

# STUDY ON BEAM STEERING IN INTERMEDIATE- $\beta$ SUPERCONDUCTING QUARTER WAVE RESONATORS

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## Abstract

Superconducting quarter wave resonators, due their compactness and their convenient shape for tuning and coupling, are very attractive for intermediate- $\beta$  beam acceleration up to about 100 MeV/u. A drawback of this kind of cavities is the beam steering caused by transverse magnetic and electric field components; this can create emittance growth and beam spill, especially in high intensity proton linacs. We did analytical and numerical studies on beam steering in quarter wave resonators and, in particular, in different 352 MHz quarter wave geometries for  $\beta=0.15-0.45$  beams. The results suggest that, in this case, the solution of alternative cavity orientation could nearly cancel the steering.

## 1 INTRODUCTION

Superconducting linear accelerators in the energy range from 5 MeV up to about 100 MeV are being widely studied in many laboratories for acceleration of protons and heavy ions. The main advantages of using short, independently phased superconducting cavities, instead of large, normal-conducting DTLs, are the high accelerating gradient, the high efficiency and the possibility of accelerating particles with different  $q/A$  in the same linac. Different geometries, like Spoke, Half-wave (HW), Quarter-wave (QWR) and Reentrant [1][2][3][4] have been proposed for beam velocities up to  $\beta=0.5$ , where multicell type superconducting cavities start to achieve a good efficiency.

Simplicity, accessibility and low construction cost could make QWRs preferable in comparison to other geometries. Pioneering work on intermediate- $\beta$  QWRs has been done in the last decade at Argonne [1]. Superconducting QWRs at frequencies from 80MHz up to 240 MHz have been developed at LNL [5]; intermediate beta QWRs can be designed and constructed with the same technology used successfully for low- $\beta$  cavities; different optimum velocities can be obtained by simply modifying the resonator length and frequency [6].

A drawback of these cavities is the beam steering due to the lack of symmetry with respect to the beam axis. Both magnetic and transverse electric fields produce steering in the direction of the resonator axis; the beam deflection, moreover, depends on the particle position. This non-homogeneous beam steering is often weak in heavy ion accelerators, where low charge beams are transported and where the beam size is very small compared to the rf wavelength. On the contrary, it can be significant in high frequency QWRs and in the case of

proton or high charge state ion beams, where it can cause emittance growth and beam losses.

## 2 BEAM STEERING IN QUARTER WAVE RESONATORS

In QWRs the drift tube is located near the top of the inner conductor, where magnetic field is weak but still present; in this region, moreover, the electric field contains transverse components. The field distribution, calculated with the ANSOFT HFSS code, is shown in Fig. 1 for a "LNL-type cylindrical" 352 MHz cavity (see below).

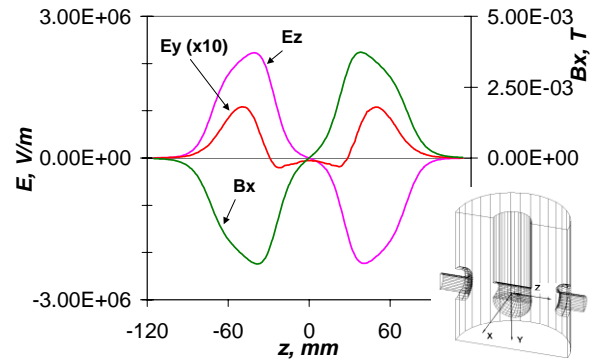


Figure 1: Field distribution along the beam axis in a "LNL type", 352 MHz QWR, at the accelerating field of 1 MV/m.  $B_x$  is displayed with  $90^\circ$  rf phase delay (for clarity,  $E_y$  is magnified in the figure). The origin of the coordinates is at the center of the resonator drift tube (see small figure).

Compared to the accelerating component  $E_z$ , the transverse magnetic one  $B_x$  is shifted by  $90^\circ$  in rf phase and have similar antisymmetric distribution and similar gap-to-gap (barycentre) distance; the distribution of  $E_y$  is different along the gap, and symmetric. All components act to the beam with different strength as a function of beam velocity and rf phase.

A possible source of emittance increase is the change of these field components along the resonator axis,  $y$ . The average values as a function of  $y$  (calculated in one gap and normalized to the accelerating field on the axis,  $\underline{E}_z$ ) are shown in Fig. 2; here  $K_{Ey}(y) = \underline{E}_y(y)/\underline{E}_z$  and  $K_{Bx}(y) = \underline{B}_x(y)/\underline{E}_z$ ,  $K_{Ez}(y) = \underline{E}_z(y)/\underline{E}_z$ .

These components appear to be rather proportional to  $y$  and can be conveniently expressed in a linear form:  $K_{Ey}(y) \approx E_y/E_z + y \partial(E_y/E_z)/\partial y$ ,  $K_{Bx}(y) \approx B_x/E_z + y \partial(B_x/E_z)/\partial y$ .

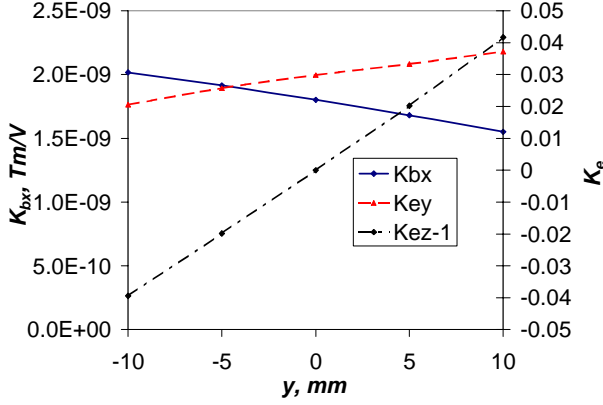


Figure 2: Average field components along the resonator axis, normalized to the accelerating field on the beam axis.

A simple analytical expression of the beam steering caused by transverse field components, based on the homogeneous gap and constant velocity approximations, is shown in Eq. 1.

$$\Delta y' = -\frac{\Delta U}{\gamma mc^2 \beta} \tan \varphi \left( \frac{\cos\left(\frac{\pi d_y}{\beta \lambda}\right)}{\beta \sin\left(\frac{\pi d}{\beta \lambda}\right)} K_{EY}(y) + c K_{BX}(y) \right) \quad (1)$$

Here  $\Delta y' \equiv \Delta p_y/p$  is the deflection angle produced by the resonator;  $\Delta U$ ,  $m$ ,  $\varphi$  are the particle energy gain, rest mass and rf phase;  $\lambda$  is the rf wavelength. While  $d$  is the gap-to-gap (center) distance,  $d_y$  is an effective gap-to-gap distance for the transverse electric field  $E_y$ .

We can observe that at  $\varphi = 0$  there is maximum acceleration and no steering, while at  $\varphi = -90^\circ$  (bunching) there is no acceleration and maximum steering (and finite:  $\Delta U \propto \cos \varphi$ ). The electric deflection, proportional to  $1/\beta^2$ , decays faster than the magnetic one when the beam velocity increases.

The deflection angle as a function of the particle velocity for the “cylindrical” cavity, calculated with eq. 1, is shown in Fig. 3; analytical results are compared with numerical calculations performed using ANSOFT HFSS electromagnetic field simulation data. The particle tracking was done in the approximation of constant beam velocity along the cavity. Analytical and numerical results show a good agreement.

It can be noted that, in the range of velocity acceptance of this cavity, the magnetic deflection is always dominant over the electric one.

Since the steering angle is proportional to the charge to mass ratio  $q/A$ , the effect can be particularly severe for high  $q/A$  particles; in the case of protons, the transverse kick of such a cavity working at 6 MV/m would be above 1 mrad. Due to the rather linear variation of the field that can be observed in Fig. 2, however, it is easy to see from Eq. 1 that the anomalies in the field can be nearly cancelled if QWRs are mounted on the beam line in couples of units with opposite orientation. This solution

seems to be particularly easy to apply, especially in high frequency cavities due to their compactness, and seems to eliminate most of the QWR beam dynamics drawbacks.

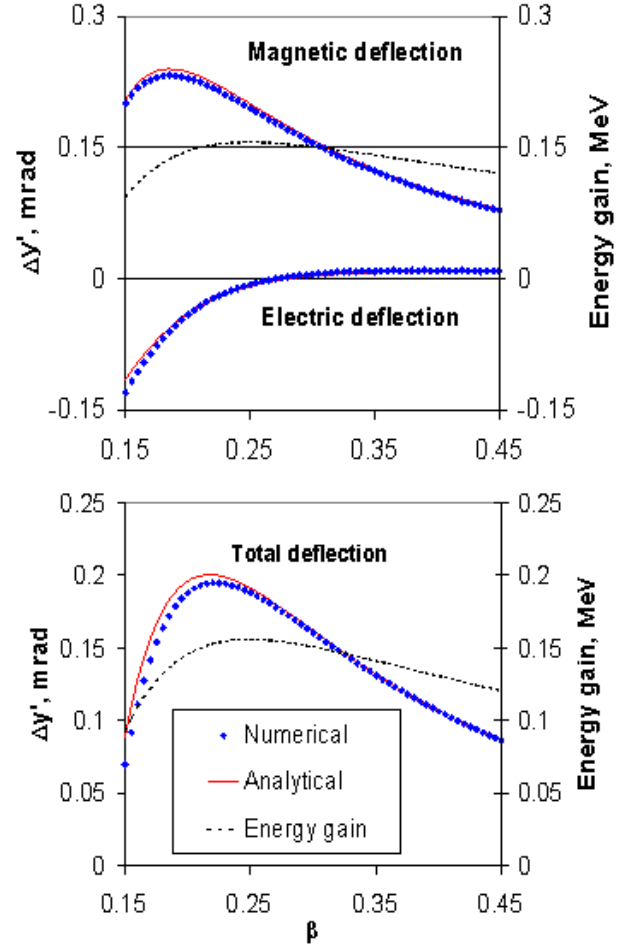


Figure 3: Numerical and analytical calculation of proton beam deflection in a “cylindrical” QWR (see text) at 1 MV/m and  $\varphi = -30^\circ$ . Energy gain is also displayed. Top: Electric and magnetic deflection. Bottom: total deflection.

### 3 COMPARISON OF DIFFERENT GEOMETRIES

We have studied three shapes of 352 MHz quarter wave resonators (see Fig. 4). For all of them the effective acceleration length is 180 mm, like in the Legnaro ALPI-PIAVE cavities, while the bore radius was changed from 10 to 15 mm to increase the transverse acceptance for high intensity beams. The basic shape has a cylindrical inner conductor. In the second cavity (“squeezed”) the inner conductor has been flattened to reduce the optimum  $\beta$ . The third shape was studied to improve the field axial symmetry and has a donut-shaped drift tube, a shorter gap and a conical inner conductor to reduce the maximum surface magnetic field which is nowadays the main limiting factor in the maximum achievable gradient.

All 3D field simulations have been performed with ANSOFT HFSS 7 electromagnetic 3D code. Beam deflection is present in all three shapes (see Fig. 5). Little

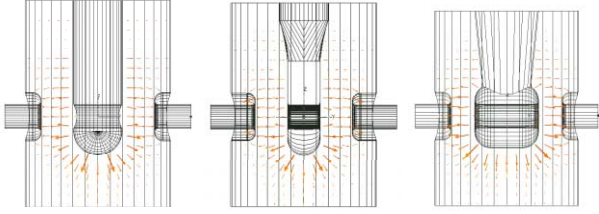


Figure 4: Quarter wave resonators for 352MHz with “cylindrical”, “squeezed” and “donut” type inner conductors.

difference is found between the “cylindrical” and the “squeezed” resonators, while the “donut” type one shows a moderately lower (by about 20%) deflection, at the cost of a higher complexity and a slightly higher ( $\approx 10\%$ ) peak magnetic field ratio  $H_p/E_a$ . In this set of cavities, the “squeezed” one shows the worst characteristics, as a consequence of the compromise between low optimum velocity and high frequency.

Table 1. Calculated parameters of 352 MHz quarter wave resonators.

Inner conductor option	Cylinder	Squeezed	Donut
Optimum $\beta_0$	0.25	0.22	0.27
Height, mm	256	254	245
Gap length, mm	40	40	30
Gap to gap, mm	100	80	110
$E_p/E_a$	5.6	6.2	5.4
$H_p/E_a$ , G/(MV/m)	104	119	115
$U/E_a^2$ , J/(MV/m) <sup>2</sup>	0.039	0.042	0.042
$\Gamma$ , $\Omega$	53.8	52.1	51.1
$E_y/E_z$	0.03	0.030	0.011
$\partial(E_y/E_z)/\partial y$ , m <sup>-1</sup>	$7.7 \times 10^{-4}$	$9.8 \times 10^{-4}$	$4.4 \times 10^{-4}$
$B_x/E_z$ , T m/V	$1.8 \times 10^{-9}$	$1.6 \times 10^{-9}$	$1.5 \times 10^{-9}$
$\partial(B_x/E_z)/\partial y$ , T/V	$2.3 \times 10^{-11}$	$2.2 \times 10^{-11}$	$2.5 \times 10^{-11}$

## 4 CONCLUSIONS

We have studied three different geometries of quarter wave resonators with  $\beta_0=0.22, 0.25$  and  $0.27$  respectively, for possible applications in proton accelerators, with particular attention to beam steering. This can be described with good approximation by means of a simple equation. The steering, in this range of velocity, is mainly caused by magnetic field, and changes almost linearly with the particle position along the resonator axis; this suggests that alternative orientation by  $180^\circ$  of subsequent cavities should allow steering cancellation.

Superconducting quarter wave resonators could be efficiently used in intermediate velocity linacs; simplicity,

accessibility and low cost make them a valuable alternative to other cavity geometries.

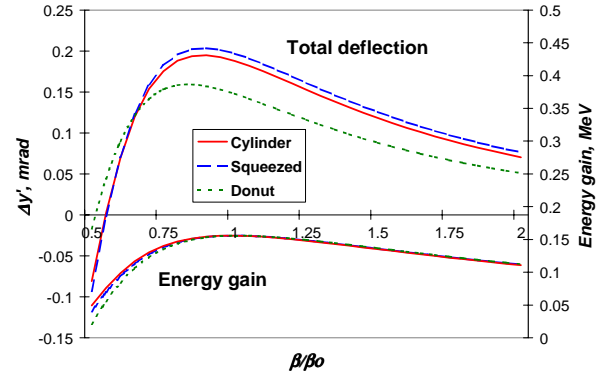


Figure 5: Energy gain and deflection angle  $\beta/\beta_0$  for the three resonators, at  $E_a=1$  MV/m and synchronous phase  $\varphi = -30^\circ$ .

## 5 ACKNOWLEDGMENTS

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## 6 REFERENCES

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