BEAM LOADING AND PHASE MOTION OF PARTICLES IN THE SELF-CONSISTENT RF FIELD OF LINAC

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Abstract

The dynamics of high intensity electron beam in the travelling wave structure is studied. The equation for self –consistent RF field are devised to the more general case in which the series resistance, wave phase velocity and damping coefficient are slowly varying functions of the longitudinal co-ordinate. Stationary solutions of nonlinear equations for RF field amplitude, for bunched beam energy gain, and phase are found. The recommendations to optimise the efficiency of low energy electron accelerator are done.

1 INTRODUCTION

In the design of high-intensity linear electron accelerators it is important to maximize the efficiency and minimize the energy spread of the accelerated beam. A correct study of the dynamics of an intense low energy beam in a waveguide requires to consider both the change of the particle velocity and the change of the amplitude and the phase velocity of the wave which interacts with the beam in the accelerated structure. This nonlinear task must be solved in a self-consistent manner. In Ref. [1] this nonlinear problem was solved for the case of uniform waveguide structure; the conditions for optimizing a structure parameters in terms of the efficiency are found there. The basic assumption used in Ref. [1] is that the beam can be treated as a train of well-grouped (point) bunches. It was shows that if the bunches have a finite phase dimension θ (a relative value of the first current harmonic I_1=2) the energy gain is essentially independent of θ. But the phase of the bunch particles ψ in the self-consistent field must be larger than zero in the initial acceleration region.

It should be noted that the use of uniform structure is limited at high currents because of the BBU effect and the low acceleration efficiency. For this reason, non-uniform waveguides should be used in high-intensity accelerators. Our purpose in the present work is to generalize the results of Ref. [1] obtained for uniform structure, to non-uniform waveguides. The assumptions used in Ref. 1 are legitimate for an analysis for the optimization of non-uniform sections in terms of the efficiency of these sections. We assume that the parameters of the waveguide section are such that the series resistance $R_n(z)=E^2/2P$ and the wave phase velocity in the absence of the beam, β_{ph}, and the damping coefficient, $w=\alpha\lambda$, are functions of the longitudinal co-ordinate $\xi=z/\lambda$ and vary smoothly.

2 SELF-CONSISTENT EQUATIONS OF PARTICLE MOTION

The system of equations for the average energy of the particles, $\gamma = W/mc^2$, for the dimensionless amplitude, $A = e\lambda E/mc^2$, and a phase of the RF field ψ is :

$$\frac{d\gamma}{d\xi} = A\cos\psi\,, \tag{1}$$

$$\frac{dA}{d\xi} + w_1 A = -2B(\xi)\cos\psi\,, \tag{2}$$

$$\frac{d\psi}{d\xi} = 2\pi\left(\frac{1}{\beta_{ph}} - \frac{1}{\beta}\right) + \frac{2B}{A}\cdot\sin\psi\,. \tag{3}$$

The last two equations can be derived by using the Vainshtein method [2], generalized to the case of an inhomogeneous structure [3]. In this case $w_1 = w - \frac{d\ln R_n}{2d\xi}$ and the dimensionless parameter $B(\xi) = \frac{eJ_0 R_n(\xi)}{2mc^2}\lambda^2$ determines the coupling of the beam with the wave in the accelerating structure with a positive dispersion (β_g>0). In equation (3) a phase changes due to dynamic slipping (first item) and reactance beam loading (second one).

Eliminating the phase ψ for Eqs. (1) and (2) we find an equation relating the amplitude of the self-consistent field, $A(\xi)$ and beam energy gain in the section:

$$\frac{A^2(0)}{4B(0)} - \frac{A^2(\xi)}{4B(\xi)}e^{2X(\xi)} = \int_{\gamma_0}^{\gamma} d\gamma(\xi_1)e^{2X(\xi_1)}\,;\; X = \int_0^{\xi} w d\xi_1\,. \tag{4}$$

This equation expresses conservation of energy. With pronounced beam loading, in which case the damping at the walls can be neglected, we find:

$$\frac{A^2(0)}{4B(0)} - \frac{A^2(\xi)}{4B(\xi)} = \gamma(\xi) - \gamma(0)\,. \tag{4a}$$

3 CONSTANT GRADIENT STRUCTURE

To find the absolute value of $\gamma(\xi)$ and $A(\xi)$, we must solve system (1)-(3) for given functions B(ξ), $\beta_{ph}(\xi)$ and $w(\xi)$. Let us examine the solution of system (1)-(3) for structure with a slight non-uniformity, in which ohmic damping is cancelled by the change in the geometric dimensions, so that the field amplitude remains constant

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in the absence of the beam (a "constant gradient structure").

It follows from Eq. (2) that in this case the function $B(\xi)$ is related to $X(\xi)$: $B(\xi) = B_0\, e^{2X(\xi)}$.

At first, we assume that dynamic slipping of the phase is small in a section ($\beta=\beta_{ph}$). Integrating (2) and (3) with the initial conditions $A(0)= A_0$, $\psi(0)=\psi_0$, we find

$$\left[A_0^2 - 4\int_{\gamma_0}^{\gamma} B\,d\gamma\right]^{1/2} \sin\psi = A_0 \sin\psi_0 \,. \qquad (5)$$

If $\psi_0 = 0$ at injection, the phase of the bunch in the self-consistent field is a constant, equal to zero. This case is of no practical interest for low electron energies, since the phase stability of the particles in the bunch is disrupted, and the energy spread increases rapidly. We assume that the initial phase is $\psi_0 > 0$; then, according to (5), $\psi(\xi)$ is a monotonically increasing function. At $\psi=\pi/2$, the beam energy reaches a maximum value. The output field amplitude here is minimal, being given by $A(\xi_k) = A_0 \sin\psi_0$.

In the simplest case, $w = cte$, the solution of (1) can be writen

$$\Delta\gamma = \gamma - \gamma_0 = \left(A_0 \cos\psi_0 + \frac{B_0}{w}\right)\xi - \frac{B - B_0}{2w^2}\,. \qquad (6)$$

Correspondingly, the maximum efficiency of the section is

$$\eta_{max} = \frac{4B_0}{A_0^2}\left(\gamma_{max} - \gamma_0\right). \qquad (7)$$

For a large current, we have

$$\eta_{max} = \cos^2\psi_0 . \qquad (8)$$

The efficiency of the accelerator section can be increased only by reducing the phase of the bunch in the self-consistent field. As already mentioned, however, for acceleration near $\psi=0$ the energy spread of the beam is degraded; this degradation is more pronounced, the lower the particle energy and the higher the RF field. Accordingly, the injection should always be carried out near $\psi=\pi/2$. In practice, this situation can be arranged by injecting the beam into a section with $\beta_{ph}>\beta_0$. Then even at large values of ψ_o the "dynamic slip" of the beam with respect to the wave makes it possible to achieve $A\approx 0$ and $\psi=0$.

4 EFFECT OF DYNAMIC SLIP

As an example we consider a section with $\beta_{ph}=cte$. Equation (7) is now replaced by

$$\left[A_0^2 - 4\int_{\gamma_0}^{\gamma} B\,d\gamma\right]^{1/2} \sin\psi = f(\gamma) - f(\gamma_0) + A_0 \sin\psi_0 \,, \qquad (9)$$

where $f(\gamma)=2\pi(\gamma/\beta_{ph}-\beta\gamma)$.

Eliminating the phase from (1) and (9) we find an equation for γ:

$$\left(\frac{d}{d\xi}\gamma\right)^2 + 2U(\gamma) = A_0^2 \,, \qquad (10)$$

$$U = \frac{1}{2}\left\{f(\gamma) - f(\gamma_0) + A_0 \sin\psi_0\right\}^2 + 2\int_{\gamma_0}^{\gamma} B\,d\gamma \,.$$

This equation has the same form as that derived in [1] for the case B = *cte*. If the amplitude A and phase ψ are to approach zero following acceleration, the following two conditions must hold:

$$A_0^2 = 4\int_{\gamma_0}^{\gamma_{max}} B\,d\gamma \,, \qquad (11)$$

$$f(\gamma_{max}) - f(\gamma_0) + A_0 \sin\psi_0 = 0 \,. \qquad (12)$$

If one can find γ_{max} and substitute it into (12), we can find the value of the phase velocity at which the efficiency is maximized. However, the section cannot be optimized in terms of the efficiency for all values of β_{ph} found in this manner (these values are functions of ψ_o, as follows from (12)). The reason is that for small values of γ and with $\beta_{ph}< 1$ the equation $H_1 - 2U(\gamma) = 0$, where $H_1 = A_0^2$, can have two additional roots, which lie between γ_o and γ_{max}. In this case the maximum beam energy is γ_1, rather than γ_{max} (Fig. 1). For a given injection energy γ_o and a given input RF power there exists a limiting value $\beta_{ph,n}$ above which these intermediate roots do not exist.

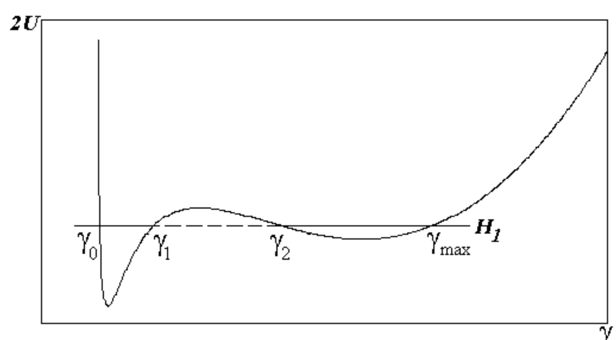

Figure 1: General form of the function $2U(\gamma)$

To determine the basic features of the beam acceleration in an optimised section we consider the phase trajectories in the (γ,ψ) plane for the case $w = 0$ (Fig. 2), when the particles are injected at various initial phases. We see from this figure that there are two kinds of phase trajectories. In one case, with $\beta_{ph} < \beta_{ph,n}$, the beam energy $\gamma<\gamma_{max}$ (the trajectories are nonclosed). If, on the other hand $\beta_{ph} > \beta_{ph,n}$, energy reaches γ_{max} and $\eta\approx 1$(the trajectories are closed). The final choice of an optimum version depends on the requirements imposed on the output properties of the beam, primarily on its energy spread.

5 OPTIMISATION FOR OTHER NON-UNIFORM SECTIONS

At section with high intensity beam slight non-uniformity cannot balance the beam loading in order to increase the gradient. Accordingly, we now assume that $B(\xi)$ and $w(\xi)$ are arbitrary independent functions of ξ.

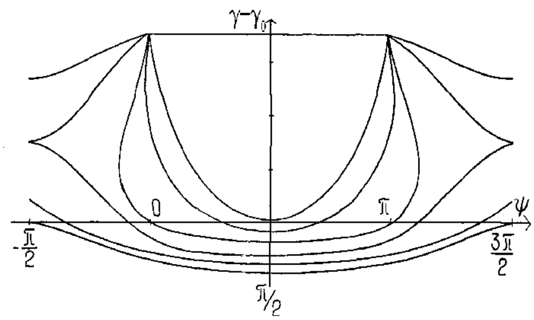

Figure 2: Phase trajectories in the γ,ψ plane for sections which are optimized in terms of the efficiency

If dynamic slipping of the phase is small ($\beta\approx\beta_{ph}$), the maximum efficiency of such a section is governed by the initial injection conditions and depend on the nature of the function $B(\xi)$. Specifically, for high intensity beam, calculating the section efficiency, we find:

$$\eta_{max} = \frac{4B_0}{A_0^2}(\gamma_{max}-\gamma_0) = \cos^2\psi_0. \qquad (13)$$

Accordingly, as in a section with a "constant gradient structure", the quantity η can be increased by reducing ψ_o. Another way to increase the efficiency without degrading the energy spread of the beam is to use sections with $\beta_{ph}>\beta$. This circumstance can be demonstrated in a straightforward manner by using (2) and (3) to express the amplitude and phase of the self-consistent field in terms of the quantity $Y(\xi) = \int_0^\xi \mu d\xi_1 = 2\pi\int_0^\xi (1/\beta_{ph}(\xi) - 1/\beta)d\xi_1$ and $X(\xi)$, i.e., in terms of the total dynamic slip of the bunch and wave damping:

$$A = \left(A_c^2 + A_s^2\right)^{1/2} e^{-X(\xi)},\ \psi = Y + \psi_0 - \mathrm{arctg}\frac{A_s}{A_c}, \qquad (14)$$

where

$$A_c = A_0\sqrt{\frac{B}{B_0}} - 2\sqrt{B(\xi)}\int_0^\xi d\xi_1\sqrt{B(\xi_1)}e^{X(\xi_1)}\cos(Y(\xi_1)+\psi_0),$$

$$A_s = 2\sqrt{B(\xi)}\int_0^\xi d\xi_1\sqrt{B(\xi_1)}e^{X(\xi_1)}\sin(Y(\xi_1)+\psi_0)\ .$$

The efficiency can be approximately unity even at large value of ψ_0, provided that A_c and A_s vanish simultaneously

$$A_0 = 2\sqrt{B_0(\xi)}\int_0^{\xi_f} d\xi_1\sqrt{B(\xi_1)}e^{X(\xi_1)}\cos(Y(\xi_1)+\psi_0), \qquad (14a)$$

$$\int_0^{\xi_f} d\xi_1\sqrt{B(\xi_1)}e^{X(\xi_1)}\sin(Y(\xi_1)+\psi_0) = 0\ . \qquad (14b)$$

In the simplest case B = *cte*, w=*cte* the slip is linear function of ξ. The slip rate μ can be expressed in terms of the working frequency: $\mu = \frac{2\pi}{\beta_{gr}}\frac{\omega-\omega_0}{\omega_0}$. Here ω_o is the nominal frequency at which the phase velocity in the system without the beam is unity and the group velocity $\beta_g\neq0$.

In the low damping limit, using (14a) we have $\xi_f = -2\psi_0/\mu$. Substituting this value into (14b) we can find optimal slip rate:

$$\mu = \frac{4B_0}{A_0}\sin\psi_0. \qquad (15)$$

The same result can be found by using Eq.(12) and setting $\gamma_{max}=\gamma_0+A_0^2/4B_0$.

6 CONCLUSION

The self-consistent equations of particle motion in the travelling wave non-uniform structure has been solved. The recommendation for choice of the phase velocity β_{ph} and the coupling parameter $B(z)$ to optimise the efficiency of the electron acceleration and to reduce energy spread was been given.

7 REFERENCES

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