ can be neglected. However, the Vlasov equation is not concerned with the absolute amplitude of functions, but rather it is obsessed throughout with derivatives. Thus it is not the relative size of ψ_0 and ψ_1 that matters, but rather the relative size of their derivatives. For the case of a very localized perturbation the derivatives of ψ_1 may be overwhelmingly large compared with those of ψ_0 . In this case, the perturbation decouples (almost) completely from the steady-state.

3.3 Dispersion relation

The stationary ψ_1 obeys equation (2) but with ψ_1 substituted in place of ψ_0 . The dispersion relation is analogous to (3) but with ψ_1 substituted in place of ψ_0 . Evidently, ψ_1 must be an inverted bell-shape, that is a *hole*, for the dispersion relation to be fulfilled (assuming $Z > 0$ and below transition). The number of particles involved is $n = \int\psi_1 dpdq$. The dispersion relation differs in two respects from Keil-Schnell: whereas the latter contains the full beam current ($\propto N$) and the full momentum spread of the beam, the former contains only the current in the hole (proportional to n) and the momentum width of the hole.

3.4 $\psi_1 = P(p, t)Q(q, t)$

We consider a more general impedance $R + iZ$ with resistive and reactive parts and $i = \sqrt{-1}$. Let σ be a range parameter. We substitute the trial solution

$$\psi_1 = \frac{e^{-(p-\gamma t)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}\lambda_1(q - \gamma t^2/2), \quad (4)$$

into the time-dependent Vlasov equation. One finds the average momentum loss rate $\gamma = -R\int\lambda_1^2(q)dq/\int\lambda_1(q)dq$,

and the residual

$$Z\frac{\partial\lambda_1}{\partial q}\frac{1}{\sigma^2}\lambda_1(q) + \frac{\partial\lambda_1}{\partial q} = 0. \quad (5)$$

Hence λ must be the inverted ‘‘top hat’’ function within some range of $|q| \leq \Delta q$ and zero outside that range. $\lambda_1(q)$ is a negative constant: $\lambda_1 = -\sigma^2/Z$. This condition implies a very strict relation between the momentum width σ of a hole and its depth λ_1 , such that progressively narrower holes ($\sigma < 1$) become very shallow and hard to detect.

Despite the decoupling of ψ_0 and ψ_1 , the steady state distribution does impose some constraints on the perturbation. Because $\psi_0 \geq 0$, the phase-space density must be high enough to support the perturbation. Thus for a hole of given momentum width, there is a critical density below which ψ_0 cannot act as a transport medium for solitons. The density ψ_1 is of order $n/(4\sigma\Delta q) = -\sigma/(4Z)$. The density ψ_0 is of order $N/(4\pi\Delta p)$ where Δp is the r.m.s. width of ψ_0 . Hence the critical density is $N/(\pi\Delta p) \geq \sigma/Z$.

4 CONCLUSION

There is much experimental evidence at the CPS Booster of near-stationary longitudinal voids introduced accidentally by the linac or deliberately by high-harmonic rf. There is strong supporting evidence from computer simulations and theoretical analysis that the holes are perpetuated by space-charge forces. We have shown that the usual Keil-Schnell criterion does not explain the voids, but rather suggested they decouple from the background distribution. We have derived conditions for stationarity of holes that satisfy the requirement of self-consistency – with the caveat that the phase-space density be high enough to support the solitons. However, we have not determined stability. A more complete discussion of these topics is given in Ref.[12]

5 REFERENCES

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